The friction channel test case for LES codes using wall laws

Code comparison based on velocity profiles for a channel flow test case

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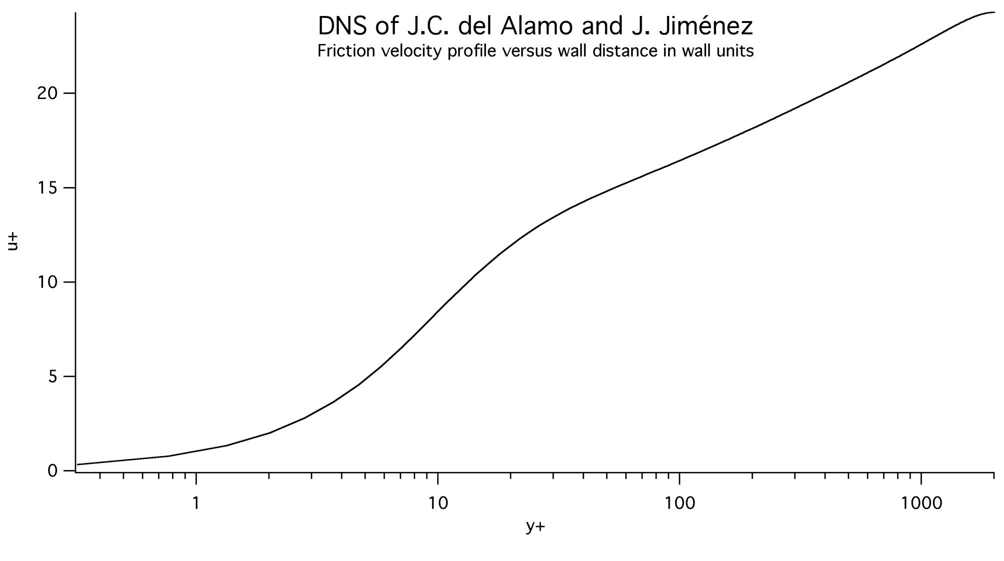

Contents

1 Introduction

The aim of this test case is to compare the performances of numerical simulations performed with various LES codes. The configuration is a bi-periodic turbulent channel with isothermal walls. Wall treatment is done using wall laws. Two meshes are suggested for the study, aiming for two values of y^+ which depend on the numerical methods (cell-centered, node-centered...) but in the order of 50 to 200 in order for the wall-law to apply. The friction Reynolds number is set to 2003, and the expected velocity profiles are those of the DNS by Alamo & Jiménez [1]. This data is available freely at : `http://torroja.dmt.upm.es/channels/data/statistics/Re2000/profiles/`. Velocity profile comparisons including rms values should give an excellent view of the quality of the numerical methods in reproducing the correct turbulent parameters under friction.

2 Theoretical background

2.1 Friction quantities

The following is freely inspired from a reference textbook in turbulence by Pope [3]. Let us consider a rectangular duct, and h be the half height of the duct. The channel is open and periodic in width and length, and it is bounded by walls on top and bottom ($y = 0$ and $y = y_{max}$). Bulk quantities are defined for this flow along the y axis, since the flow is statistically symmetric along the x and z directions. They are described by the subscript b, as opposed to wall values which are described by subscript w, and average values which have no subscript. Hence, bulk quantities can be defined for the flow and a variable Φ, as :

$$\Phi_b = \frac{1}{h} \int_0^h \langle \Phi \rangle \, dy \tag{1}$$

thus enabling the definition of the bulk velocity (u_b), bulk temperature (T_b), etc... yielding the definition of the bulk Reynolds number :

$$Re = \frac{u_b h}{\nu_b} \tag{2}$$

It can be deduced from the lateral mean-momentum equation that the mean axial pressure gradient is uniform across the flow. This leads to rewriting the axial mean-momentum equation as :

$$\frac{d\tau}{dy} = \frac{dp_w}{dx} \tag{3}$$

with :

$$\tau = \rho\nu\frac{d\langle U\rangle}{dy} - \rho\langle uv\rangle \tag{4}$$

The resolution of Eqn. 3, considering that τ is a function only of y and p_w only of x, as well as the fact that the *wall shear stress* τ_w is antisymmetric about the mid-plane, yields :

$$-\frac{dp_w}{dx} = \frac{\tau_w}{h} \tag{5}$$

Accordingly, close to the wall, the importance of viscosity and friction suggests the definition of the *friction velocity* :

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \tag{6}$$

as well as a *friction Reynolds number* based on this quantity :

$$Re_\tau = \frac{u_\tau h}{\nu} \tag{7}$$

and a famous quantity, the distance from the wall measured in viscous lengths, also called *wall units* :

$$y^+ = \frac{u_\tau y}{\nu} \tag{8}$$

Accordingly, a velocity can be defined in wall units as :

$$u^+ = \frac{\langle U\rangle}{u_\tau} \tag{9}$$

2.2 Source terms

Momentum and energy equations can be written as :

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + S_{qdm,i} \tag{10}$$

$$\rho\frac{DE}{Dt} = -\frac{\partial q_i}{\partial x_i} + \frac{\partial}{\partial x_j}(\tau_{ij}u_i) - \frac{\partial}{\partial x_i}(pu_i) + \dot{\omega}_T + S_e + u_i S_{qdm,i} \tag{11}$$

where $S_{qdm,i}$ and S_e are the momentum and energy source terms, respectively.

2.3 Testing strategy

The configuration is chosen to match the DNS by Alamo & Jiménez [1]. This configuration is characterized by the parameters :

Re_b	Re_τ
43530	2003

In order to compare a code to this DNS, one set of parameters must be fixed, and the other compared. A periodic channel requires a pressure gradient to be imposed numerically in order for the flow to be established and the friction losses compensated. It is not possible to impose directly Re_b, as the velocity is only a result characterizing the established flow and depending on the link between the imposed pressure gradient and the corresponding velocity profile. For these reasons, the friction Reynolds number is set to 2003 in order to match exactly the DNS. Comparisons will be made with the velocity profiles of the DNS. The corresponding momentum source term is computed using eqs. 5, 6 and 7 :

$$\begin{aligned} S_{qdm,x} &= -\frac{dp_w}{dx} \\ &= \frac{Re_\tau^2\ \nu_w^2 \rho}{h^3} \end{aligned} \tag{12}$$

The wall temperature is set to $T_b/1.1$, in order to establish a small heat flux. This should have very little influence on Re_b since according to Kays *et al.*[2] the temperature correction on the C_f given by the Karman-Nikuradse correlation can be written as :

$$\begin{aligned} C_{f\ corrected} &= C_f * \left(\frac{T_b}{T_w}\right)^{0.1} \\ &= 1.1^{0.1}\ C_f \\ &\sim 1.01\ C_f \end{aligned} \tag{13}$$

Hence, the energy source term S_e reasonably be set to 0. The heat produced by the viscous dissipation will be eliminated through the walls, and the previous argument shows that the bulk temperature should settle very close to the target. Any difference with the DNS in bulk velocity over 1% will be attributed to errors produced by the code.

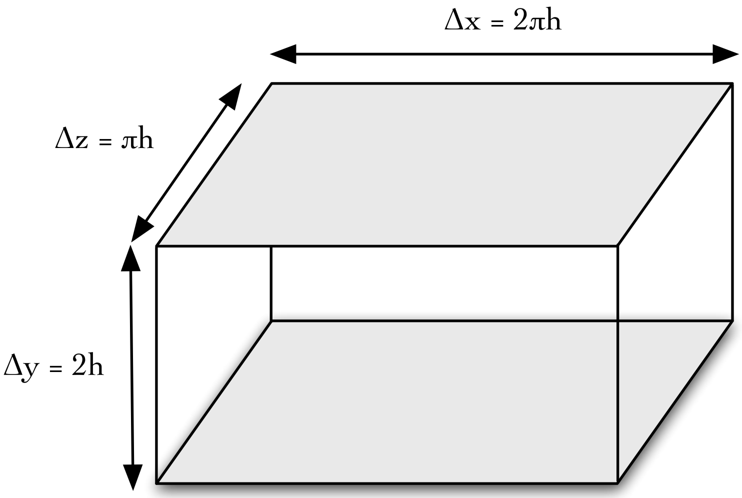

Figure 1: View of the computational domain

3 Test cases

3.1 Common parameters

The computational domain is chosen to be a rectangular duct with Δx the length of the duct in the direction of the flow, Δy the height of the duct (smallest of the 3 space dimensions) and Δz it's width. The dimensions above are related by :

$$\Delta x = 2\pi h \tag{14}$$

$$\Delta y = 2h \tag{15}$$

$$\Delta z = \pi h \tag{16}$$

A representation of the domain is given in figure 1.

The only parameter for which two values are imposed is the mesh. Two meshes with different grid refinement are determined and used for all solvers. They are fully hexahedral, and completely homogenous in size along any given direction. They aim towards an expected value of y^+, according to the velocity profile of the DNS. Descriptions of these meshes are available in table 1.

Moreover, the Mach number is set to 0.2, thus minimizing compressibility effects.

Dynamic viscosities are inferred from Sutherland's law :

$$\mu_{b,w} = \mu_{ref} \frac{T_{ref} + S}{T_{b,w} + S} \left(\frac{T_{b,w}}{T_{ref}} \right)^{\frac{3}{2}} \tag{17}$$

Table 1: Summary of the mesh parameters

Mesh size (stream wise, wall normal, spanwise)	41 x 49 x 41	21 x 25 x 21
Expected y^+ for node-centered codes	100	200
Expected y^+ for cell-centered codes	50	100

and supposing a constant pressure :

$$\rho_{b,w} = \frac{P}{r\ T_{b,w}} \qquad \nu_{b,w} = \frac{\mu_{b,w}}{\rho_{b,w}} \tag{18}$$

A summary of all common parameters for initialization is given in table 2.

Table 2: Common initial parameters

Re_b	T_b (K)	M_b	ρ_b	γ	r (J/kg.K)	μ_{ref} (Pa.s at 273K)	S (K)
43530	293	0.2	1.165	1.4	296.79	1.716×10^{-5}	110.6

Run time The run time can be determined by defining a dimensionless number t^* which characterizes the total run time Δt by comparing the friction velocity and the channel half-height :

$$\Delta t = \frac{ht^*}{u_\tau} \tag{19}$$

therefore representing the number of times a particle traveling at the friction velocity would go from the wall to the centerline of the flow. DNS suggests a reasonable statistical convergence after $t^* = 10$. We choose to run for $t^* = 33$, in order to average approximately over the last $t^* = 20$.

3.2 Evaluation of the codes

Only 2 runs, that is for both meshes, are needed of each code to compare results. However, each code will be free to exhibit several numerical methods. Comparisons will be made against the DNS values of Hoyas & Jiménez, available freely on the internet. The comparisons will include :

- u^+ vs y^+ profiles;
- u^+_{rms}, v^+_{rms} and w^+_{rms} vs y/h profiles

where for example :

$$u^+_{rms} = \frac{\sqrt{\langle u'^2 \rangle}}{u_\tau}$$

References

[1] J.C. del Alamo and J. Jiménez. Direct numerical simulation of the very large anisotropic scales in a turbulent channel. *Center for Turbulence Research Annual Research Briefs. Stanford University*, pages 329–341, 2001.

[2] WM Kays and EY Leung. Heat transfer in annular passages–hydrodynamically developed turbulent flow with arbitrarily prescribed heat flux. *International Journal of Heat and Mass Transfer*, 6(7):537–557, 1963.

[3] S. B. Pope. *Turbulent flows.* Cambridge University Press, 2000.