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Efficient Triangular Solution in Sparse Out-of-Core Solvers

Sparse Days

CERFACS, Toulouse
June 2006

Overview

- Introduction
- System Based Method
- Direct IO Method
- Concluding Remarks

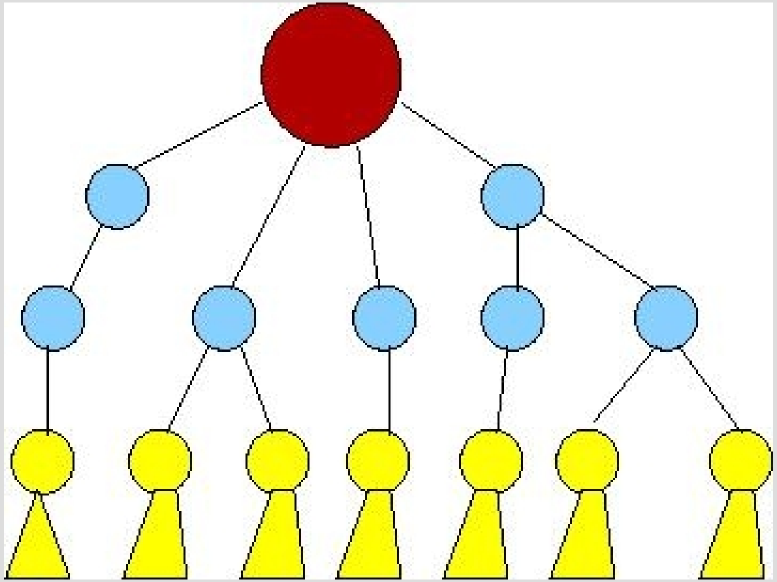
Objective: Reduce time for solution of $Ax=b$
in a parallel limited memory environment

Context:

- Direct Method $A=LU$ or $A=LDL^T$
- Distributed Memory Parallel Solution
- Factors stored on disk (Out-Of-Core, OOC)
- MUMPS (distributed memory multifrontal solver)

Parallelism in MUMPS Solver

3 levels of parallelism for both factorization and solve phases



- 2D block cyclic parallelism

- irregular 1D decomposition

- sequential processing of subtree

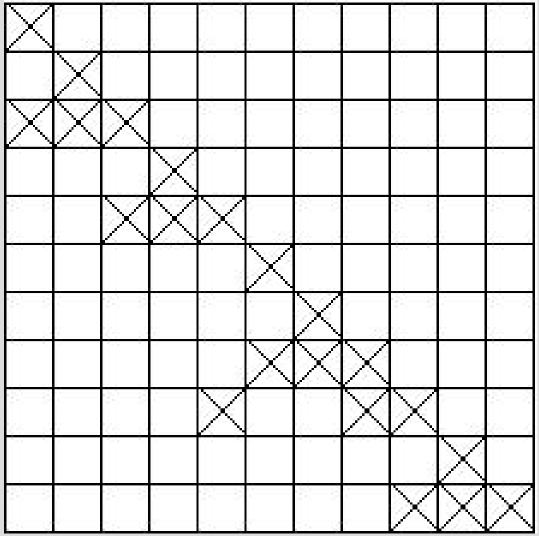
For a symetric case: $Ax=b \Rightarrow LD\,y = b$ (forward)

$$L^T x = y$$ (backward)

For a symetric case: $Ax=b \Rightarrow LD\,y = b$

$L^T x = y$

matrix pattern



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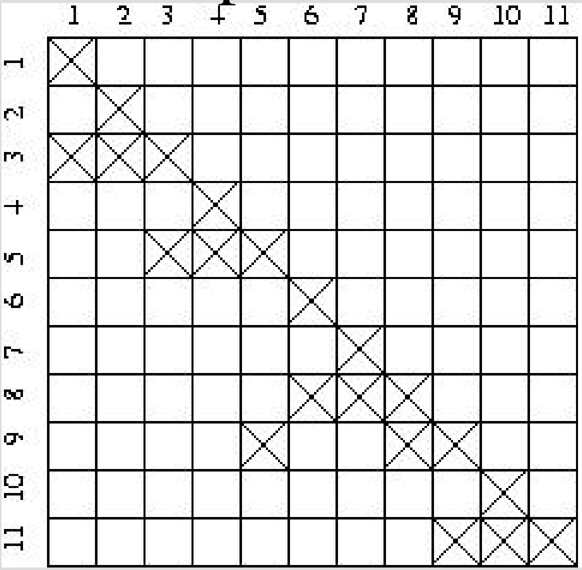
For a symetric case: $Ax=b \Rightarrow LD\,y = b$

$$L^T x = y$$

matrix pattern



	1	2	3	4	5	6	7	8	9	10	11
1	X										
2		X									
3	X	X	X								
4				X							
5			X	X	X						
6						X					
7							X				
8						X	X	X			
9					X			X	X		
10										X	
11									X	X	X

For a symetric case: $Ax=b \Rightarrow LD\,y = b$

$L^T x = y$

matrix pattern

	1	2	3	4	5	6	7	8	9	10	11
1	X										
2		X									
3	X	X	X								
4				X							
5			X	X	X						
6						X					
7							X				
8						X	X	X			
9					X			X	X		
10										X	
11									X	X	X

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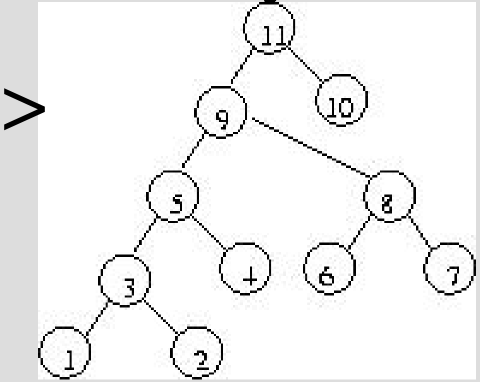
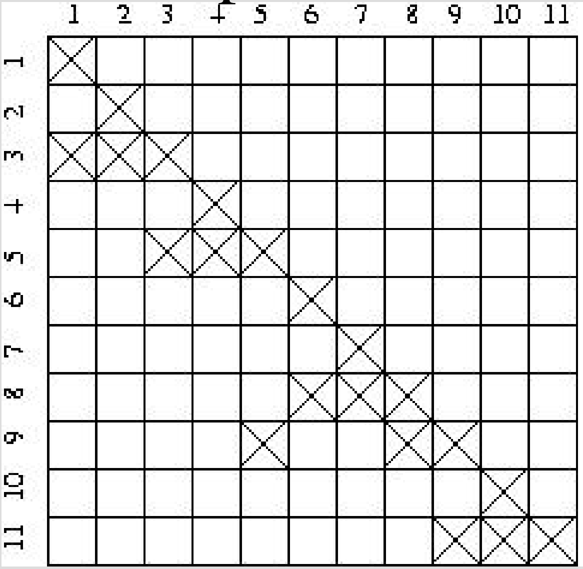
assembly tree



For a symetric case: $Ax=b$ => $LD\,y = b$

$$L^T x = y$$

matrix pattern



	1	2	3	4	5	6	7	8	9	10	11
1	X										
2		X									
3	X	X	X								
4				X							
5			X	X	X						
6						X					
7							X				
8						X	X	X			
9					X			X	X		
10										X	
11									X	X	X

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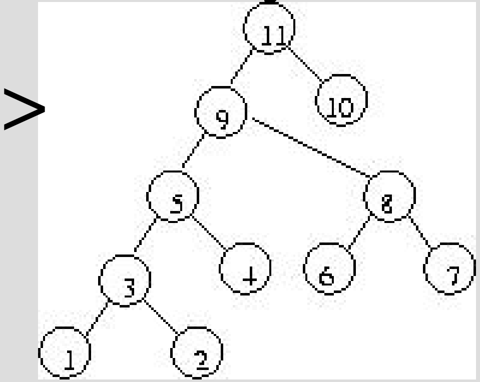
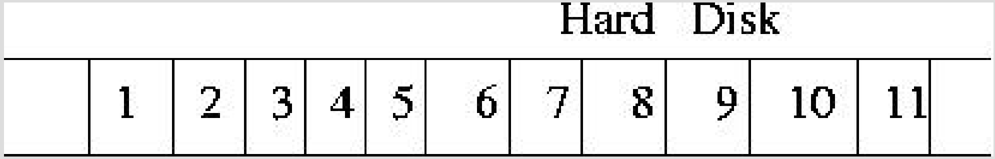
assembly tree



Factorization



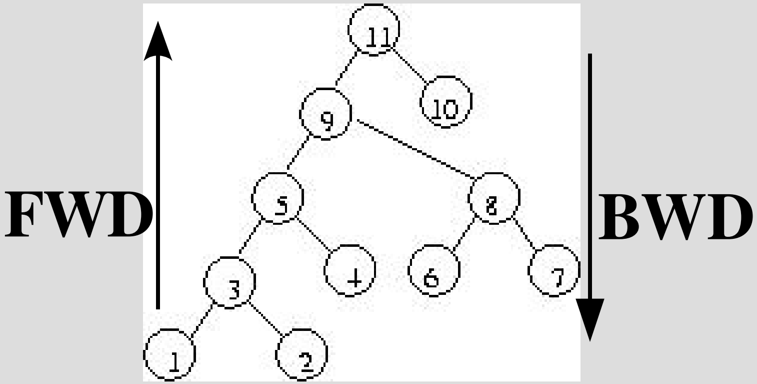
Tree traversal during solve:

for the **sequential case:**

--> forward step(Fwd) : postordering

as in the factorization phase

assembly tree



--> backward step(Bwd):

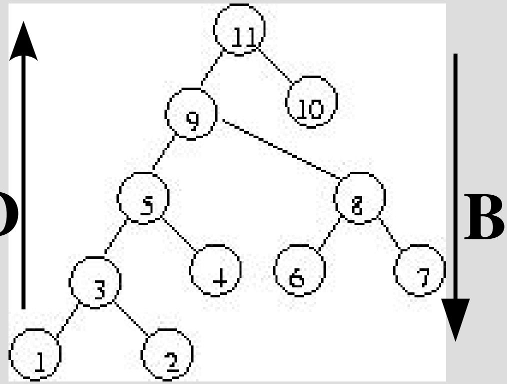
in the reverse order

Tree traversal :

for the **parallel case:** ***no guarantee of the order***

in which the nodes are processed

assembly tree



FWD

BWD

-->Fwd: topological ordering

(not necessarily postordering)

--> Bwd: top-bottom traversal

Implementation issue: the **pool of tasks**

definition: list of all tasks ready to be executed

(In core: Last-In-First-Out strategy to extract tasks)

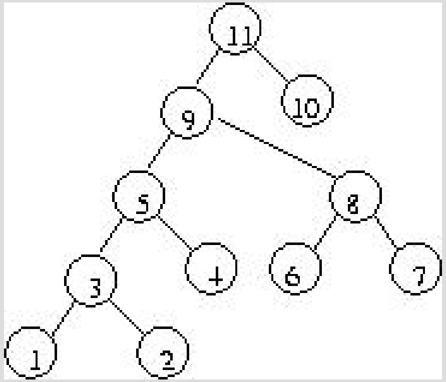
Out-Of-Core issues:

How to process the pool

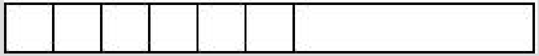
to have a reasonably "*regular*" access to disk in both

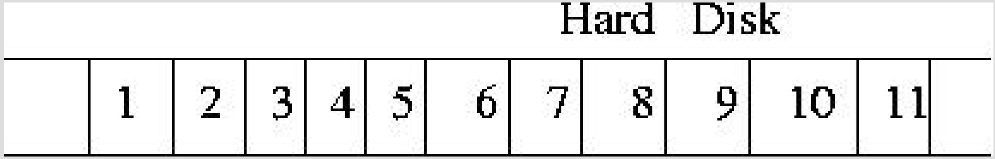
- sequential and parallel context

Illustration: sequential processing of the **tree**



Pool at the beginning of FWD

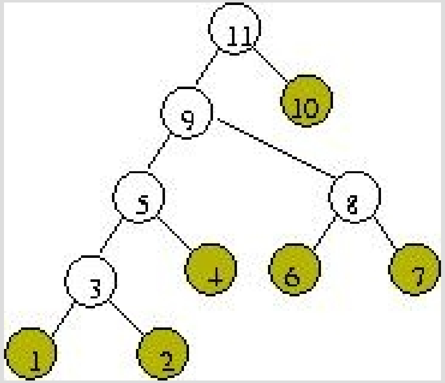




Hard Disk

	1	2	3	4	5	6	7	8	9	10	11	

Illustration: sequential processing of the **tree**



Pool at the beginning of FWD

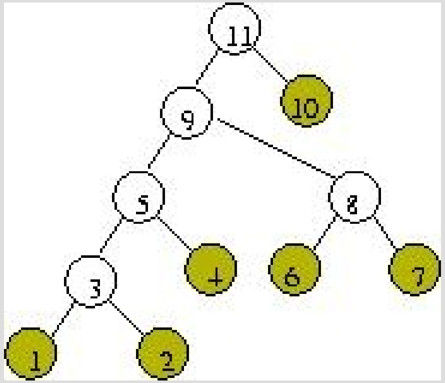
I step

10	7	6	4	2	1	

Hard Disk

	1	2	3	4	5	6	7	8	9	10	11	

Illustration: sequential processing of the **tree**



Pool at the beginning of FWD

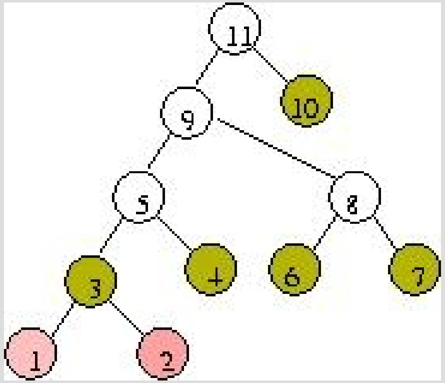
II step

10	7	6	4	2		

Hard Disk

	1	2	3	4	5	6	7	8	9	10	11	

Illustration: sequential processing of the **tree**



Pool at the beginning of FWD

I - II step

10	7	6	4	2	1	

III step

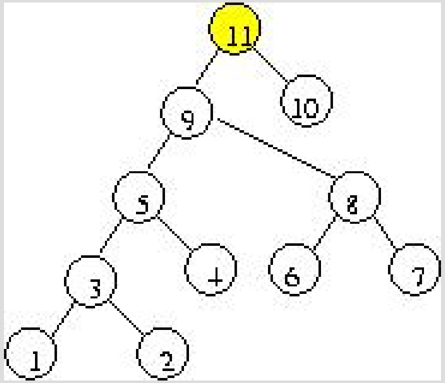
10	7	6	4	3	

FWD →

Hard Disk

	1	2	3	4	5	6	7	8	9	10	11	

Illustration: sequential processing of the **tree**



Pool at the beginning of FWD

I - II step

10	7	6	4	2	1	

III step

10	7	6	4	3	

Pool at the beginning of BWD

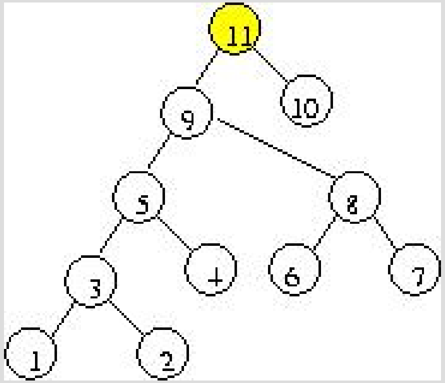
11						

FWD

Hard Disk

	1	2	3	4	5	6	7	8	9	10	11	

Illustration: sequential processing of the **tree**



Pool at the beginning of FWD

I - II step

10	7	6	4	2	1	

III step

10	7	6	4	3	

Pool at the beginning of BWD

I step

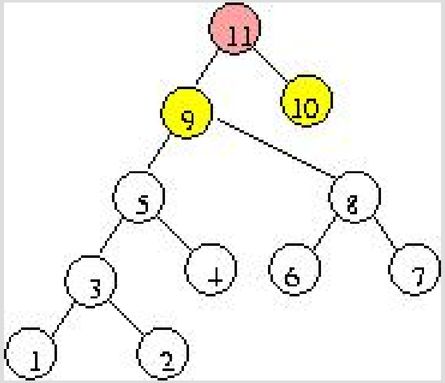
11						

FWD →

Hard Disk

	1	2	3	4	5	6	7	8	9	10	11	

Illustration: sequential processing of the **tree**

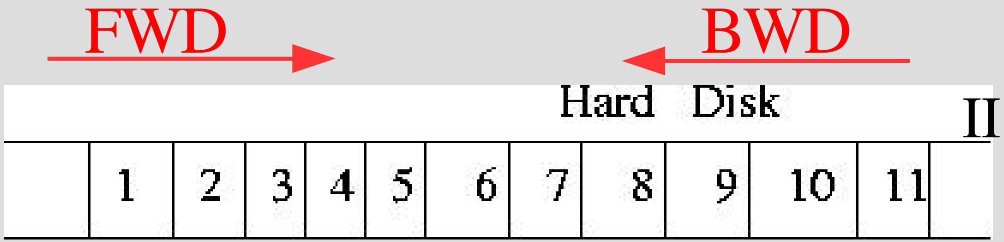


Pool at the beginning of FWD

I - II step	10	7	6	4	2	1	
III step	10	7	6	4	3		

Pool at the beginning of BWD

I step	11						
II - III step	9	10					



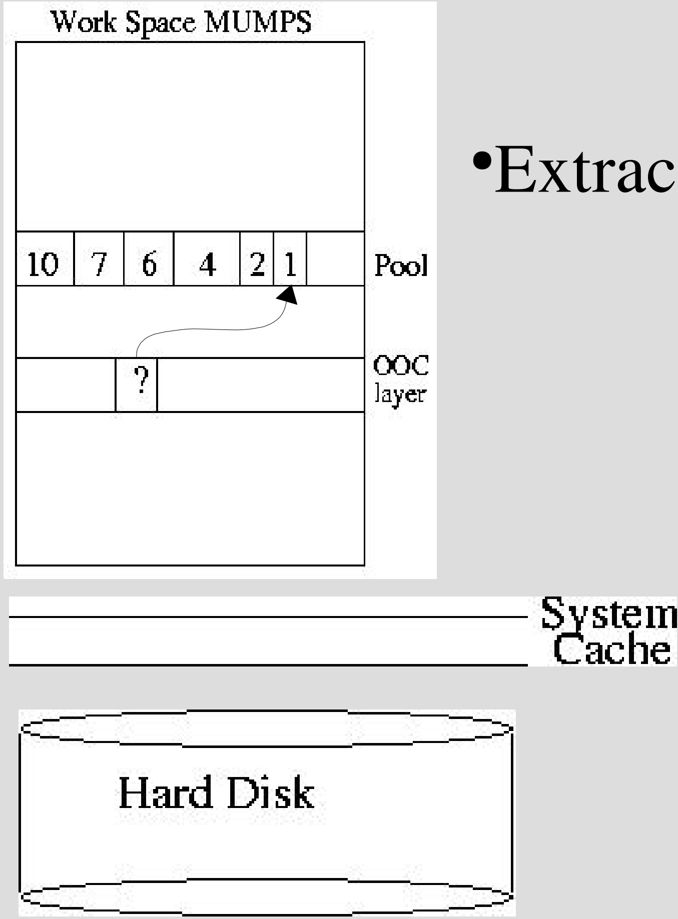
OOC context

- During factorization **ALL** factors are written to local disks
- **No factors kept in memory** at the beginning of the solve part (**and between** forward and backward steps)
- We load data from the local disks to the main memory
- Two possibilities to access data on disk:
  - The simple/natural/default way: do nothing and exploit system properties so called **System Based Method**
  - One user defined way, more critical to implement so called **Direct IO Method**

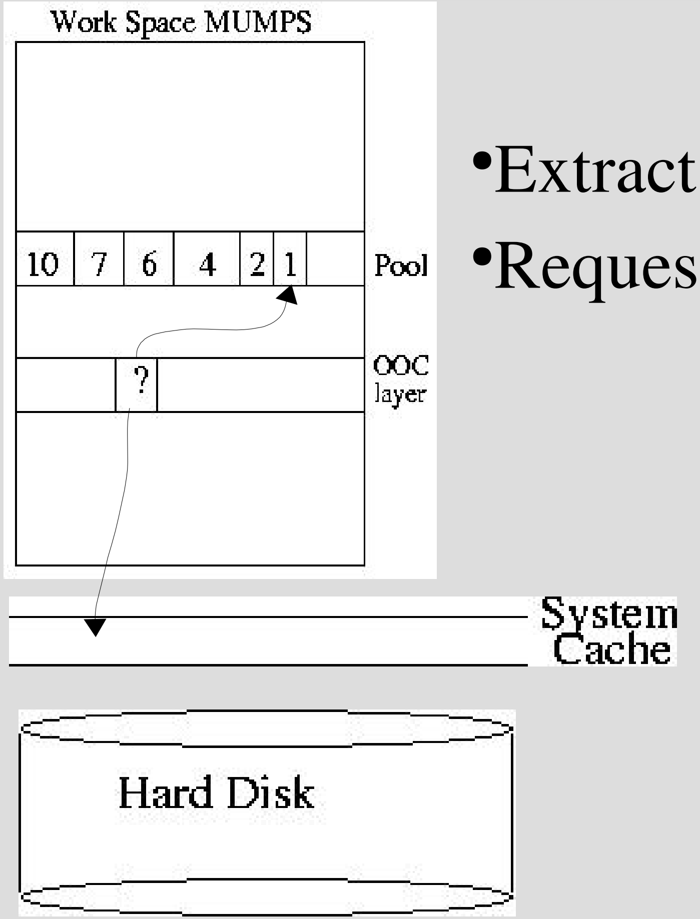
Testing environment

- Multiprocessor Cray XD1 (CERFACS)
- 58 nodes with 2 processors per node
- 4 GB memory per node
- IDE disk managing for each node
- Reiserfs file system(max bandwidth for read operation: 16 MB/sec)
- 2 matrices will be used:
  - Grid 300-100-10 (factor size **748 MB, order: 300 000)**
  - Qimonda07, from Qimonda company (factor size **2 534 MB order: 8 613 291)**

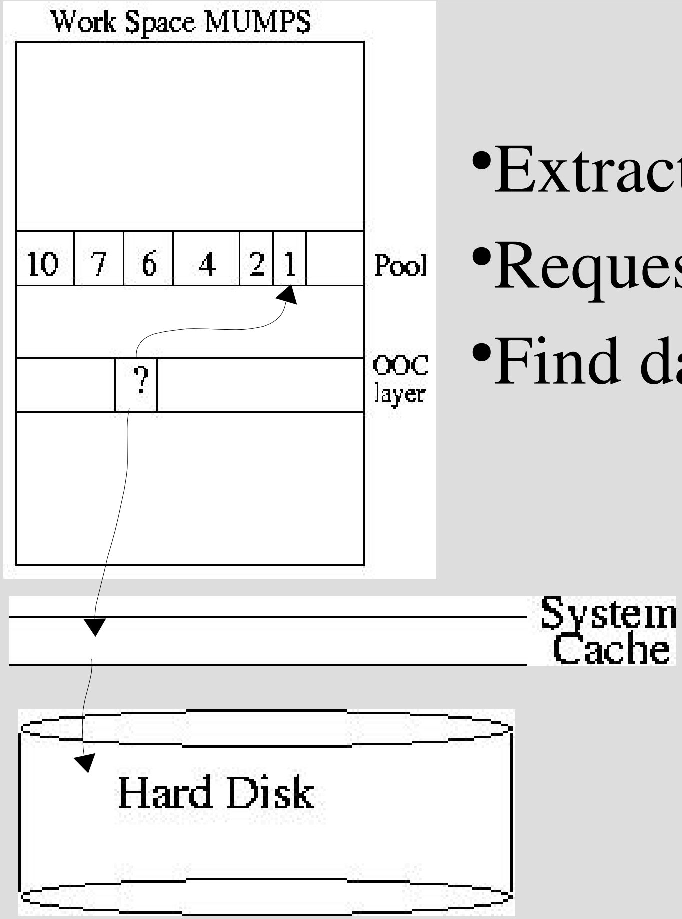
System Based Method



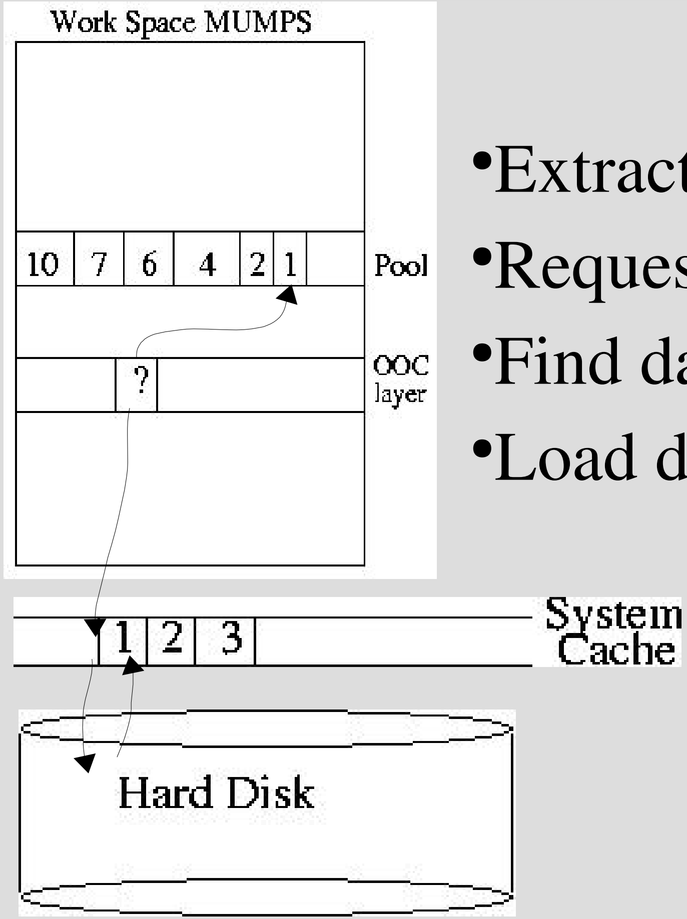
- Extract task from pool



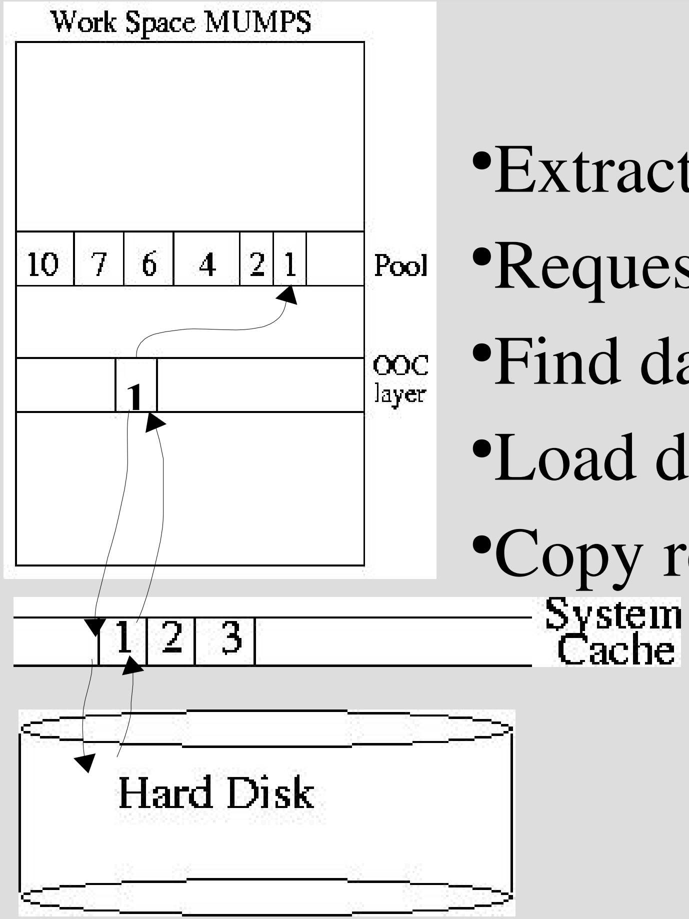
- Extract task from pool
- Request to load data



- Extract task from pool
- Request to load data
- Find data on the disk



- Extract task from pool
- Request to load data
- Find data on the disk
- Load data in the system cache (+ look-ahead)



- Extract task from pool
- Request to load data
- Find data on the disk
- Load data in the system cache (+ look-ahead)
- Copy requested data to the work space

Advantage

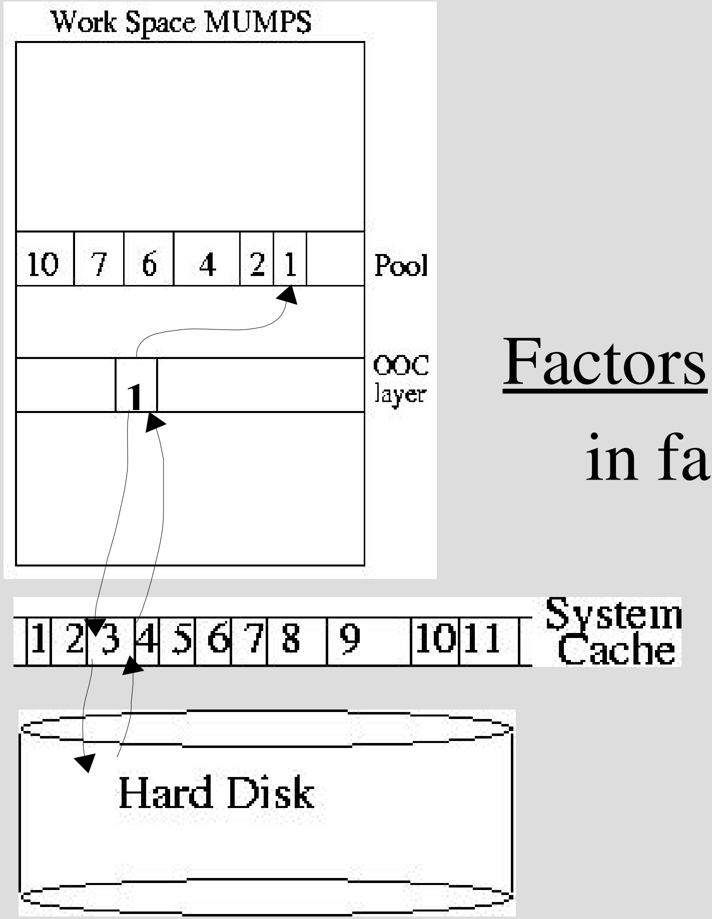
- Easy to implement, demand driven strategy

Strategy	Factor	Solve		
	Time (sec)	Fwd (sec)	Bwd (sec)	Disk access (MB/s)
Grid 300-100-10				
In-core	34.91	0.39	0.37	
OOC	34.90	1.26	1.17	616

Table 1-a) System Based Approach;

Remark: Average speed of disk access is 616 MB/sec.

616 MB/sec >> **16** MB/sec WHY ?



Factors written on disk are in fact kept in the system cache.

How will this work **when memory is critical** ?

	Factor	Solve		
Strategy	Time (sec)	Fwd (sec)	Bwd (sec)	Disk access (MB/s)
	Qimonda07			
In-core	40.40	0.9	0.9	
OOC	191.00	186.4	207.7	13

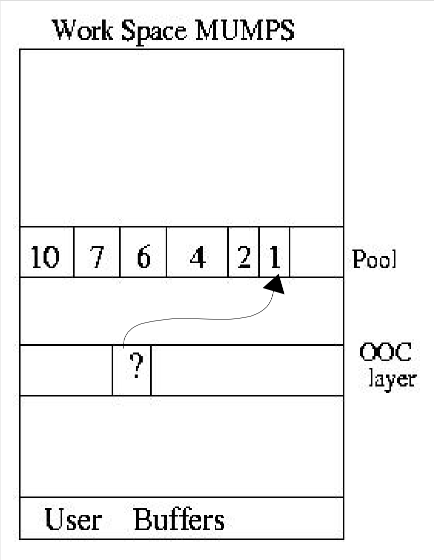
Table 1-b) System Based Approach

Remark: Average speed of disk access is 13 MB/sec

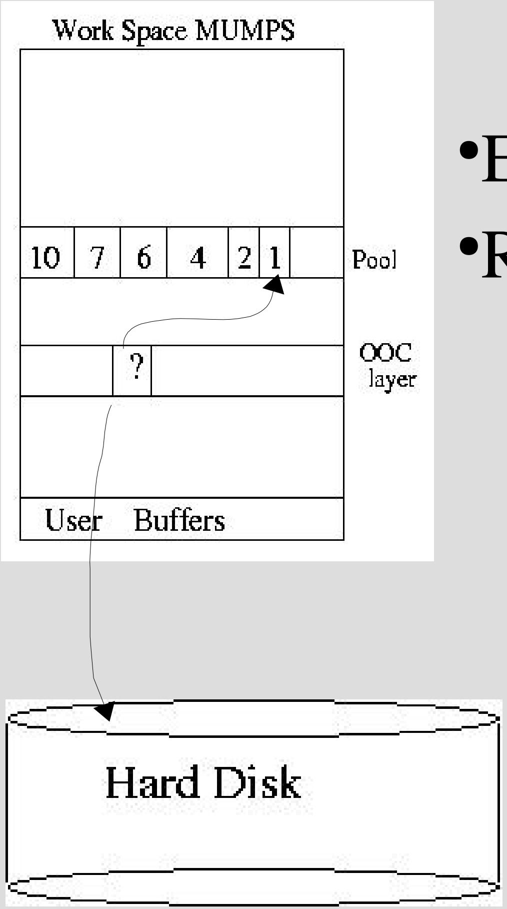
Drawback

- The system cache grows at each disk (read/write) access
- Impossible to control memory use (size and speed)
- Problem for large matrices, when the volume of disk access is larger than memory size
- Swapping effect
  - System decision often based on a variant of Least Recently Used strategy
  - System has no knowledge of data access pattern
  - System cache management may lead to user space swap

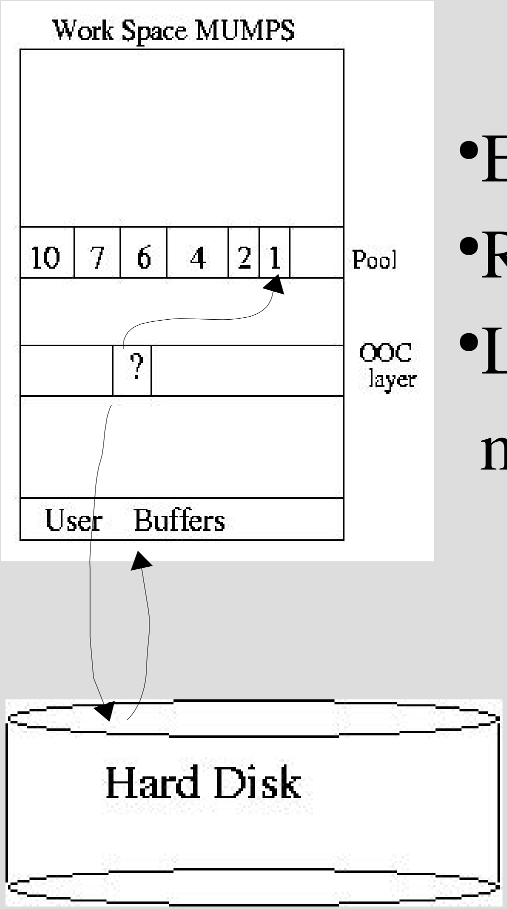
Direct IO Method



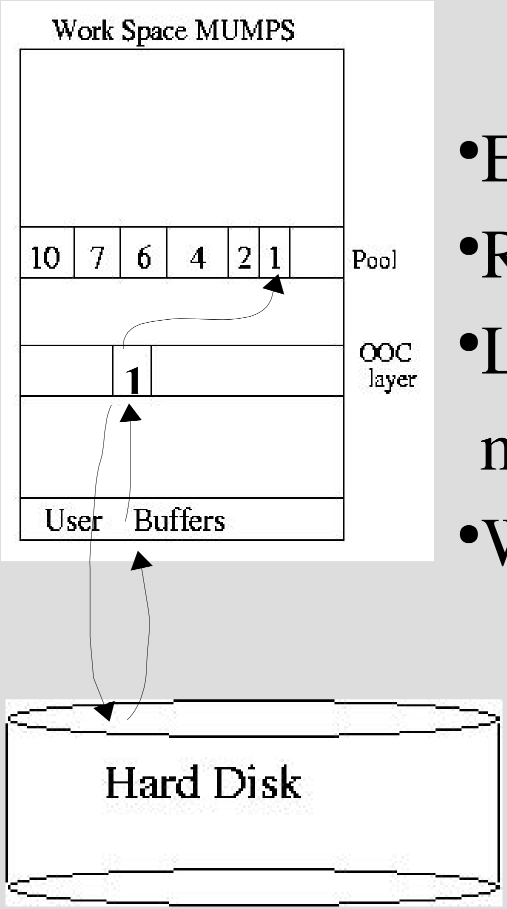
- Extract task from pool



- Extract task from pool
- Request to load data

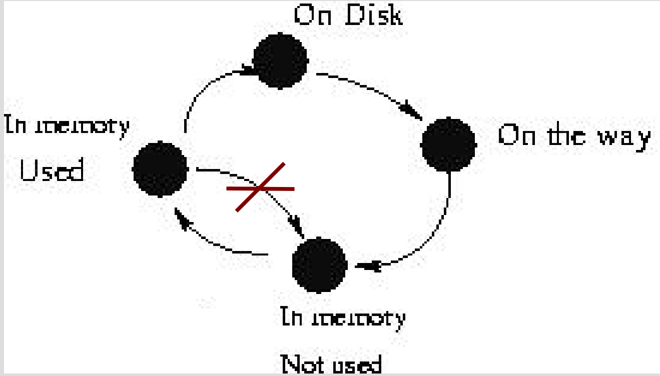


- Extract task from pool
- Request to load data
- Load data in the user's buffer (+some look-ahead mechanisms)



- Extract task from pool
- Request to load data
- Load data in the user's buffer (+some look-ahead mechanisms)
- Wait for availability of data

Implementation details



- Each factor-block is loaded only once
- User control of number and size of buffers
- One Emergency buffer (EMG), to hold largest front (demand driven)
- Other buffers used to automatically prefetch data with look-ahead mechanism

Introduce user buffers to improve performance

Method	User Buffers	T_factor (sec)	T_fwd (sec)	T_bwd (sec)	Nb_Req 1 buffer FWD	Nb_Req EMG zone FWD	Nb_Req 1 buffer BWD	Nb_Req EMG zone BWD
System Based	-	191.0	186.4	207.7	-	-	-	-
Direct IO	EMG	107.8	1 149.2	1 279.2	0	3 083 998	0	3 083 998
	EMG+1buffer	107.5	174.0	183.7	543	0	494	1

EMG buffer = **1 MB**, 1 buffer = **10 MB,** factor size = **2 534 MB**

- EMG- significant number of requests to the disk => overhead
- EMG+1 buffer- anticipate mechanism
  - Stable performance; almost suppress the EMG use

Influence of Scheduling

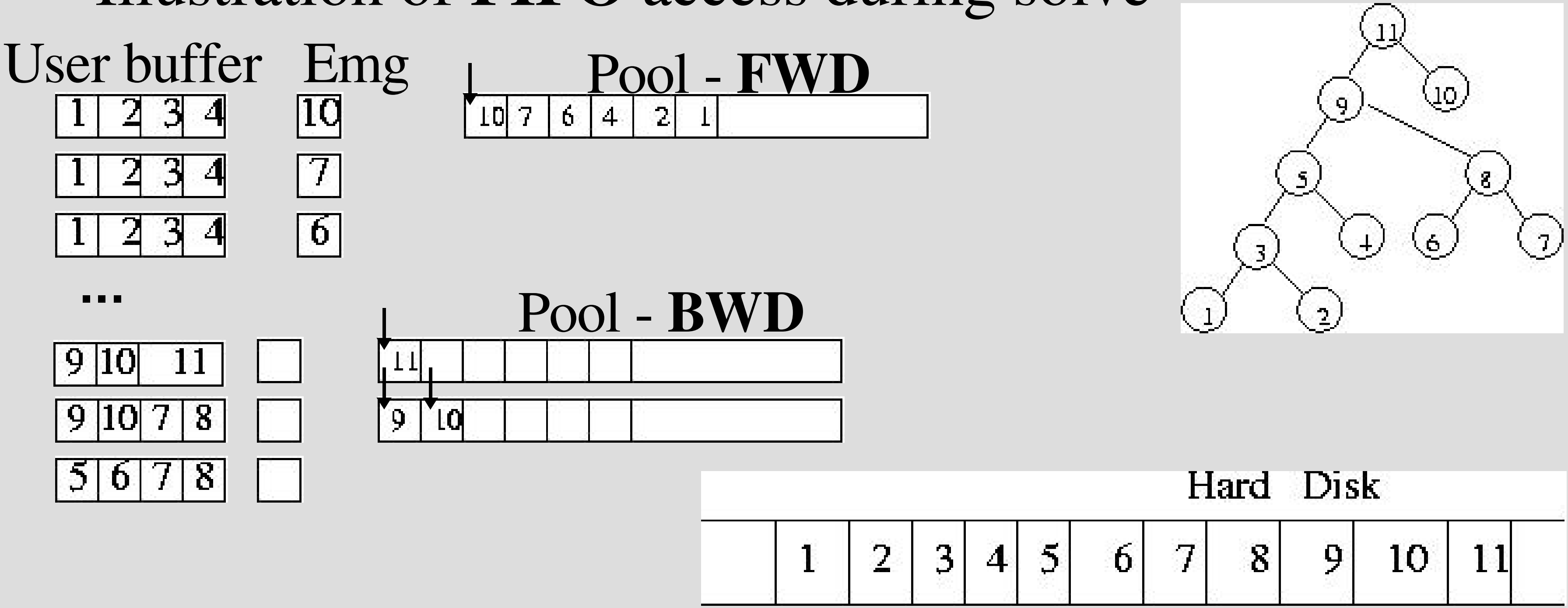
- Influence of pool management strategy on sequential performance

Strategy	Nb of Procs	T_factor (sec)	T_min (sec)	T_fwd (sec)	T_bwd (sec)	Nb_Req 1 buffer FWD	Nb_Req EMG zone FWD	Nb_Req 1 buffer BWD	Nb_Req EMG zone BWD
LIFO	1	107.5	158.4	174.0	183.7	543	0	494	1
FIFO	1	107.8	158.4	2307.9	1403.7	87	3 054 879	252	3 073 355

Table 3: Influence of the scheduling of the tasks on Qimonda07. T_min is the min time to load factors from disk. EMG=**1 MB**, 1 buffer = **10 MB** per processor

- Note that FWD is slower than the BWD

- FWD step might be slower than BWD for FIFO !
  - Illustration of **FIFO** access during solve



Advantage

- User has full control of:
  - Memory Used
  - User buffers
  - Strategies to prefetch (to exploit algorithm properties)
- Stable behaviour for small as well as for large problems

Drawback

- System based cache mechanism not available
- More complex to implement
- Algorithmic effort required (asynchronous and look-ahead mechanisms)

Influence of parallelism (LIFO strategy)

- In parallel, the normal/natural order to access is not respected

	Nb of	T_factor	T_min	T_fwd	T_bwd	Nb_Req	Nb_Req	Nb_Req	Nb_Req
Strategy	Procs					1 buffer	emg zone	1 buffer	emg zone
		(sec)	(sec)	(sec)	(sec)	FWD	FWD	BWD	BWD
LIFO	2	76.0	**78.8**	90.0	97.4	267	0	240	4 520
LIFO	6	51.2	**24.3**	33.8	**365.7**	75	0	71	**426 014**
LIFO	8	42.2	**20.8**	24.7	**212.0**	61	0	32	**195 990**

Table4: Influence of the scheduling of the tasks on Qimonda07. T_min is the min time to loads factors from disk. EMG=**1 MB**, 1 buffer = **10 MB** per processor

BWD: written sequence not respected -->

poor performance and high nb of EMG calls

Concluding remarks

- Time for solve is critical
- Scheduling is also critical
- Future work:
  - Scheduling for parallel case (exploit slave task write sequence)
  - Overlapping computation and disk access in a context of multiple RHS
- *On going work* .....