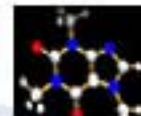




# SciDAC

Scientific Discovery through Advanced Computing



# Enabling Petascale Science through Combinatorial Algorithms

**Alex Pothen**

**CSCAPES Institute**

[www.cscapes.org](http://www.cscapes.org)



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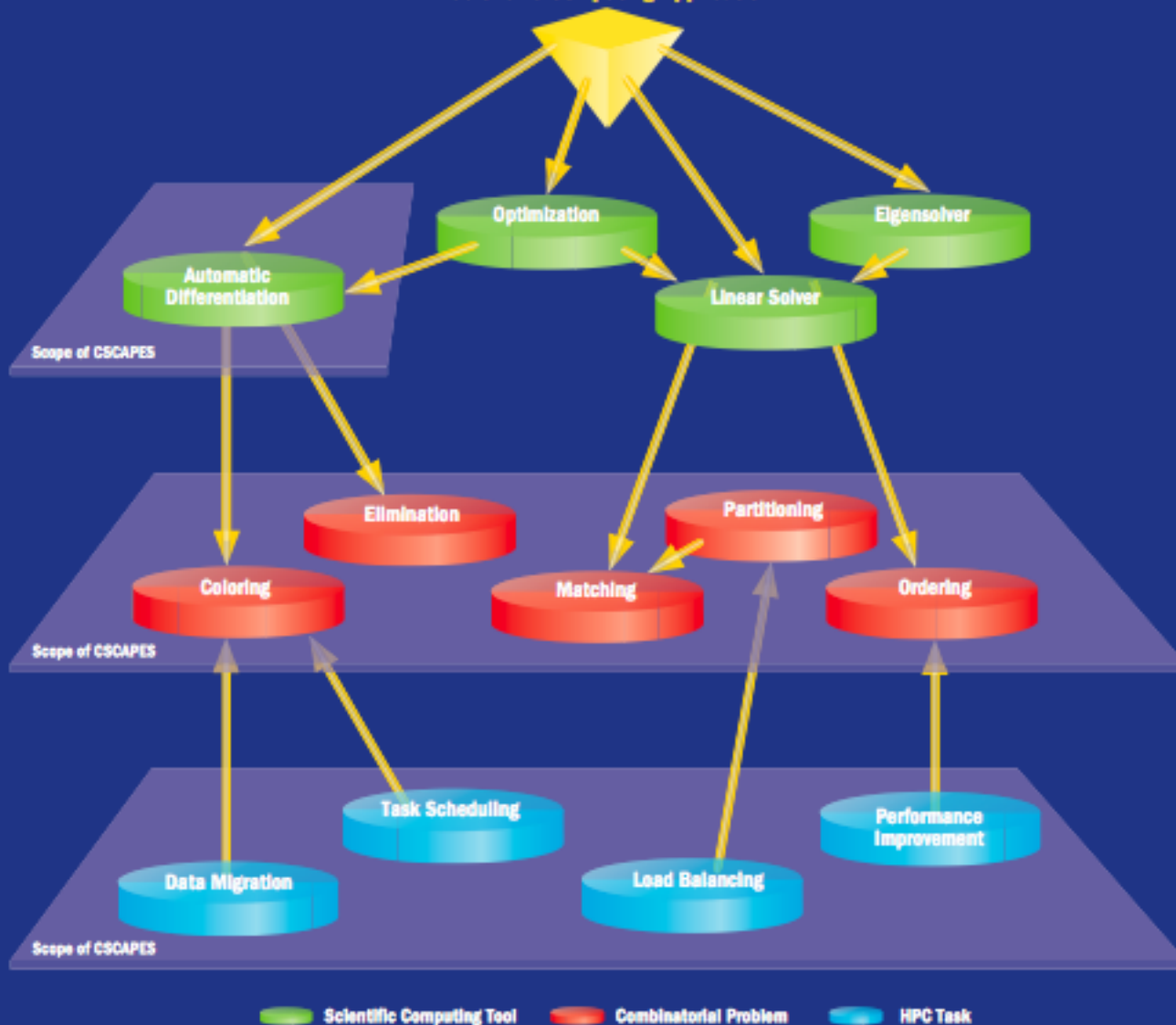
# CSC Community Thanks CERFACS

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- CSCAPES Institute is a natural outgrowth of the CSC community, which has been fostered by **CERFACS** for the past 20 years.
- Thank you to **CERFACS researchers**, who have led the way on sparse solvers, parallel computing, matchings, mathematical software, and many other topics!
- Thanks for organizing the **CSC05 Workshop** at CERFACS!
- Thanks for organizing the annual **Sparse Days conferences**, a remarkable forum for discussions, good food and wine!



# Scientific Computing Application



Parallelization,  
Load Balancing



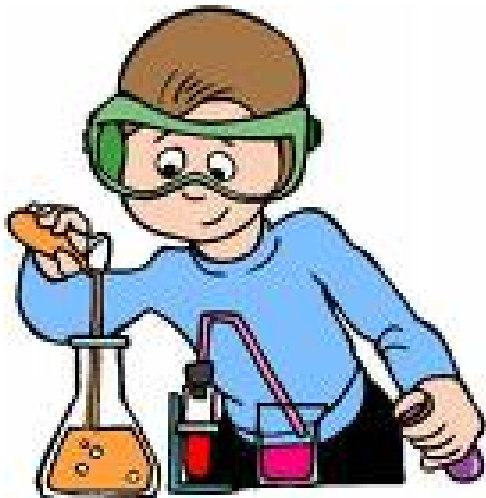
Graph Coloring



Performance



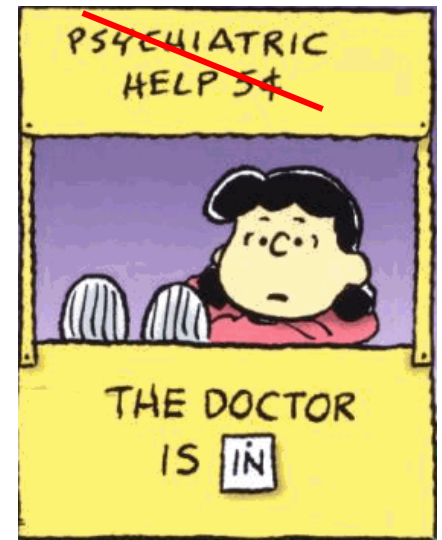
Automatic Differentiation



Graph Matching



Combinatorial problems?



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# Load Balancing

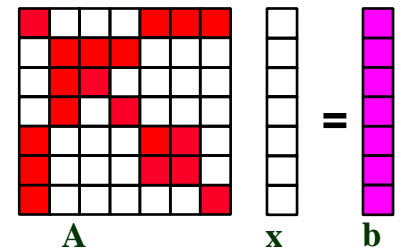
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*...enabling parallelization and fast run-times for irregular applications*

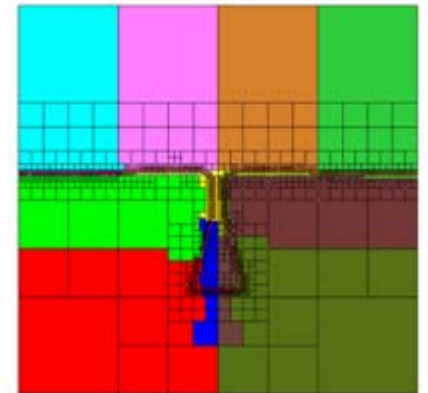


# Partitioning and Load Balancing

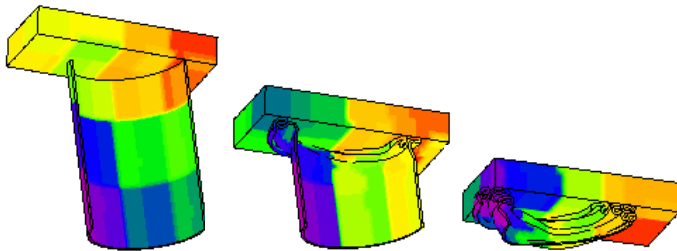
- **Goal:** assign data (and tasks) to processors to
  - minimize application runtime
  - maximize utilization of computing resources
- **Metrics:**
  - minimize processor idle time (balance workloads)
  - keep inter-processor communication costs low
- **Impacts performance of a wide range of simulations**
  - Accelerator code speeded up 3X with a geometric partitioner
- **Several partitioning and load balancing algorithms**



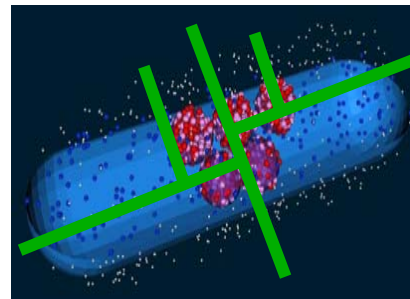
Linear solvers & preconditioners



Adaptive mesh refinement



Contact detection



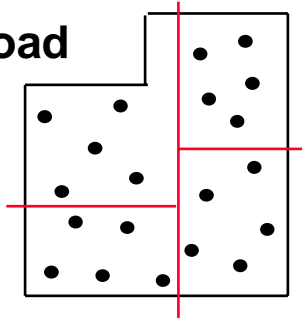
Particle simulations



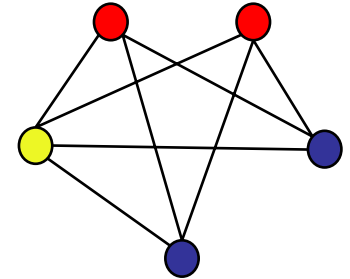


# Zoltan Toolkit: Data Services for Dynamic Applications

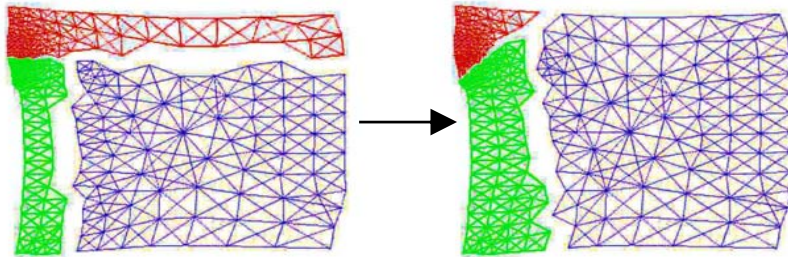
Dynamic Load Balancing



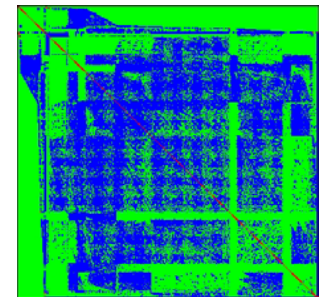
Graph Coloring



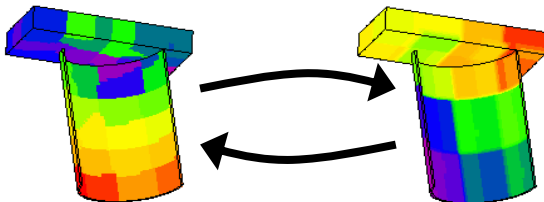
Data Migration



Matrix Ordering



Unstructured Communication



Distributed Data Directories

A	B	C	D	E	F	G	H	I
0	1	0	2	1	0	1	2	1



# Zoltan 3.0

**Zoltan 3.0 is now available ([www.cs.sandia.gov/Zoltan](http://www.cs.sandia.gov/Zoltan)). New features use hypergraphs for modeling communications accurately:**

- **Hypergraph repartitioning**
  - Reduces total communication in dynamic applications.
- **Hypergraph refinement**
  - Quickly improves an existing parallel distribution (partitioning)
- **Hypergraph partitioning with fixed vertices**
  - Allows application to fix certain data to specific processors.
- **Hierarchical partitioning**
  - 2-level partitioning, possibly using different algorithms, cost metrics
  - Useful for complex computer architectures (e.g., multi-core)





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# Automatic Differentiation

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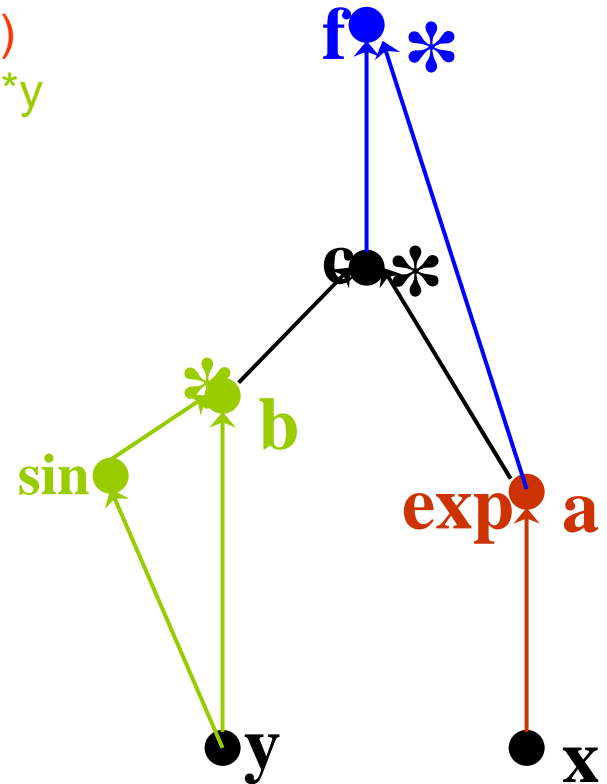
*...enabling the solution of nonlinear differential equations, optimization, sensitivity analysis, uncertainty quantification, etc.*



# AD: Introduction

- Transforms code for computing a function into code for differentiating it
- Function computed from intrinsic operations, and modeled by a directed acyclic graph (DAG)
- Compute derivatives by composing partial derivatives for each operation, using the chain rule on the DAG
- Efficiency of generated code depends on sophistication of compiler analysis and **combinatorial algorithms**

$a = \exp(x)$   
 $b = \sin(y) * y$   
 $c = a * b$   
 $f = a * c$

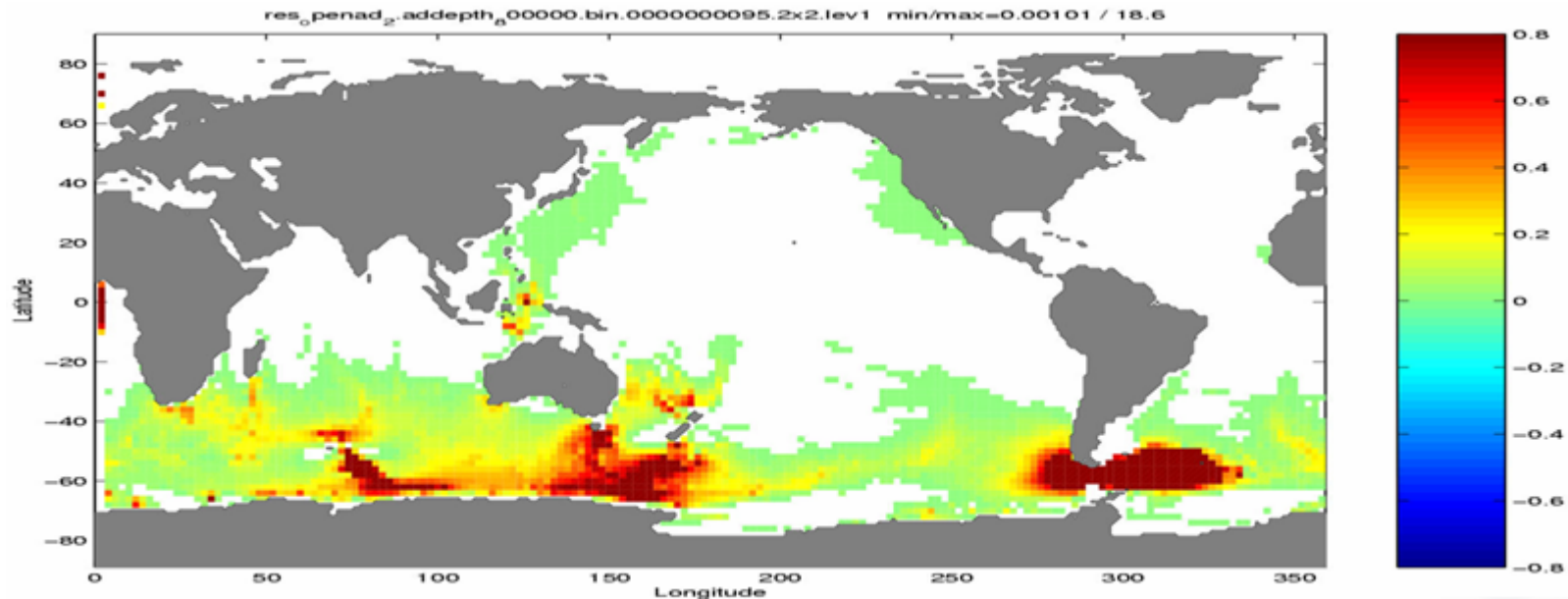


# AD: Combinatorial Problems

- **Parallel algorithms for differentiating reduction operations**
- **Reduce operations and storage needed to compute the derivatives by evaluating the DAG in suitable orders**
  - Two extreme modes: Forward and Reverse
  - Modeled as vertex and edge elimination in DAG
  - Stop at some intermediate stage to find minimum storage
- **Location of checkpoints in reverse mode**
- **Graph coloring for computing many derivatives in one AD pass through the DAG**
- **Integration with PETSc and Zoltan toolkits**

# Sensitivity analysis in climate model

- Sensitivity of flow through Drake Passage to ocean bottom topography (**P. Heimbach, MIT**)
  - Finite difference approximations: 23 days
  - Naïve automatic differentiation: 2 hours 23 minutes
  - Smart automatic differentiation: 22 minutes



# AD: Current Capabilities

- **Fortran 77: ADIFOR 2.0/3.0**
  - Robust, mature tool with excellent language coverage
  - Excellent compiler analysis
  - Efficient forward mode; adequate reverse mode
- **C/C++: ADIC 2.0**
  - Semi-mature tool with full C language coverage
  - Sophisticated differentiation algorithms
  - Efficient forward mode
- **Fortran 90: OpenAD/F**
  - New tool with partial language coverage
  - Sophisticated differentiation algorithms
  - Accurate and novel compiler analysis
  - Innovative templating mechanism
  - Efficient forward and reverse modes



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# Graph Coloring

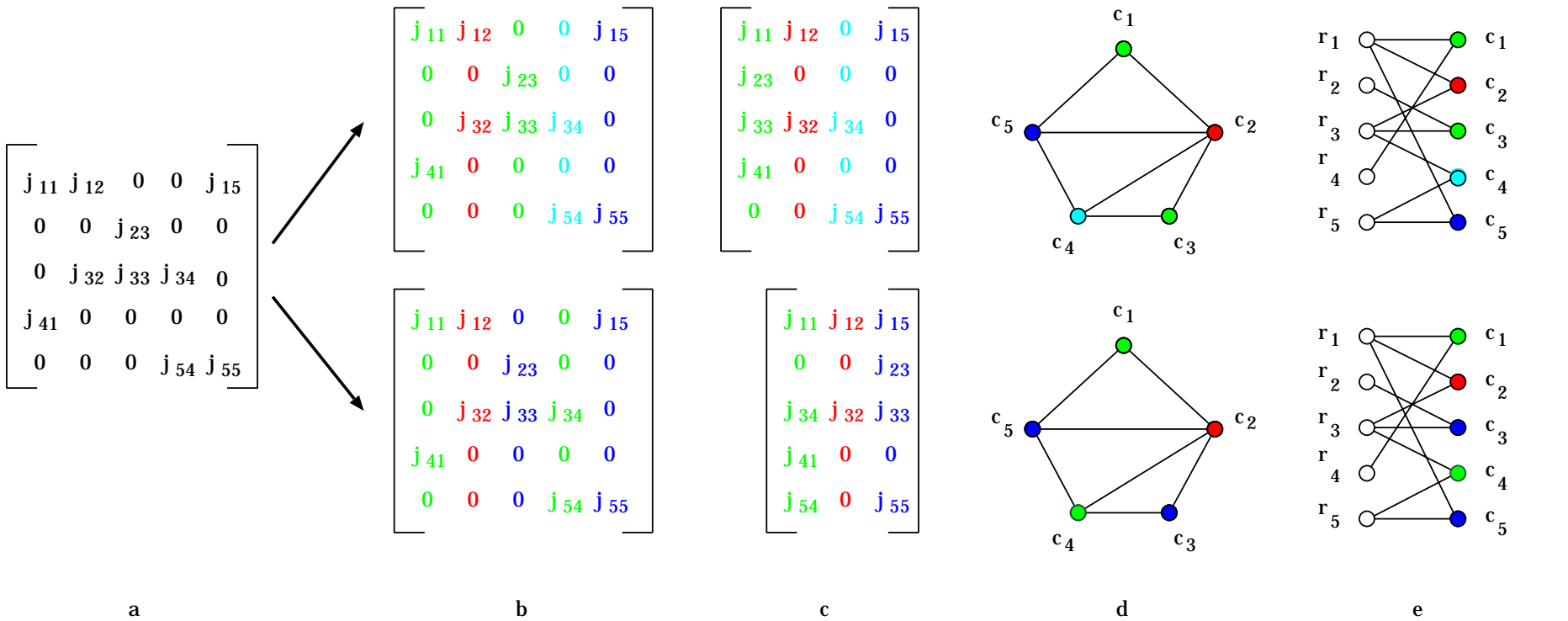
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*...reducing work in Automatic  
Differentiation; and discovering  
parallelism in computations*





# Coloring and Jacobian Computation



Original Jacobian

**Compressed** representation  
(**Structurally orthogonal**  
columns packed together)

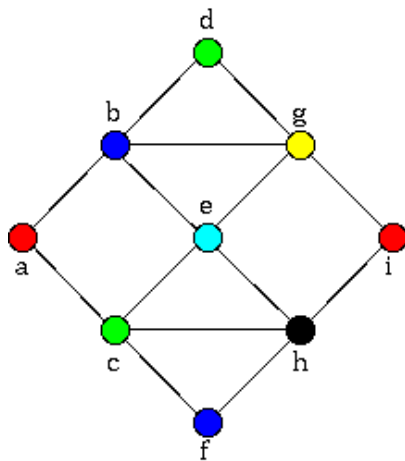
**D1 coloring**  
formulation on  
column inter. graph

**D2 coloring**  
bipartite graph

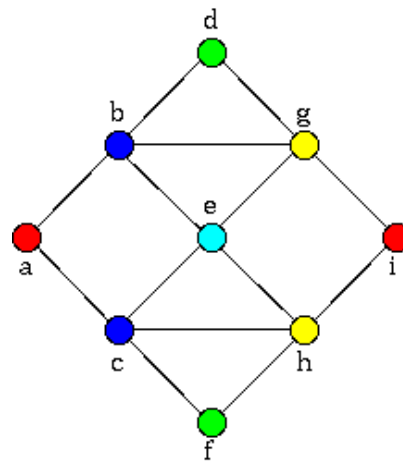
# Coloring and Derivatives: The Big Picture

- **Scenarios and coloring models:**
  - unsymmetric vs symmetric matrix
  - direct vs substitution method
  - uni- vs bi-directional partitioning
- **Developed novel sequential algorithms**
- **Future plans**
  - Develop parallel versions
  - integrate with AD tools

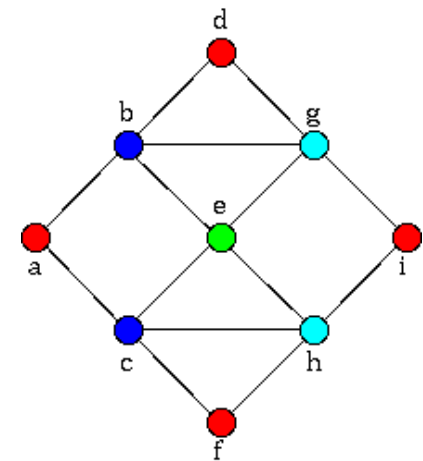
	<i>1d partition</i>	<i>2d partition</i>	
<i>Jacobian</i>	<i>Distance-2 coloring</i>	<i>Star bicoloring</i>	<i>Direct</i>
<i>Hessian</i>	<i>Star coloring</i>	<i>NA</i>	<i>Direct</i>
<i>Jacobian</i>	<i>NA</i>	<i>Acyclic bicoloring</i>	<i>Subst</i>
<i>Hessian</i>	<i>Acyclic coloring</i>	<i>NA</i>	<i>Subst</i>



D2 coloring



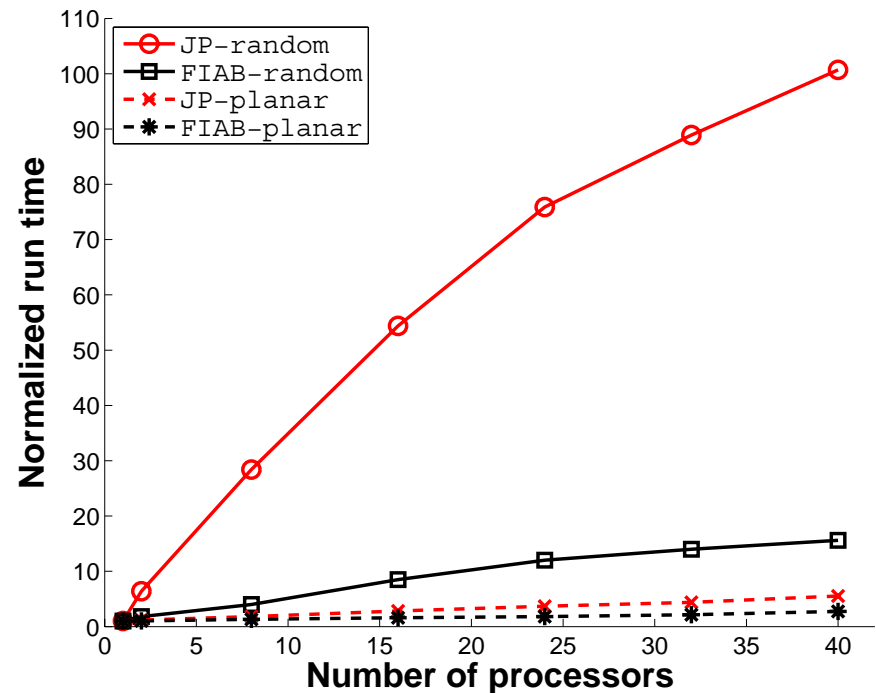
Star coloring



Acyclic coloring

# Framework for parallel coloring

- **Essential ingredients of framework:**
  - Partition graph on processors, and **speculatively color** subgraphs in rounds
  - Exchange color info after a superstep (coloring a specified no. of vertices)
  - Detect conflicts after each round, resolve using randomization, recolor when needed
- **Applied to D-1 and D-2 coloring, implemented in MPI; available in Zoltan**
- **Extending the framework to**
  - Tera- and peta-scale machines
  - Other graph problems



**Weak scalability on two families of graphs: random (unstructured); planar (structured).**



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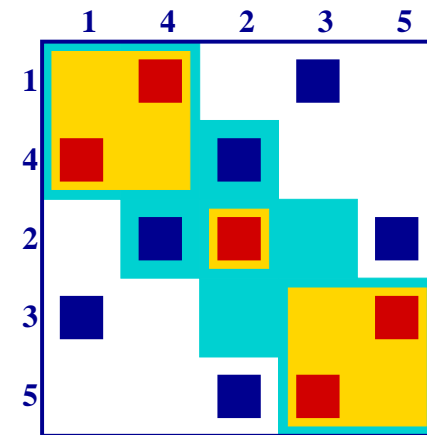
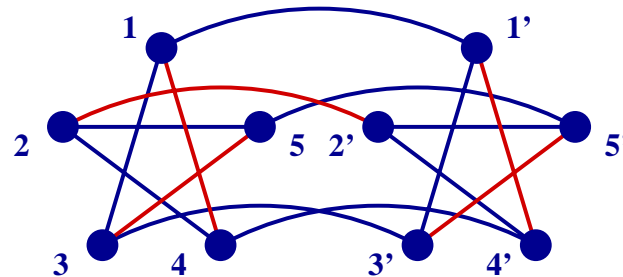
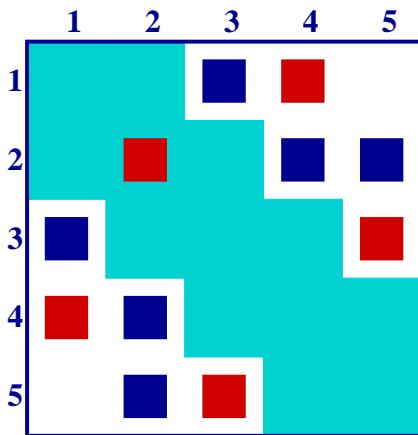
# Matchings in Graphs

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*...enabling load balancing and linear solvers*

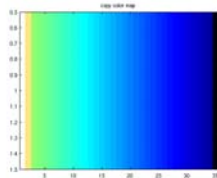
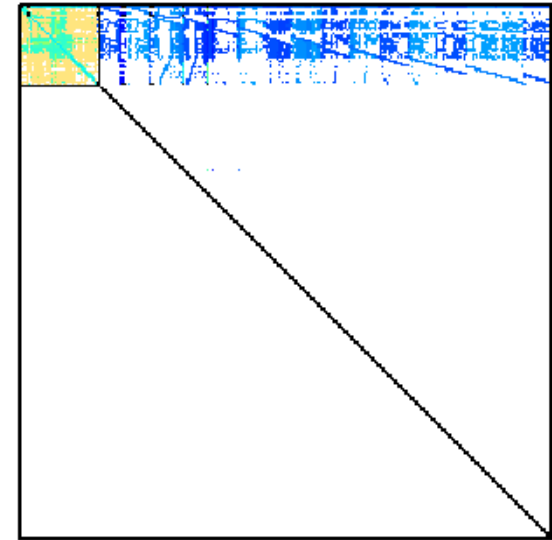
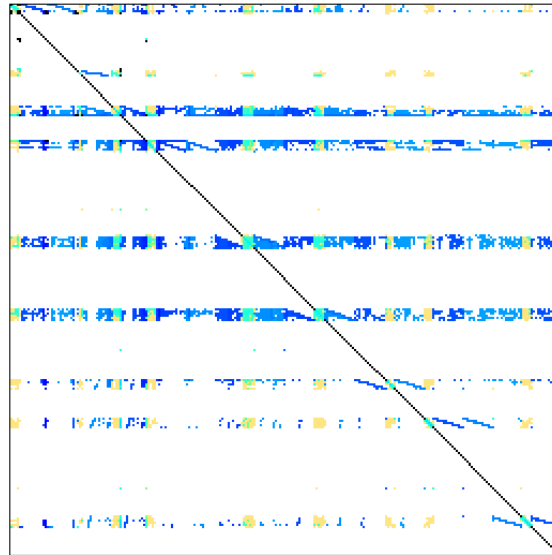
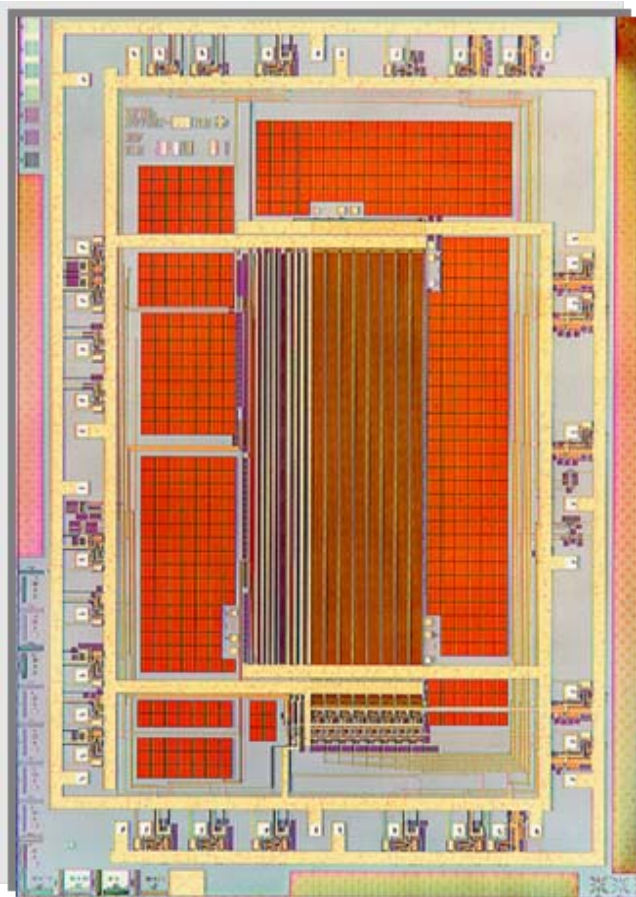


# Matchings in Graphs



- **Matching is a pairing of vertices; a vertex is paired with one neighboring vertex or none**
- **Applications**
  - Place large elements on diagonals of matrices for solvers
  - Block triangular form to reduce work in solvers, improve condition number
  - Coarsening step in multilevel graph and hypergraph partitioners

# Block Upper Triangular Form (BTF)



Circuit model from **Xyce** (Hoekstra, Day; Sandia) 683K rows, 2M nnz, 584K diag blocks  
Solved 200 times faster! 100M problem waits.



# Outreach and Training

- **Organized the SIAM Workshop on CSC in Feb. 2007. 100 attendees, 12 early career researchers supported. SIAM News article in May 2007. URL: [www.cscapes.org](http://www.cscapes.org), click on CSC07**
- **International collaborations with CERFACS, AD groups in Germany, CSC groups in Norway, Netherlands, and others.**
- **5 Postdoctoral researchers, 4 PhD students, and an undergraduate are involved in CSCAPES research, and are co-mentored by Lab scientists.**
- **Working with several enabling technology and applications groups to integrate CSC software and solve their combinatorial problems.**
- **We welcome application kernels where CSC issues are significant; tell us about your combinatorial problems!**



# CSCAPES Institute

[www.cscapes.org](http://www.cscapes.org)

**Erik Boman, Karen Devine, Bruce Hendrickson;**

**Paul Hovland, Boyana Norris, Jean Utke;**

**Alex Pothen, Assefaw Gebremedhin, Florin Dobrian, Sudip Seal;**

**Umit Catalyurek, Doruk Bozdog;**

**Michelle Mills Strout**

