Normal Degree and Krylov Sequences

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The normal degree $n(A)$ for a normalizable matrix A is the degree of the minimum degree polynomial $n(z)$ for which $A' = n(A)$, where A' is the dual of Ain some H-inner product space in which A becomes normal. Because the recurrence lengths of orthogonalizing Krylov subspace bases are exactly the normal degrees, normal degree is important for sparse matrix solvers $A\,x = b$.. Minimal normal degree may be looked at for individual data b, for given sets b to be treated in parallel, or for the whole operator A. Recent papers by Liesen, Saylor, Strakos, Faber, Tichy, perhaps others, apparently did not know that in a paper [1] I showed the following, which I will elaborate and bring up to date in my lecture.

- Normal degree is independent of the H-inner product. This corrected incompleteness in the literature, and removes certain vagueness one still finds in current papers.
- If A is unitary, then $n(A) = m - 1$. Here m is the degree of A's minimum polynomial.
- I emphasized how normal degree is fundamentally a complex interpolation problem which needed a corresponding new theory of "complex dipoles analysis". This theory has been found, is quite interesting, and I will comment on its implications.

References

[1] K. Gustafson, Normal Degree, Numerical linear Algebra with Applications 11 (2004), 661-674.

[2] K. Gustafson, Normal Degree revisited (2008), to appear.