PITFALLS OF SPECTRAL ANALYSIS

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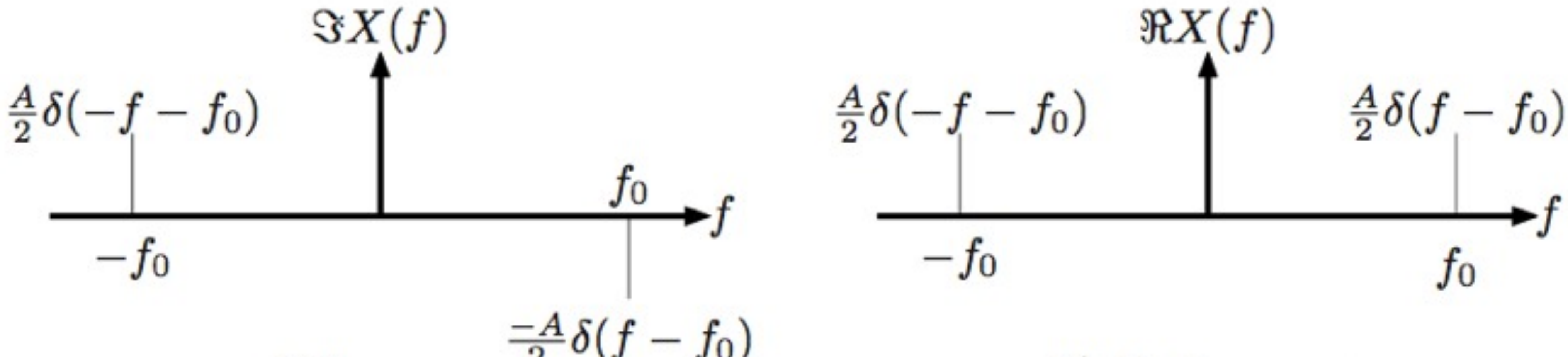
Five Exercises

- Manual evaluation of a FT (Excel)
- Statistics vs Parseval (Pen and Paper comp.)
- Filtering and Aliasing (A priori design.)
- Bias of short signals (Matlab tool)
- Spectral analysis lav (Web based tool)

Fourier transform -continuous-

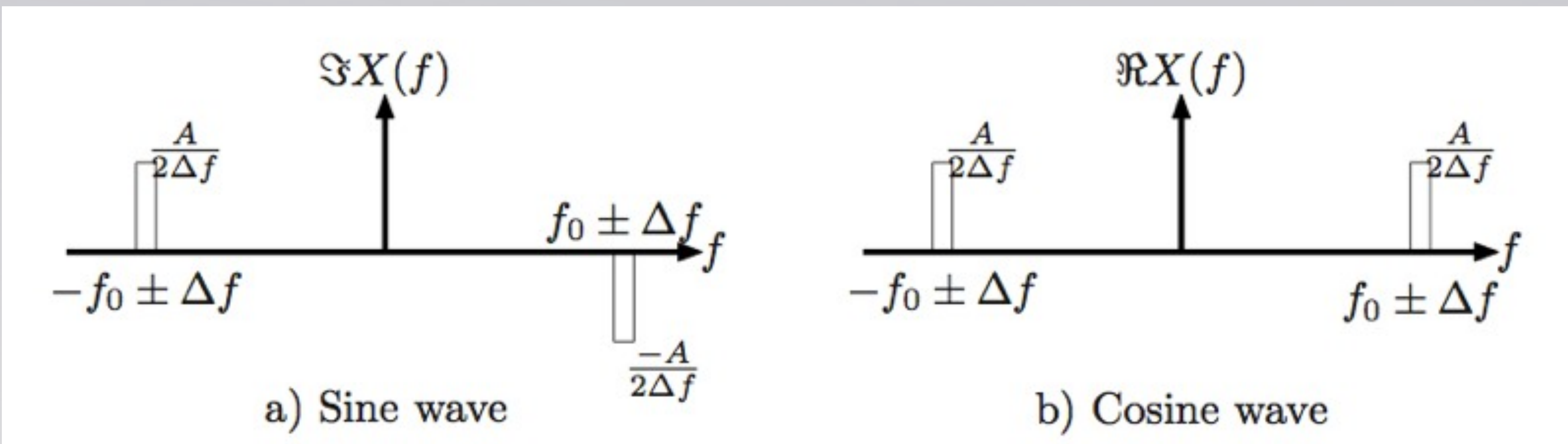
$$X(f) = \int_{-\infty}^{+\infty} x(t) \exp(-i2\pi f t) dt$$



a) Sine wave

b) Cosine wave

Fourier transform -discrete-



$$\sum_{n=-\infty}^{n=+\infty} \frac{A\delta_{n,n_0}}{\Delta f}\Delta f = A = \int_{f=-\infty}^{f=+\infty} A\delta(f_0)df$$

Amplitude spectrum -discrete-

$$F_k = Magn\left[\frac{1}{N}\sum_{n=0}^{N-1}\left(x_n \exp\left(-i\frac{2\pi}{N}nk\right)\right)\right], k = 0$$

$$= 2Magn\left[\frac{1}{N}\sum_{n=0}^{N-1}\left(x_n \exp\left(-i\frac{2\pi}{N}nk\right)\right)\right], k = 1, ..., \frac{N}{2} - 1$$

Ex 1: Manual evaluation of a Fourier Transform

- In an Excel spreadsheet, calculate each term of the Fourier serie (real part) for a 8 sample/1sec signal.
- Change the signal on this DFT. What happen for components on the Nyquist frequency (4Hz)? Beyond (>4Hz)?
- How zeropadding affect this transform?

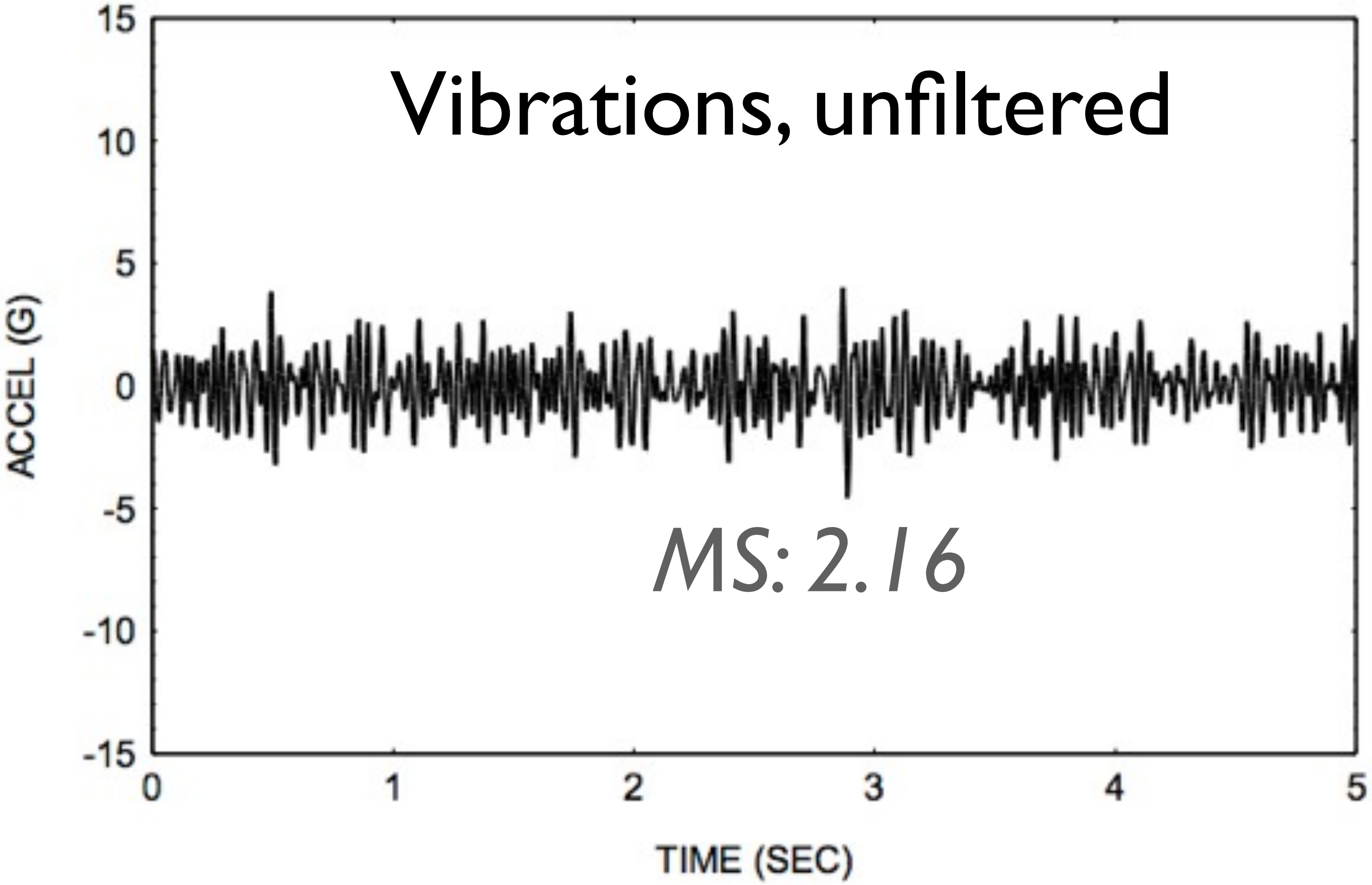
Power Spectral Density Spectrum

The Power Spectral Density is the Mean Square value of the signal on frequency bands

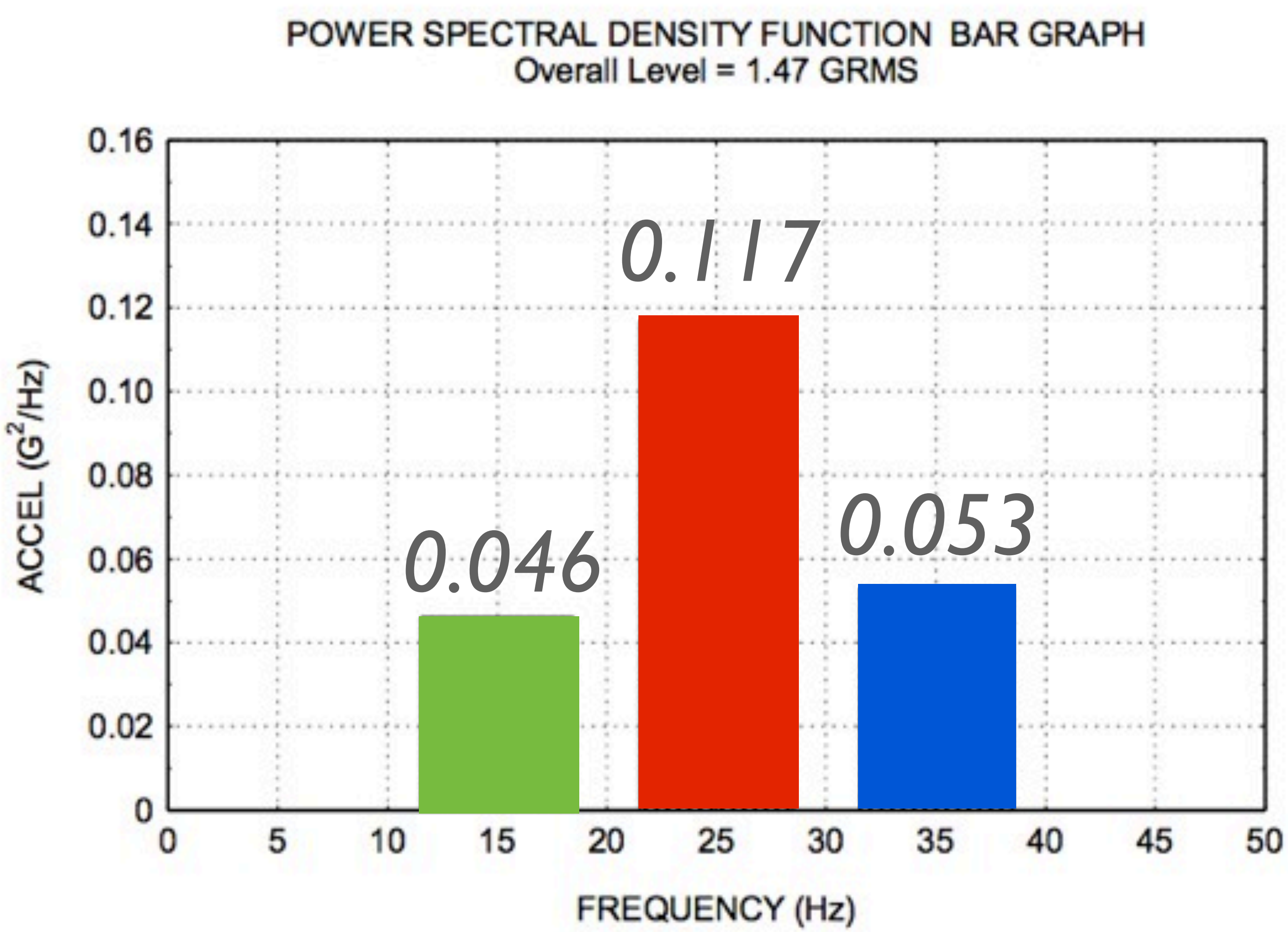
$$PSD_k = \left[\frac{F_k F_k^*}{\Delta f}\right], k = 0$$

$$= \frac{1}{2}\left[\frac{F_k F_k^*}{\Delta f}\right], k = 1, ..., \frac{N}{2} - 1$$

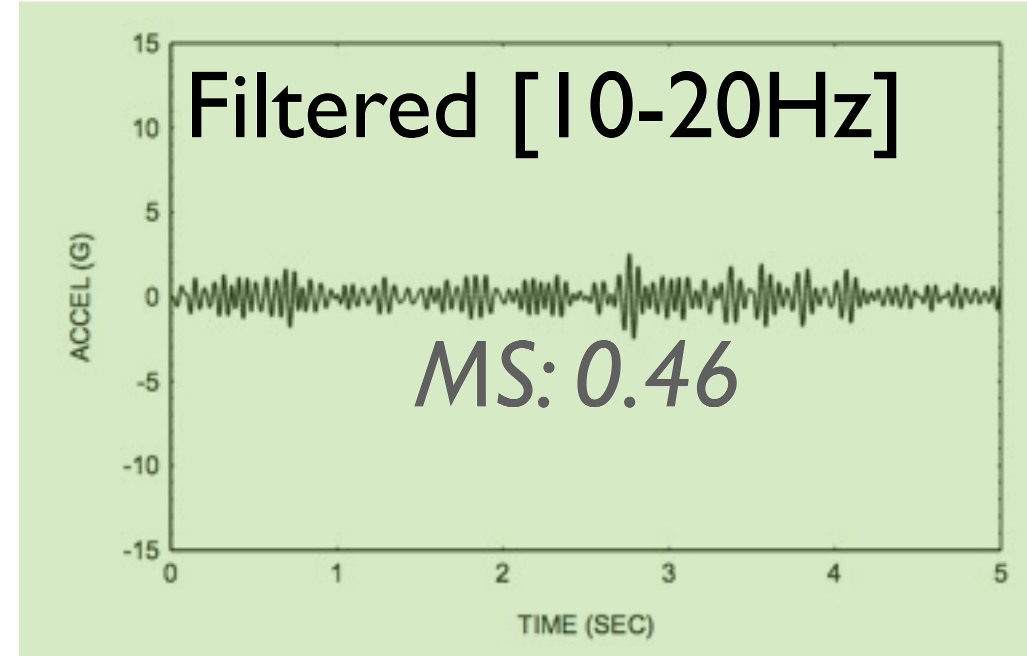
Name	Continuous form	Discrete form
Mean $\overline{X}$	$\lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)dt$	$\lim_{N\to\infty} \frac{1}{N}\sum_i^N x_i$
Mean Square $\overline{X^2}$	$\lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)^2dt$	$\lim_{N\to\infty} \frac{1}{N}\sum_i^N x_i^2$
Root Mean Square $\sqrt{\overline{X^2}}$	$\lim_{T\to\infty} \sqrt{\frac{1}{T}\int_0^T x(t)^2dt}$	$\lim_{N\to\infty} \sqrt{\frac{1}{N}\sum_i^N x_i^2}$
Variance σ^2	$\lim_{T\to\infty} \frac{1}{T}\int_0^T (x(t) - \overline{X})^2dt$	$\lim_{N\to\infty} \frac{1}{N}\sum_i^N (x_i - \overline{X})^2$



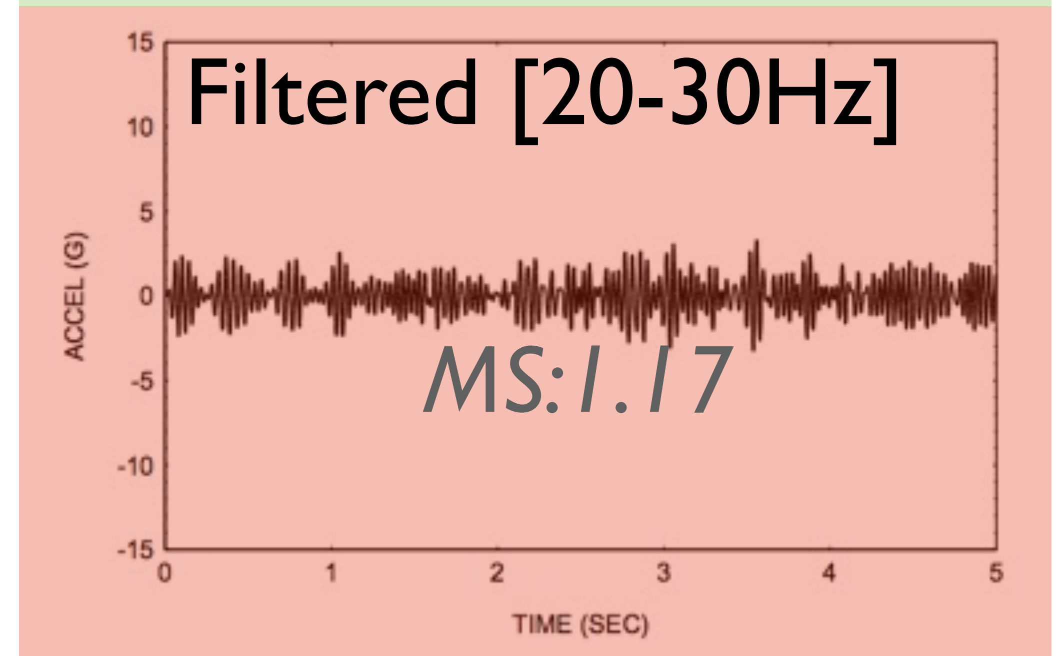
Vibrations, unfiltered
MS: 2.16
ACCEL (G)
15
10
5
0
-5
-10
-15
0
1
2
3
4
5
TIME (SEC)



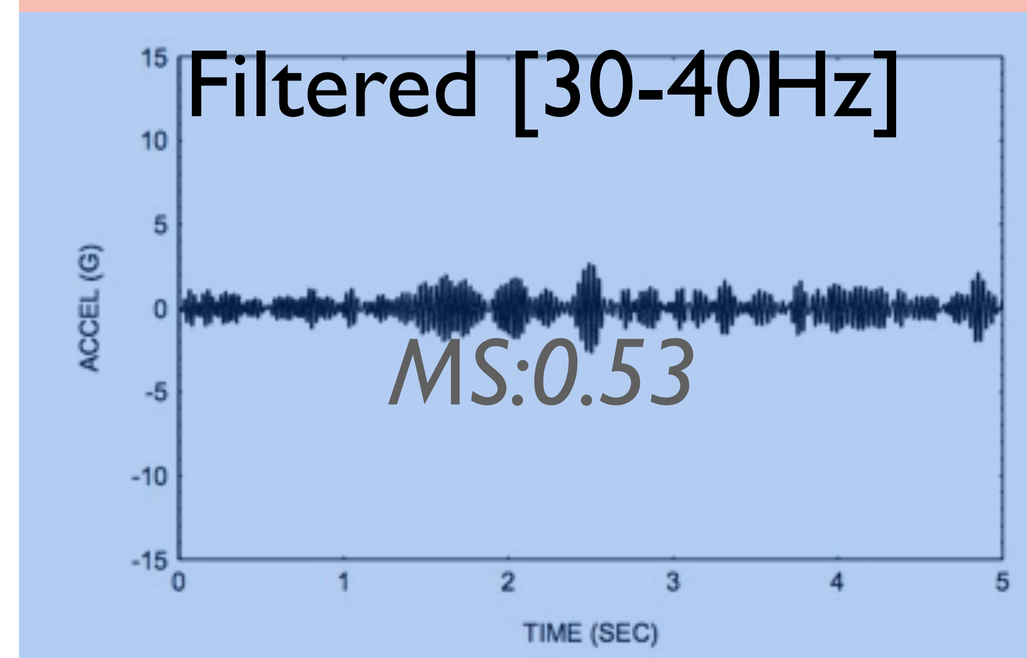
POWER SPECTRAL DENSITY FUNCTION BAR GRAPH
Overall Level = 1.47 GRMS
ACCEL (G^2/Hz)
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0
0.046
0.117
0.053
0
5
10
15
20
25
30
35
40
45
50
FREQUENCY (Hz)



Filtered [1 0-20Hz]
MS: 0.46
ACCEL (G)
15
10
5
0
-5
-10
-15
0
1
2
3
4
5
TIME (SEC)

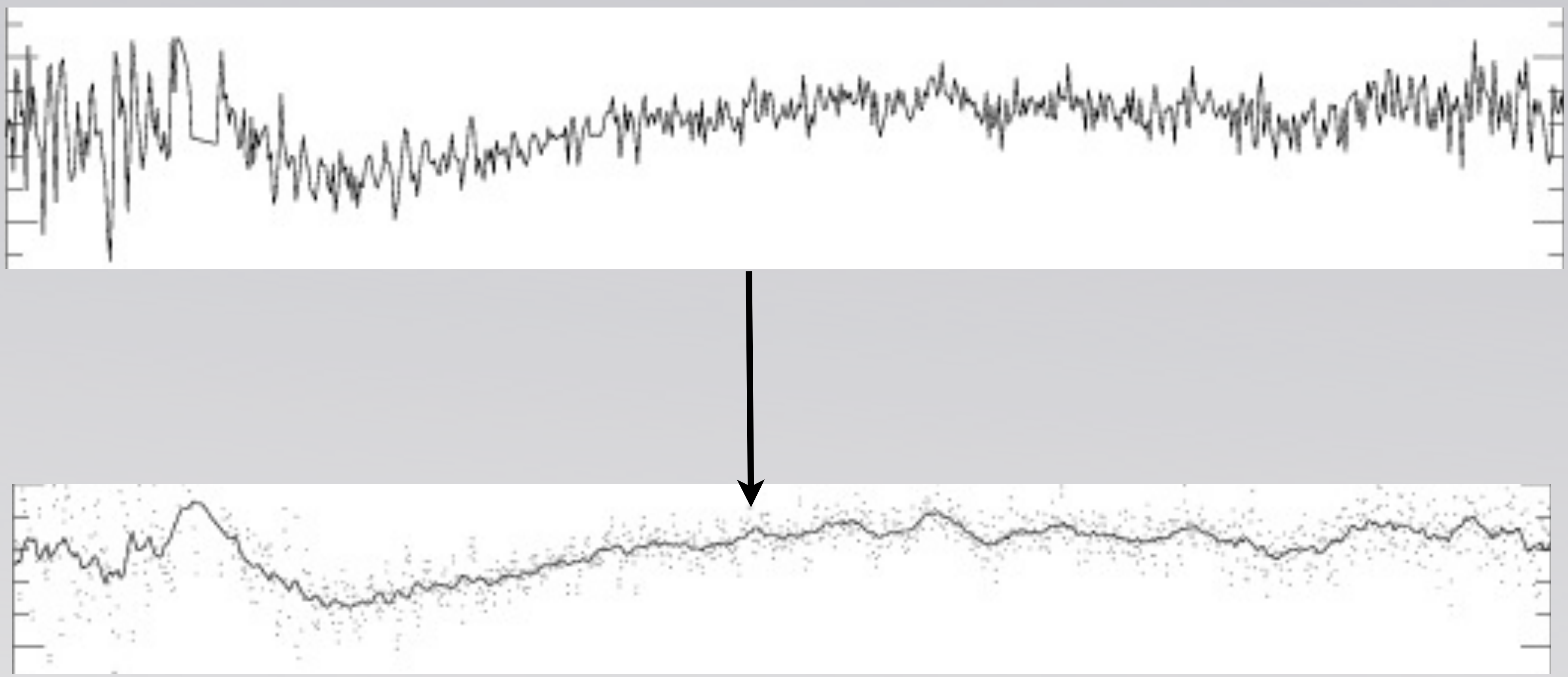


Filtered [20-30Hz]
MS:1.17
ACCEL (G)
15
10
5
0
-5
-10
-15
0
1
2
3
4
5
TIME (SEC)



Filtered [30-40Hz]
MS:0.53
ACCEL (G)
15
10
5
0
-5
-10
-15
0
1
2
3
4
5
TIME (SEC)

Why averaging?

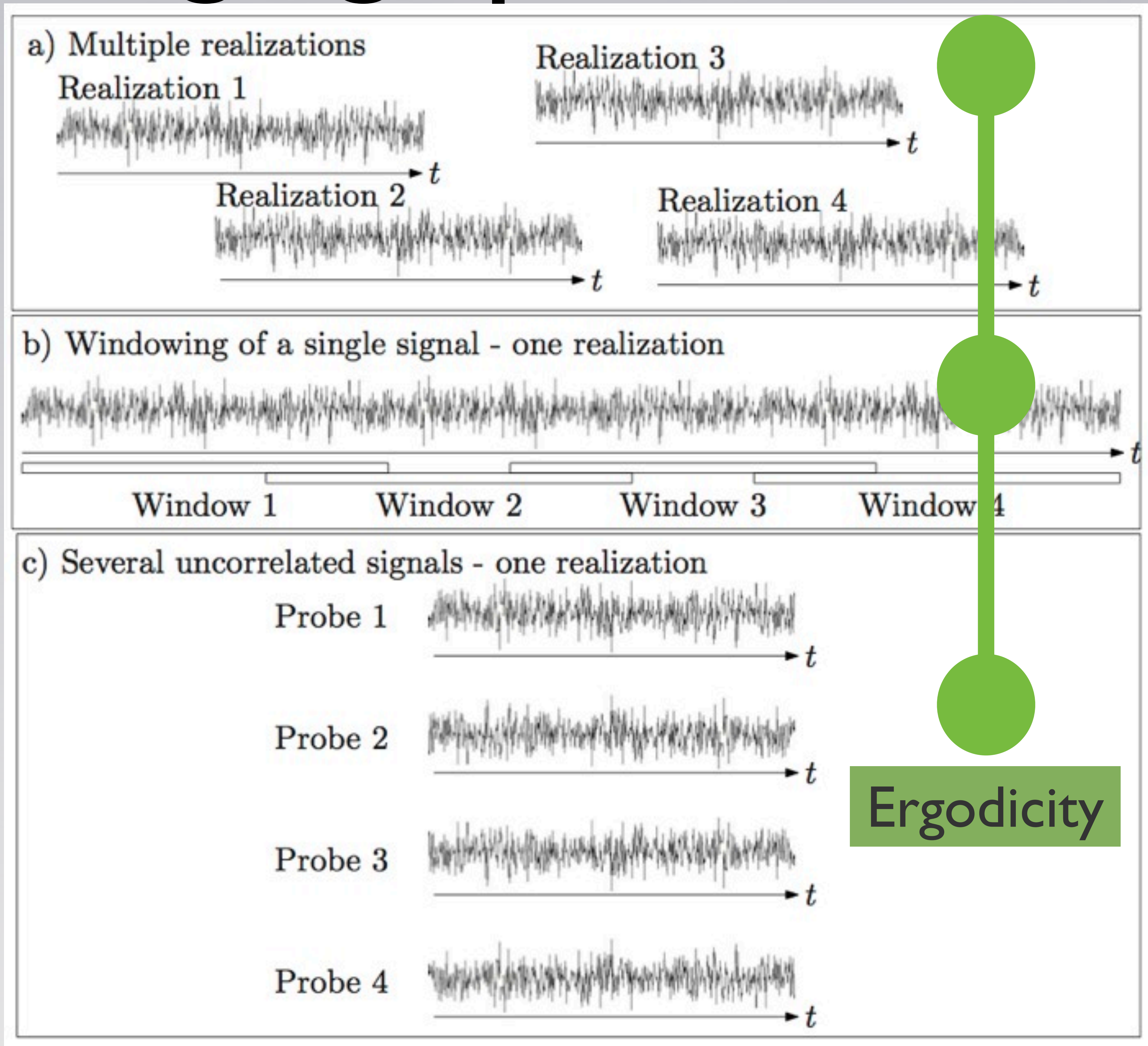


Averaging spectras

Direct averaging

Temporal averaging (Experiments)

Spatial averaging (CFD)



Ergodicity illustration: average weight of customers

Direct averaging, 1 day




Temporal averaging 4 days

Spatial averaging 1 day

Sound pressure level

$$SPL = 10\log\left(\frac{\sum_i \sigma_i^2}{p_{ref}^2}\right)$$

Name	Continuous form	Discrete form
Mean $\overline{X}$	$\lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)dt$	$\lim_{N\to\infty} \frac{1}{N}\sum_i^N x_i$
Mean Square $\overline{X^2}$	$\lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)^2dt$	$\lim_{N\to\infty} \frac{1}{N}\sum_i^N x_i^2$
Root Mean Square $\sqrt{\overline{X^2}}$	$\lim_{T\to\infty} \sqrt{\frac{1}{T}\int_0^T x(t)^2dt}$	$\lim_{N\to\infty} \sqrt{\frac{1}{N}\sum_i^N x_i^2}$
Variance σ^2	$\lim_{T\to\infty} \frac{1}{T}\int_0^T (x(t)-\overline{X})^2dt$	$\lim_{N\to\infty} \frac{1}{N}\sum_i^N (x_i-\overline{X})^2$

Ex 2: Statistics vs Parseval

- Find the relation between Mean Square and Variance.
- Draw the spectrums (Amplitude and DSP) of the Signal :

$$x(t) = A + Bcos(2\pi f_0) + \mathcal{N}(C)$$

- Discuss the cases of an increasing sampling rate; of the signal duration.

Variance and Mean Square

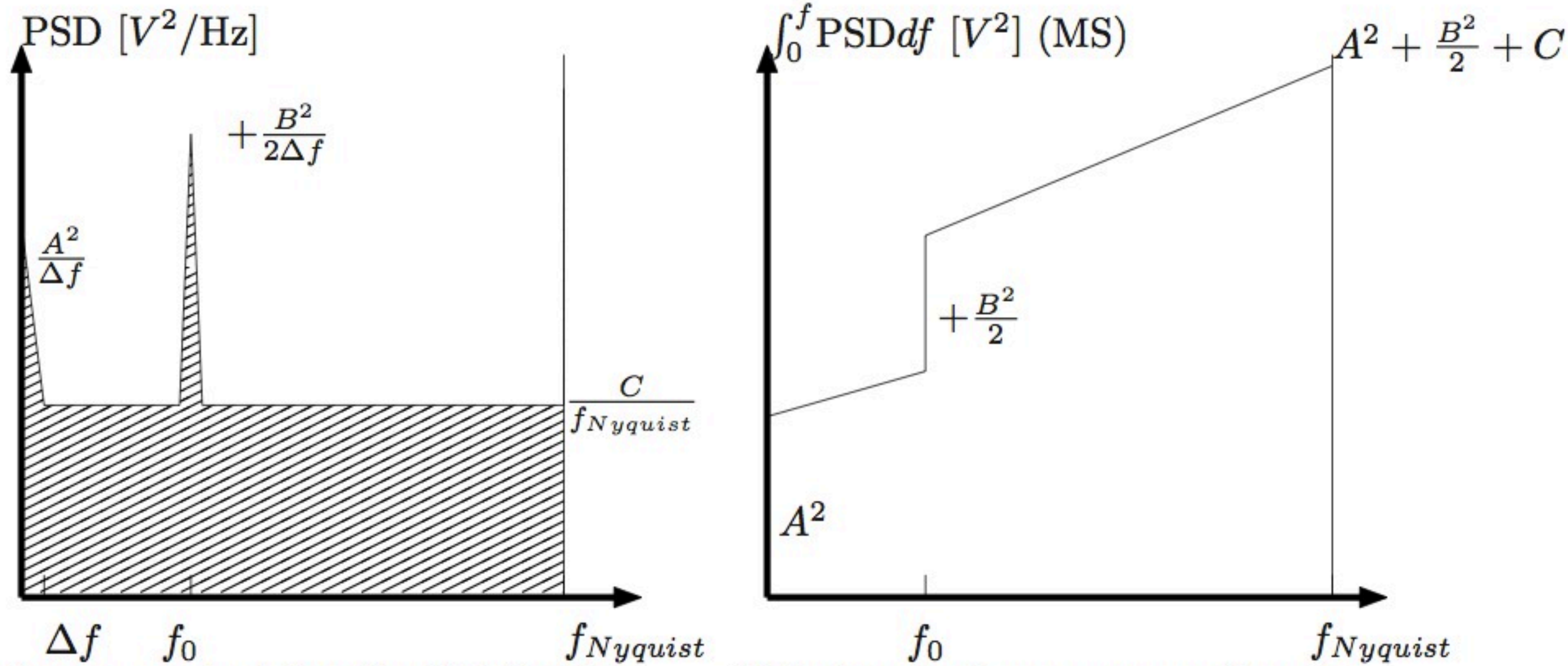
$$
\begin{aligned}
\sigma^2 &= \lim_{T\to\infty}\left(\frac{1}{T}\int_0^T (x(t)-\overline{X})^2\right) \\
&= \lim_{T\to\infty}\left(\frac{1}{T}\int_0^T x(t)^2 - 2x(t)\overline{X} + \overline{X}^2\right) \\
&= \lim_{T\to\infty}\left(\frac{1}{T}\int_0^T x(t)^2\right) - 2\overline{X}\lim_{T\to\infty}\left(\frac{1}{T}\int_0^T x(t)\right) + \overline{X}^2 \\
&= \overline{X^2} - 2\overline{X}^2 + \overline{X}^2
\end{aligned}
$$

PSD decomposition

$$x(t) = A + Bcos(2\pi f_0) + \mathcal{N}(C)$$

$$\begin{aligned} MS(x(t)) &= \sigma^2(A) + \overline{A}^2 \\ &+ \sigma^2(Bcos(2\pi f_0)) + \overline{Bcos(2\pi f_0)}^2 \\ &+ \sigma^2(\mathcal{N}(C)) + \overline{\mathcal{N}(C)} \end{aligned} \qquad \begin{aligned} &= 0 + A^2 \\ &+ \frac{B^2}{2} + 0 \\ &+ C + 0 \end{aligned}$$

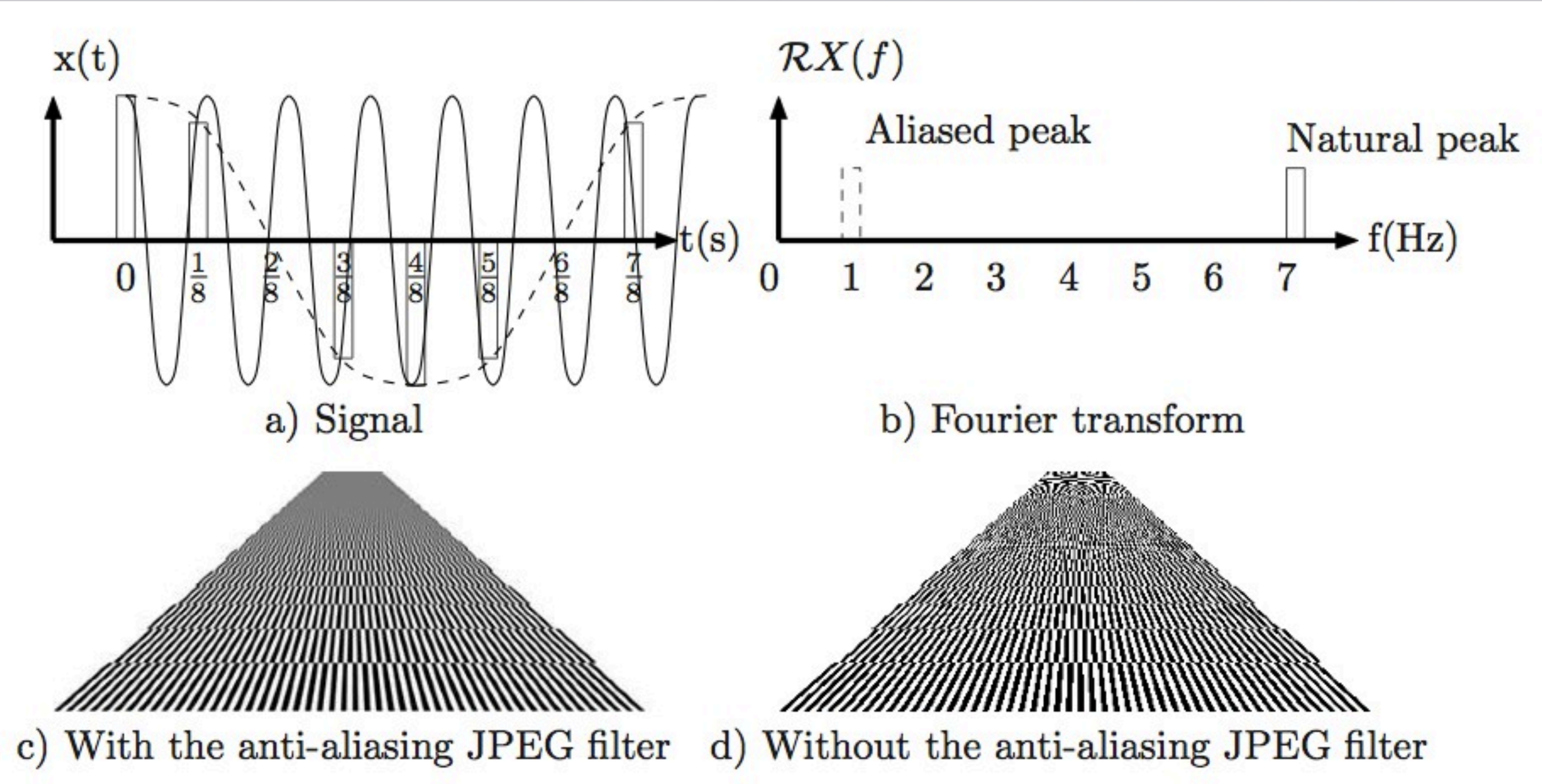
PSD decomposition



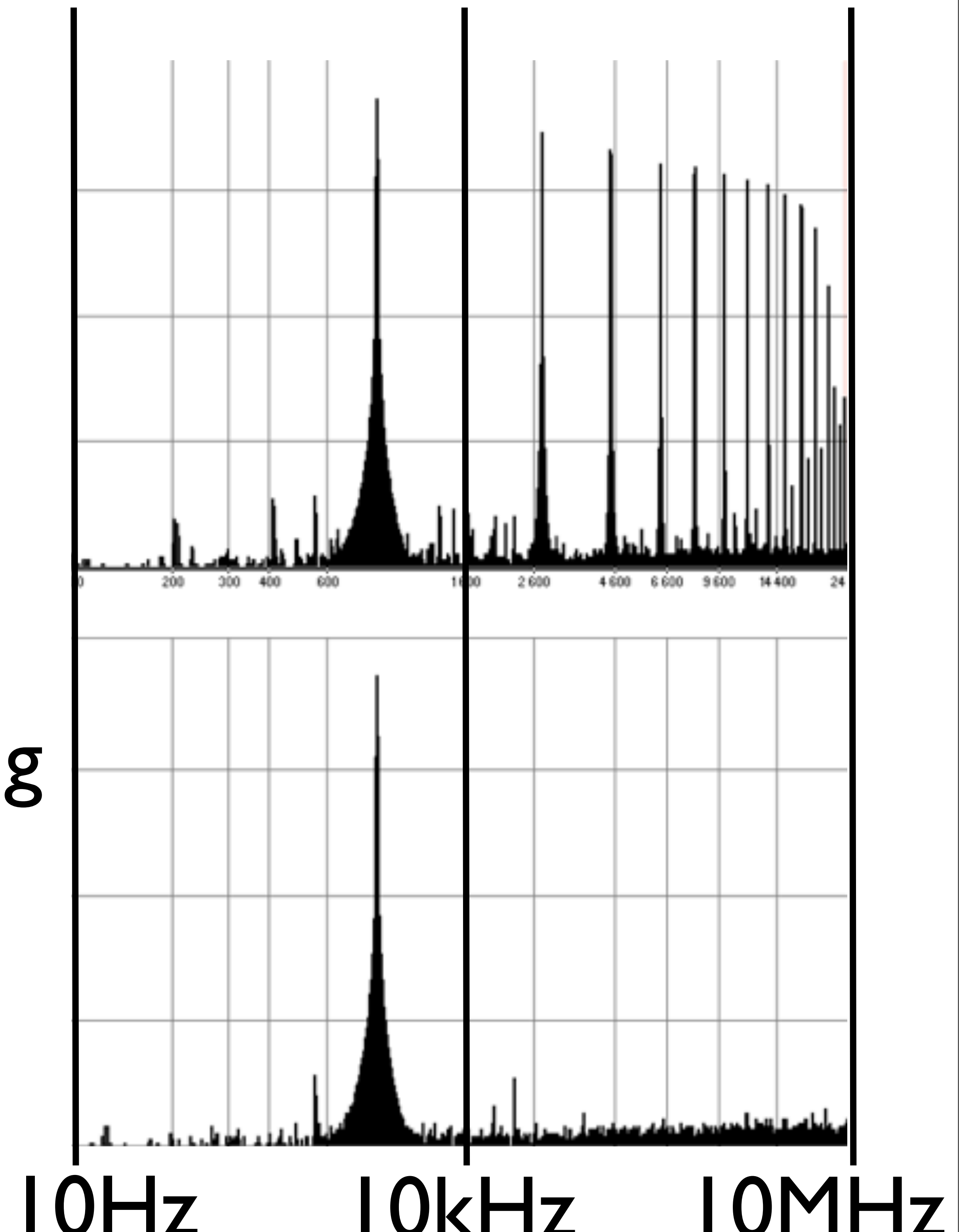
a) Power spectral density distribution

b) Integral of mean square function

Aliasing


x(t)
t(s)
0
1/8
2/8
3/8
4/8
5/8
6/8
7/8
a) Signal
RX(f)
Aliased peak
Natural peak
f(Hz)
0
1
2
3
4
5
6
7
b) Fourier transform
c) With the anti-aliasing JPEG filter
d) Without the anti-aliasing JPEG filter

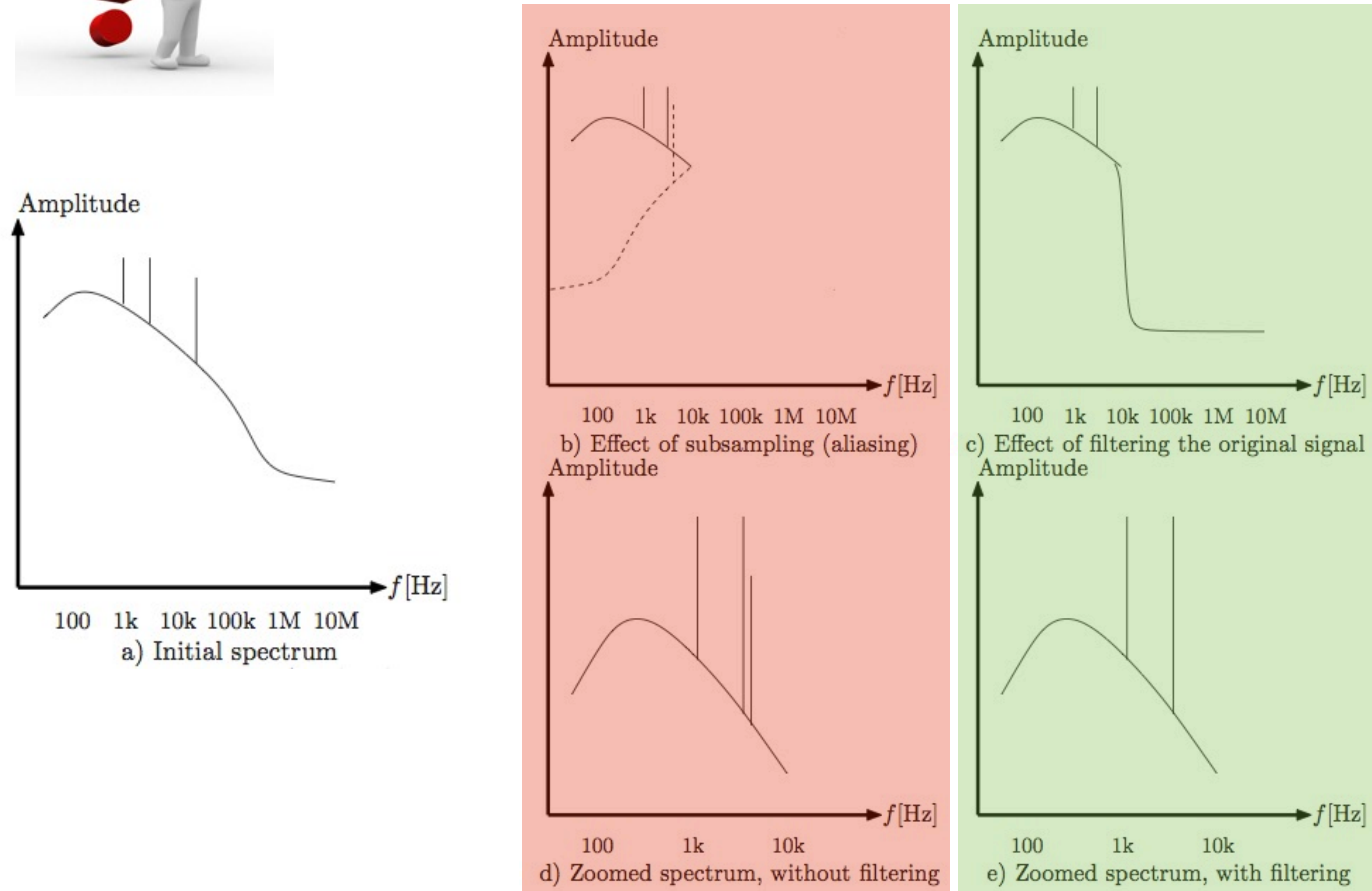
Ex 3: Filtering and Aliasing



- You have a CFD signal , duration 10ms, timestep 1e-7s, with a strong HF activity beyond 10kHz.

  Design a fast signal processing tool focusing on the low frequency range [0-10kHz]

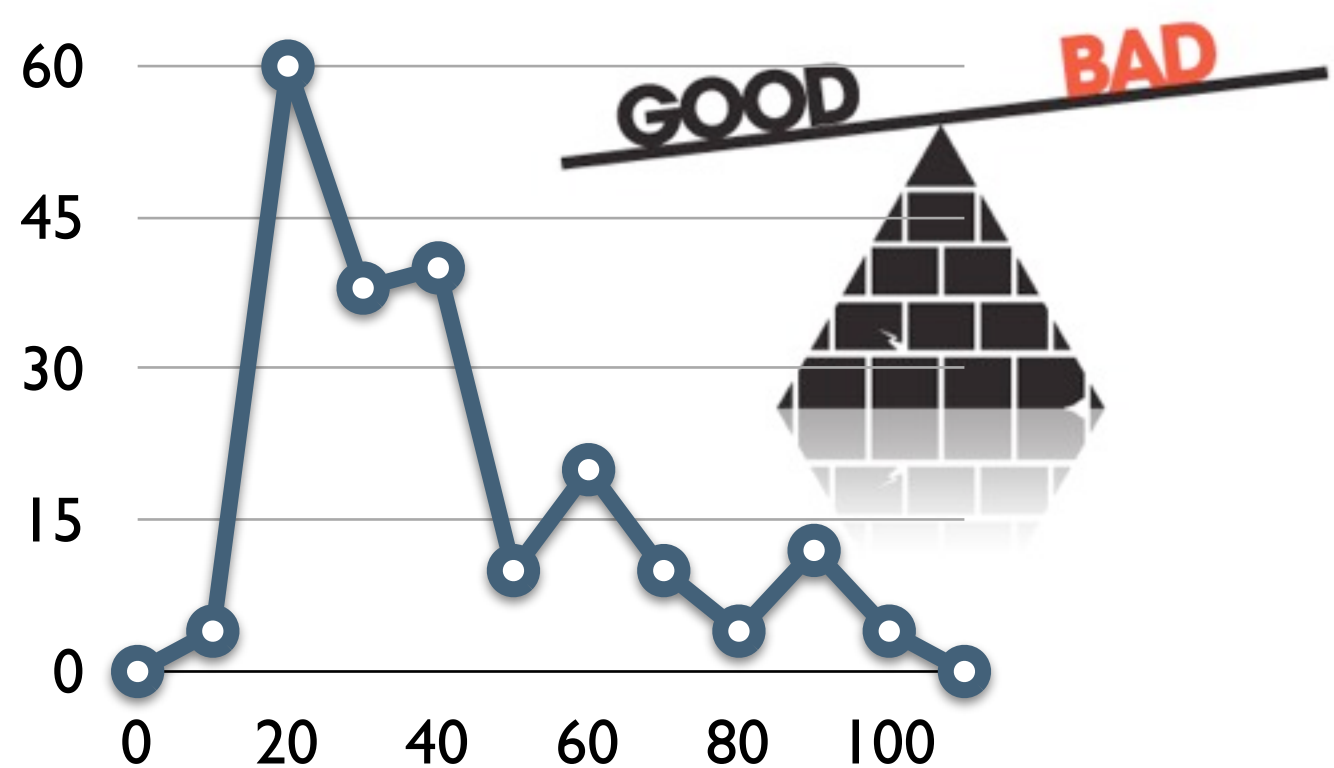
Antialiasing filter



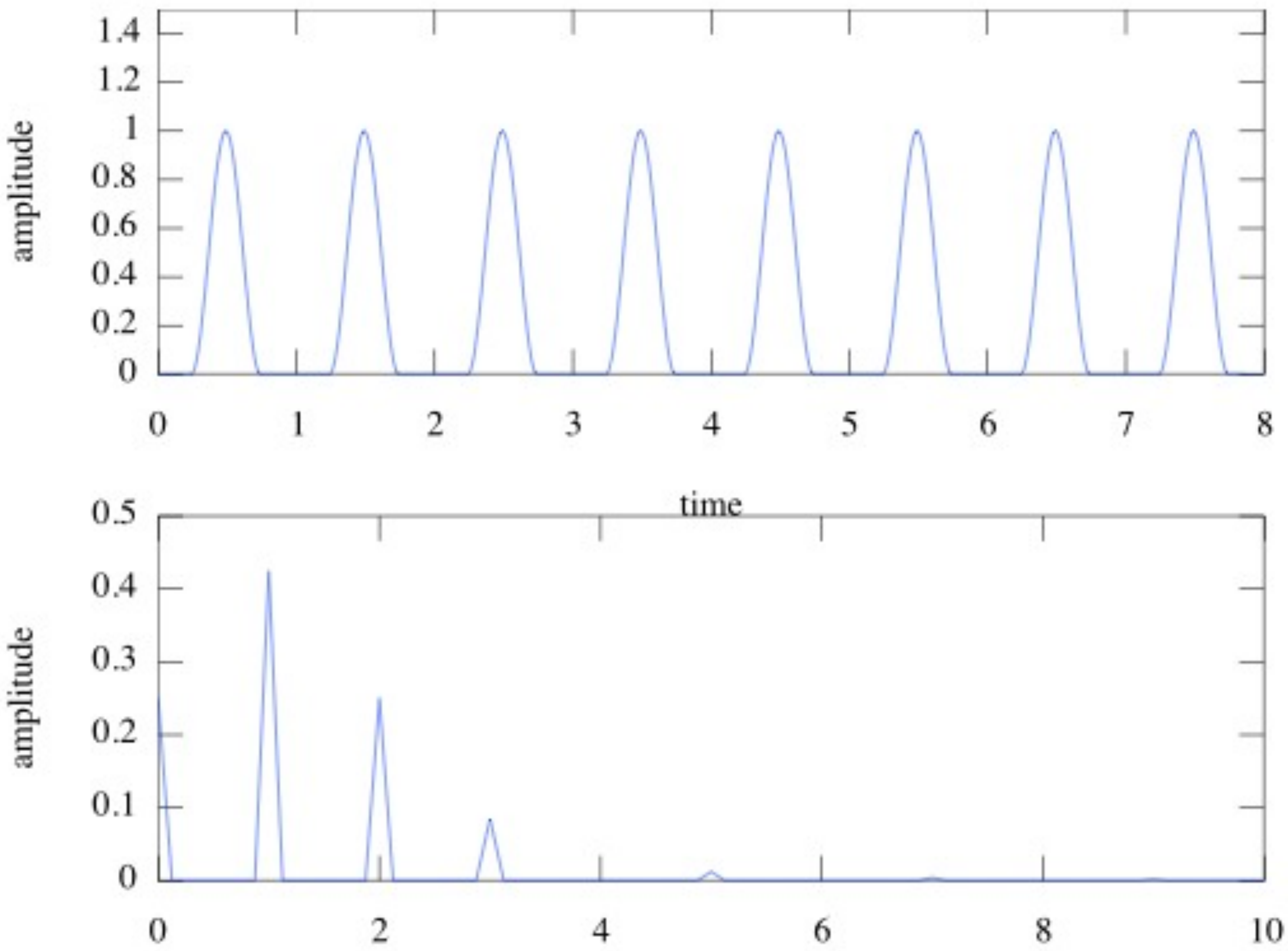
Ex 4: Bias of short Signal

- Try the Octave tool 'jammer(***N,n1,n2***)'.
  N *is the number of pulses [3-1000]*
  n1 *is a fluctuation on the amplitude [0-1]*
  n2 *is a fluctuation on the timelags [0-1]*

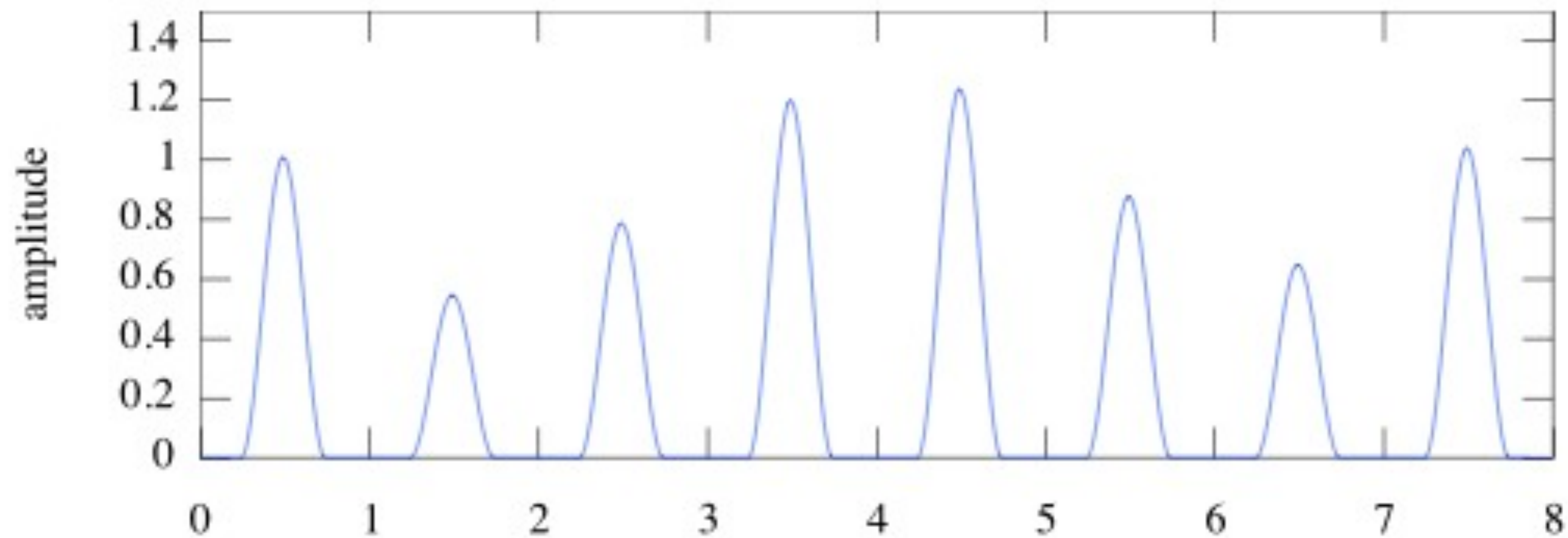
- Conclude on CFD signals. Good or bad spectra?

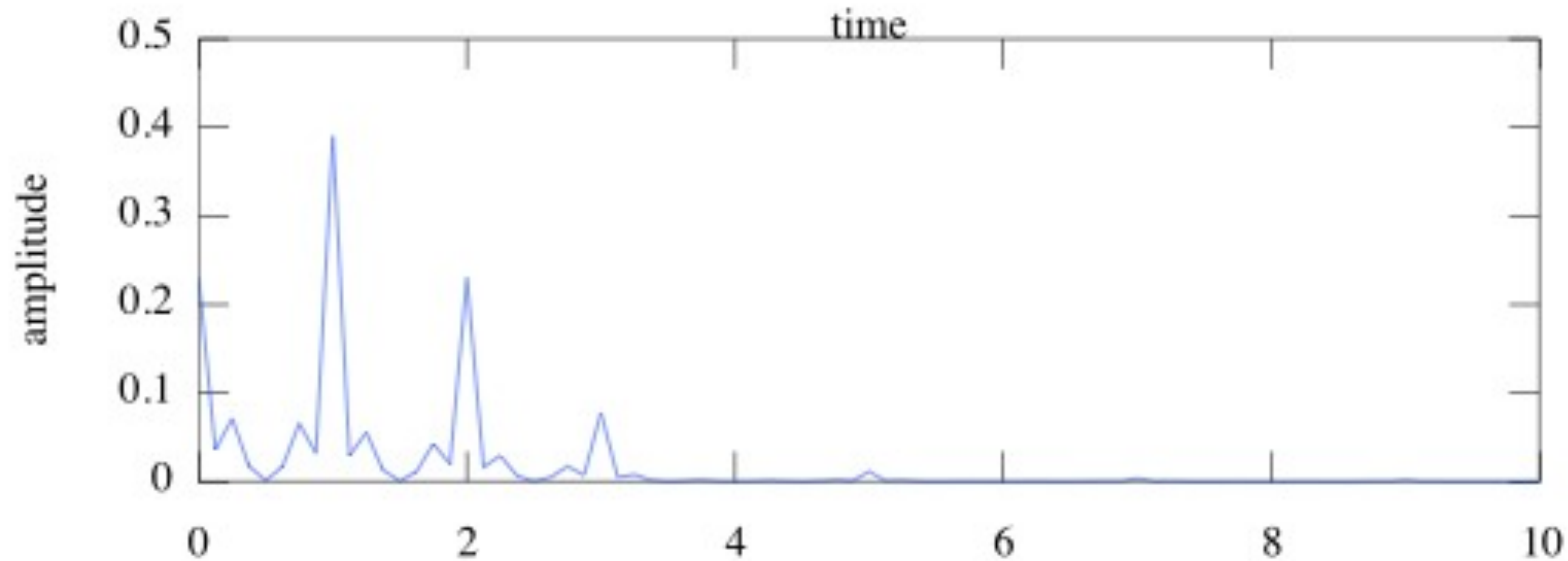


Reference signal

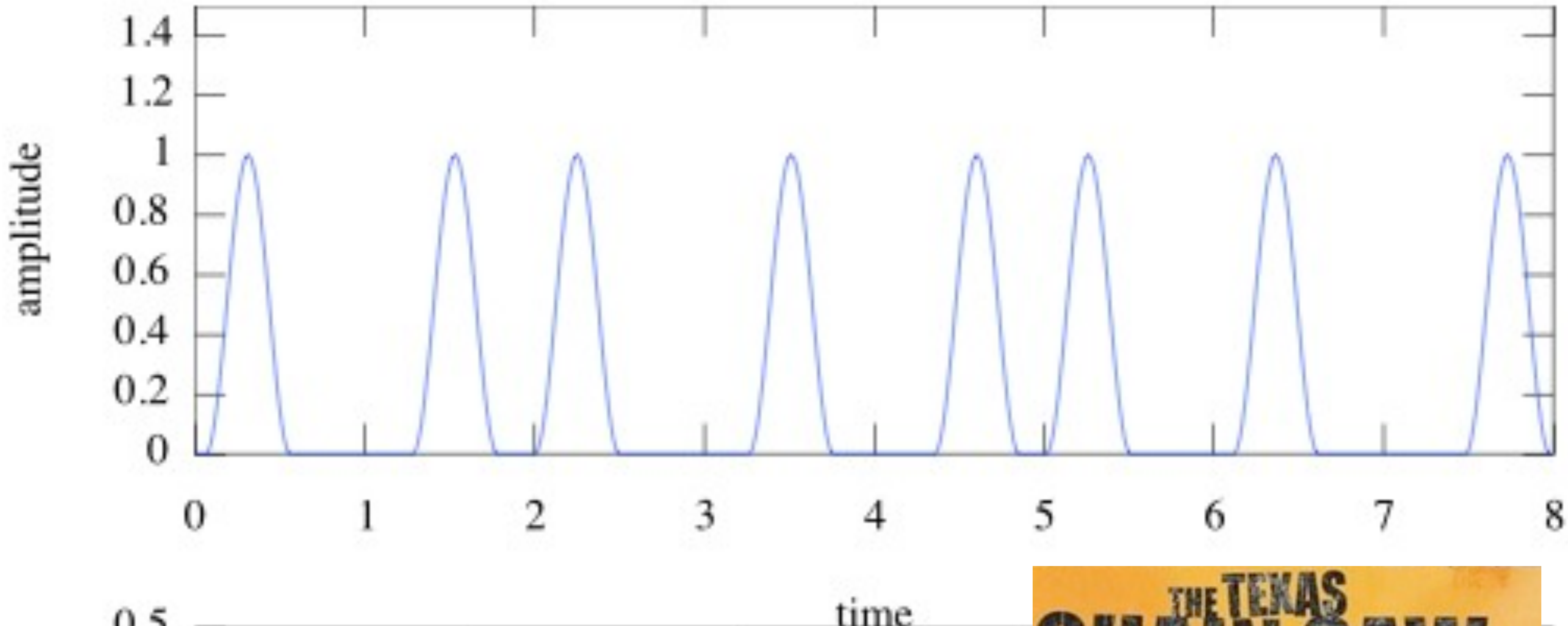


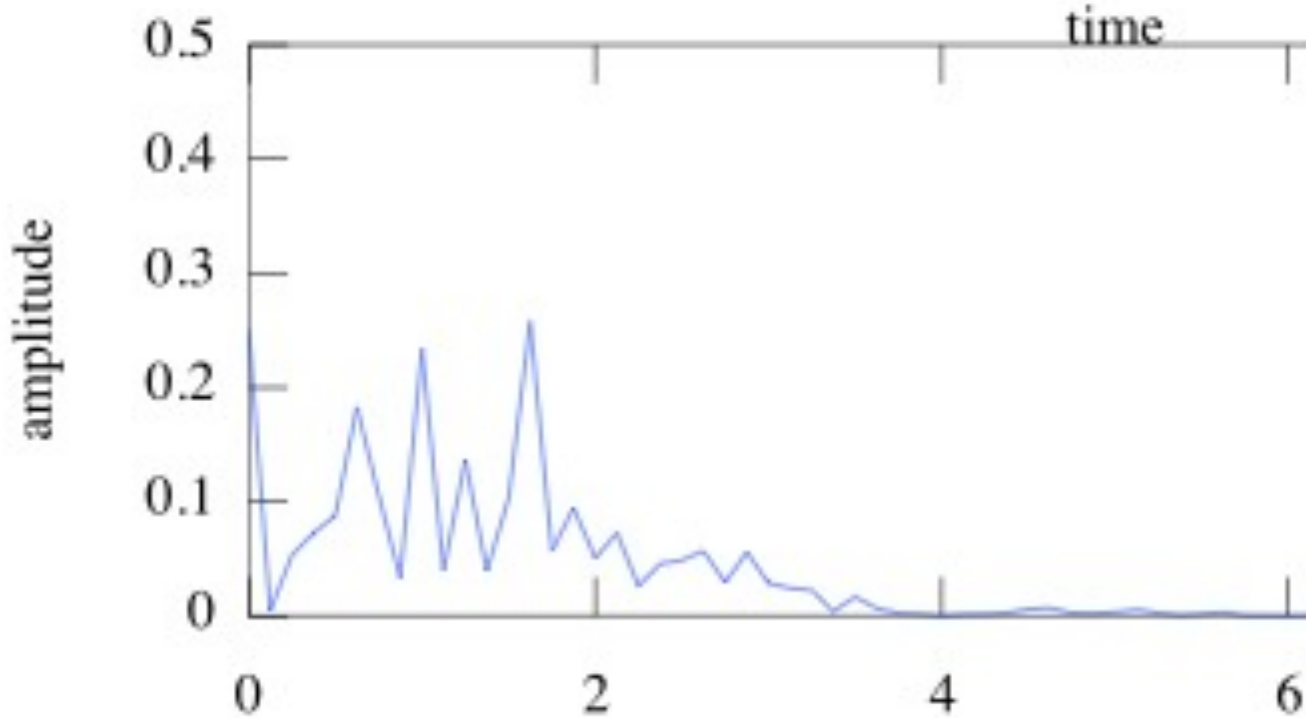
Peak fluctuation





Lag fluctuation





Ex 5 - Online spectral lab to illustrate:

- Invariance of amplitude vs signal
- Invariance of PSD vs signal
- DC component on all spectra
- Aliasing
- Noise shapes
- Parseval equality
- Noise contribution to power