Measuring the product of vectors by SVD in 2^k dimensional spaces, $k \geq 1$

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In an effort to analyze the role of nonlinearities in Computation, we look at the product of two vectors in hypercomplex Dickson algebras A_k, of dimension 2^k, $k \geq 0$. These algebras, which are *nonassociative* for $k \geq 3$, generalize the quaternions A_2 in a different way from Clifford. Their construction is a recursive complexification, starting from $A_0 = \mathbb{R}$, $A_1 = \mathbb{C}$. As k increases and classical properties of multiplication gradually vanish, *new* properties emerge which challenge classical logic for $k \geq 3$.

In this talk, we describe the phenomenon on the example of the measure of the vector x in A_k by the SVD of the linear map: $L_x : \; y \mapsto x \times y$. for $y \in A_k$. Let $\| \cdot \|$ denote the euclidean norm in A_k. For $k \leq 3$, $\|L_x\| = \|x\|$, as we expect, because of the isometry of $\| \cdot \|$: $\|x \times y\| = \|x\|\|y\|$. But for $k = 4$, the norm is anisometric and zerodivisors play havoc in the geometry of the octonions A_3 and in the logic of their computation: this is a spectacular *backward* effect.

For $k = 5$, it is possible that $x \times y \neq 0$ and $\|x \times y\| = 0$: the euclidean "norm" becomes a semi norm in a space of 32 dimensions! More transformations of classical notions occur in higher dimensions, exposing various aspects of the local clash between the linearity of SVD computation and the inherent nonlinearity of the nonassociative algebraic structure for spaces of dimension 2^k, $k \geq 4$

In conclusion, we indicate how classical logic should be complemented by an *organic* logic to account for the possible filtering of nonlinearlity by the linear SVD filter.

References: Cerfacs Reports TR/PA/07/54 and 55