Large Least-Squares problems in Meteorology

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Introduction

•The assimilation of observations at Météo-France aims at providing initial conditions for the forecast models.

• Two models are used :
-a global model on the sphere (ARPEGE) with ~25 km resolution
-a limited area model (ALADIN) with 10 km resolution

• A new model is under development (AROME) :
- with increased resolution (2.5 km) and improved physics.

• ARPEGE is associated with a 4D-Var assimilation scheme,
ALADIN with a 3D-Var assimilation scheme,
AROME will be associated with a 3D-Var scheme.

• ~10^6 observations assimilated / day in ARPEGE.

• Large spectrum of observations.

Incremental formulation in the ARPEGE 4D-Var (I)

Cost function:

$$J(\delta \mathbf{w}_k) = (\delta \mathbf{w}_k)^T \mathbf{B}^{-1} \delta \mathbf{w}_k$$
$$+ \sum_{i=0}^{n} (\mathbf{G}_i \mathbf{L}_i \delta \mathbf{w}_k^* - (\mathbf{d}_i)_{k-1})^T \mathbf{R}^{-1} (\mathbf{G}_i \mathbf{L}_i \delta \mathbf{w}_k^* - (\mathbf{d}_i)_{k-1})$$

$\delta \mathbf{w}_k$ is le « simplified » (low resolution) increment at outer loop k

and $\delta \mathbf{w}_k^* = (\mathbf{w}_k^b + \delta \mathbf{w}_k) - (\mathbf{w}_k^b + \delta \mathbf{w}_{k-1})$ with $\mathbf{w}_k^b = S_k(\mathbf{x}^b)$

$$(\mathbf{d}_i)_{k-1} = \mathbf{y}_i^o - H_i(M_i(\mathbf{x}_{k-1}))$$

Trajectory (analysis) updates:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + S_k^{-I}(\mathbf{w}_k^a = \mathbf{w}_k^b + \delta \mathbf{w}_k) - S_k^{-I}(\mathbf{w}_k^b)$$

Incremental formulation in the ARPEGE 4D-Var (II)

To pre-condition the problem, an additional change of variables is performed:

$$\chi_k = \mathbf{B}^{-1/2}\,\delta\,\mathbf{w}_k$$

CHAVAR

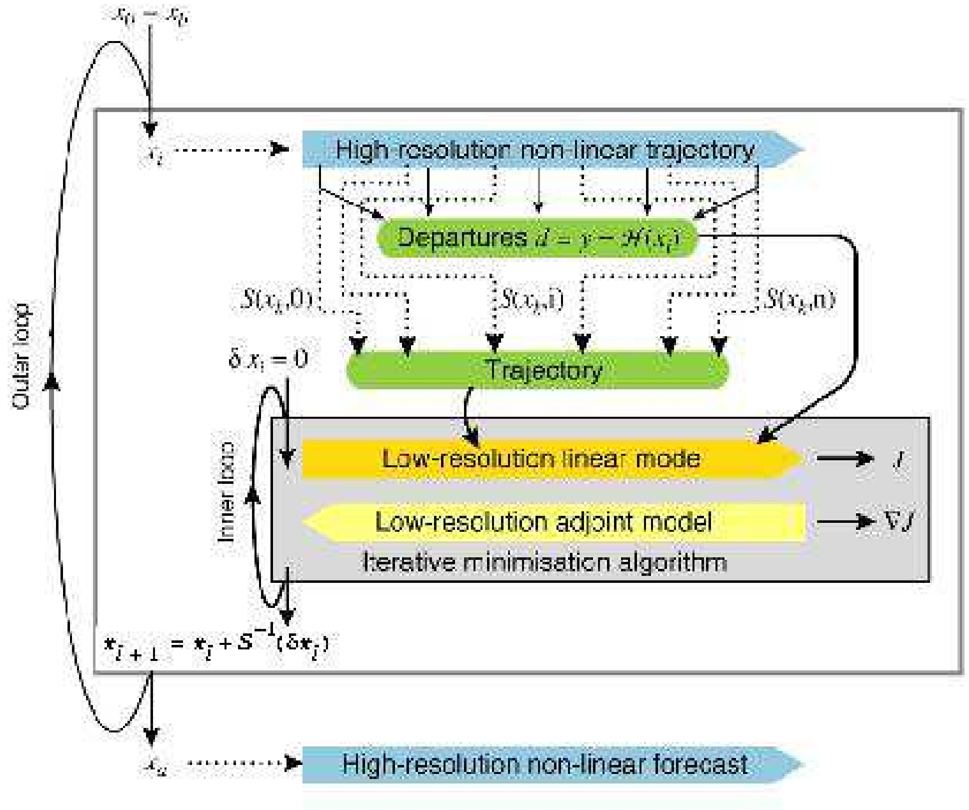
New cost function:

$$J(\chi_k) = {\chi_k}^T\,\mathbf{B}^{-1}\chi_k$$

$$+\sum_{i=0}^{n}(\mathbf{G}_i\,\mathbf{L}_i\,\mathbf{B}^{1/2}\chi_k - (\mathbf{d}_i)_{k-1})^T\,\mathbf{R}^{-1}(\mathbf{G}_i\,\mathbf{L}_i\,\mathbf{B}^{1/2}\chi_k - (\mathbf{d}_i)_{k-1})$$

CHAVARIN

Incremental 4D-Var algorithm



(ECMWF documentation)

4D-Var skeleton

Node Skeleton (4dvar.0101)

```
family 4dvar.0101
    directory YYYYMMDD
        directory HH
            directory CUTOFF
                family fc_init
                    task guess
                    task fullpos
                family obsprep
                    task bator
                    task merge
                family analyse
                    family obsscreen
                        task screening
                        task obsccma
                        task obsccmacan
                    family update_1
                        task fph2l
                        task minim
                        task fpl2hg
                        task fpl2ha
                        task traj
                    family update_2
                        task fph2l
                        task minim
                        task fpl2hg
                        task fpl2ha
                        task traj
                        task matchup
                    family surfan
                        task canari
                family fc
                    task forecast
                    task fullpos
```

Done

ARPEGE 4D-Var configuration

6-hour time window.

2 outer loops with T358 C2.4 resolution (25 km) for trajectory.

1st outer loop :
- T107 C1.0 resolution (200km)
- CONGRAD with 40 iterations
- TL/AD: vertical diffusion
- Jc-DFI

2nd outer loop :
- T149 C1.0 resolution (150 km)
- CONGRAD with 15 iterations and preconditioning
- TL/AD: vertical diffusion, gravity wave drag, large scale condensation
- Jc-DFI

Computation of Jo

$$J^o = \sum_{i=0}^{n} (\mathbf{G}_i \mathbf{L}_i \delta \mathbf{w}_k{}^* - (\mathbf{d}_i)_{k-1})^T \mathbf{R}^{-1} (\mathbf{G}_i \mathbf{L}_i \delta \mathbf{w}_k{}^* - (\mathbf{d}_i)_{k-1})$$

where

$(\mathbf{d}_i)_{k-1} = \mathbf{y}_i^o - H_i(M_i(\mathbf{x}_{k-1}))$ are the « high resolution » departures.

These high resolution departures are computed during high resolution integrations and stored in ODB files.
In practice, one computes

$$(\mathbf{z}_i)_{k-1} = (\mathbf{y}_i^o - \mathbf{G}_i \mathbf{L}_i \delta \mathbf{w}_k{}^*) - \mathbf{y}_i^o + (\mathbf{y}_i^o - H_i(M_i(\mathbf{x}_{k-1})))$$

Low Resol. depart. - Obs. + High Resol. depart.

« Warm » minimization

If NUPTRA > 0 (outer loop index k > 0), a warm start is needed.

The control variable χ_{k-1} is recovered,

and transformed back to $\delta \mathbf{w}_{k-1}$:

$$\delta \mathbf{w}_{k-1} = \mathbf{B}^{1/2} \chi_{k-1}$$

CHAVARIN

$$\mathbf{w}_k = \mathbf{w}_k^b + \delta \mathbf{w}_{k-1}$$

is the starting point of the new Low Resolution trajectory, with

$$\mathbf{w}_k^b = S_k(\mathbf{x}^b)$$

Trajectory update

Use of Full-Pos as « simplication » operator S :

- change of geometry : T358C2.4 -> T107C1.0 (T149C1.0)
- « sophisticated » interpolation.

Full-pos is also used as pseudo-inverse simplification operator S^-I.

Trajectory update:

$$\mathbf{x}_k = \mathbf{x}_{k-1} + FP_k^{-I}\left(\mathbf{w}_k^a = \mathbf{w}_k^b + \delta\,\mathbf{w}_k\right) - FP_k^{-I}\left(\mathbf{w}_k^b\right)$$

with

$$\mathbf{w}_k^b = FP_k\left(\mathbf{x}^b\right)$$

Digital Filtering Initialization (DFI) in ARPEGE 4D-Var

Use of DFI at two levels in ARPEGE 4D-Var

- Extra term Jc-DFI in the cost function:

$$J^c(\delta \mathbf{w}_0) = < \delta \mathbf{w}_{T/2} - \delta \overline{\mathbf{w}}_{T/2}, \delta \mathbf{w}_{T/2} - \delta \overline{\mathbf{w}}_{T/2} >$$

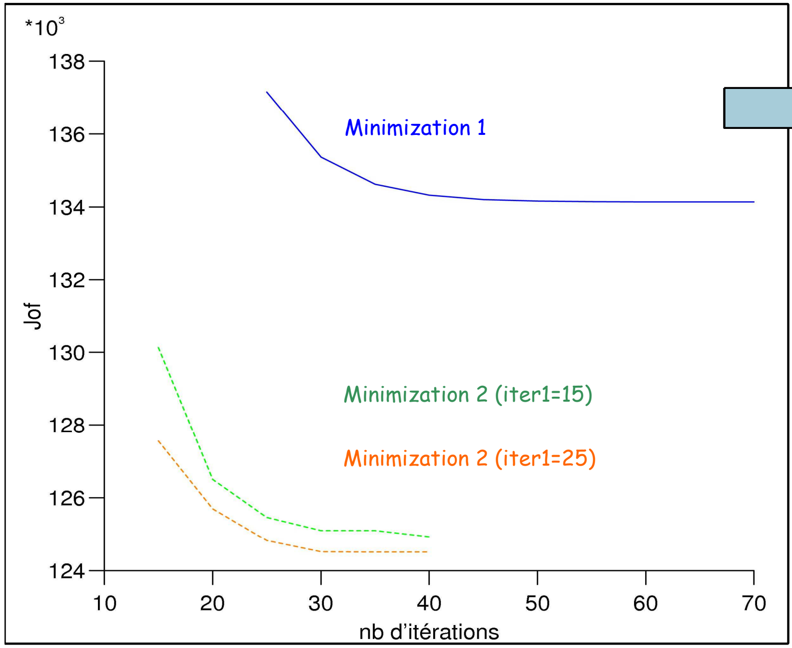
with

$$\delta \overline{\mathbf{w}}_{T/2} = \sum \alpha_i \mathbf{L}_i \delta \mathbf{w}_0$$

« Semi-external » incremental DFI initialization applied to last trajectory, in the midle of the assimilation period.

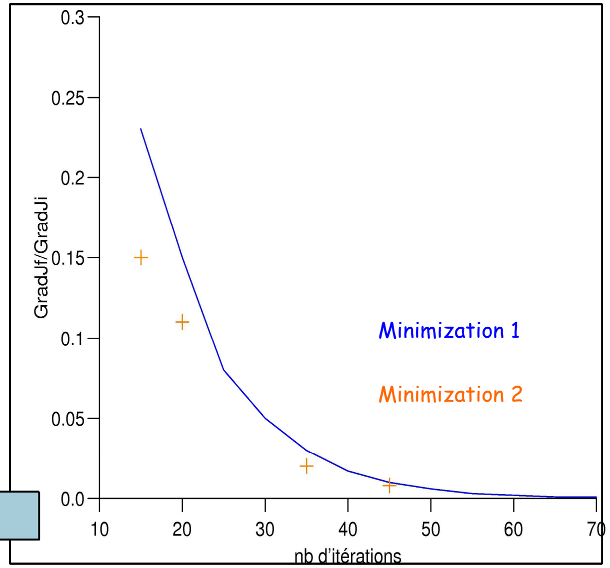
Impact of the number of iterations

- Variation from 15 to 70 of the number of iterations in the 1st minimization
- Variation from 15 to 40 of the number of iterations in the 2nd minimization



Jof=f(nb of iterations)

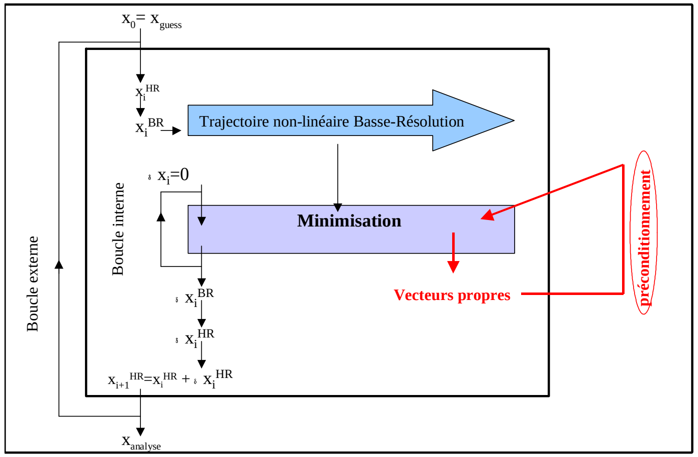
Small impact of an increase of the number of iterations beyond a given number:
- 35-40 for 1st minimization
- 25-30 for 2nd minimization.



GradJf/GradJi = f(nb of iterations)

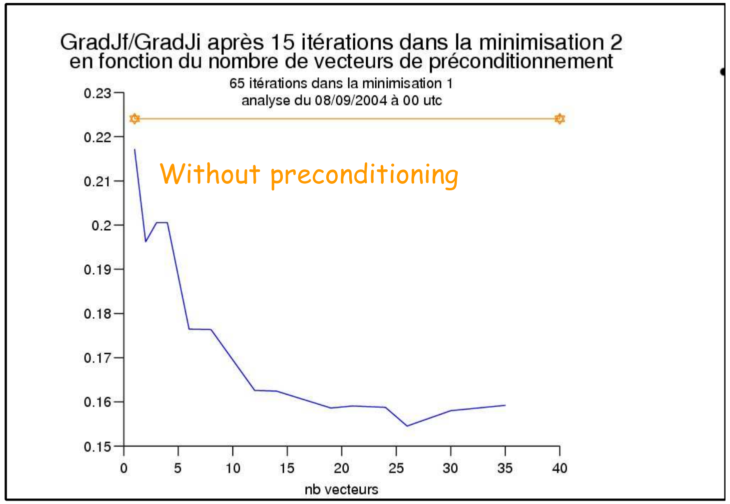
Larger efficiency of the 2nd minimization, for a given number of iterations.

Impact of preconditioning



The number of eigen vectors given by the first minimization is roughly equal to **(number of iterations /2).**

- Impact of the number of used vectors
  - 65 iterations in 1st minimization 1 → 38 eigen vectors.
  - 15 iterations in the 2nd minimization with the use of 3 to 38 eigen vectors.

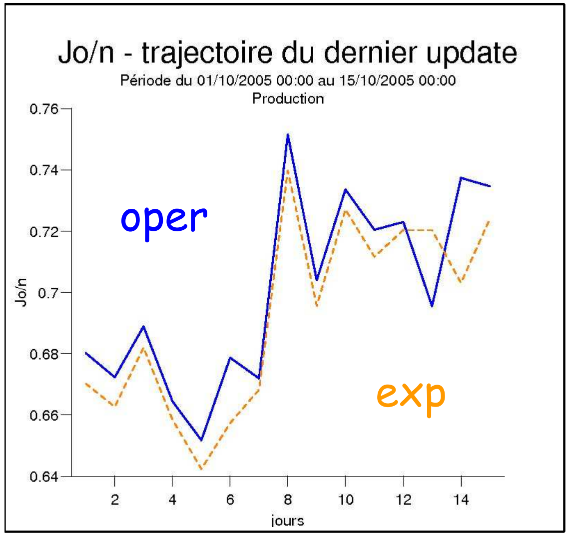


- Preconditioning improves the quality of the second minimization.
- Slight improvement beyond 10 to 12 eigen vectors used for preconditioning.

Impact of an additional outer loop

3 outer loops (instead of 2 for the operational scheme).
25 iterations for each minimization.
Resolutions: T107s1-T149rs-T149rs.
Use of the complete set of eigen vectors in the preconditioning.

Two week experiment.



Positive diagnostics in the assimilation,
but neutral impact on the scores of the forecasts!

The use of an ensemble of analyses in ARPEGE/ALADIN/AROME

(Belo Pereira and Berre 2006, Stefanescu et al 2006, Berre et al 2006,...)

- Principle of the assimilation ensemble:

Analysis state: $xa = (I-KH)\, xb + K\, y$

Analysis error: $Ea = (I-KH)\, Eb + K\, Eo$

$\Rightarrow$ The analysis error can be simulated by perturbing the information sources (obs and background), and by calculating and cycling analysis differences.

- Applications for the error covariance estimations:
  - error standard deviations and spatial correlations.
  - « climatological » and/or « flow-dependent » (in time).
  - global and/or local (in space).
  - analysis and background errors.

- Other applications and connections:
  - a posteriori diagnostics (innovation statistics).
  - observation impact.
  - ensemble prediction.

Conclusion

• Following the previous tests, we will certainly use the same number of iterations (25) in the two outer loops of the ARPEGE 4D-Var.

• An increase of the resolution of the analysis increment will also be soon tested: T149 (150km) ->T224 (100km).

• The number of satellite data should also be increased in the next future: impact on the number of iterations?

• Our main current development is concentrated on the possibility to perform an ensemble of perturbed analyses (~6) to get an information on the statistics of background errors:
is there a possibility to precondition those perturbed analyses by the un-perturbed analysis?