# Development of 4DEnVar at Météo-France

Gérald Desroziers, Etienne Arbogast
Loïk Berre, Benjamin Ménétrier
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# Outline

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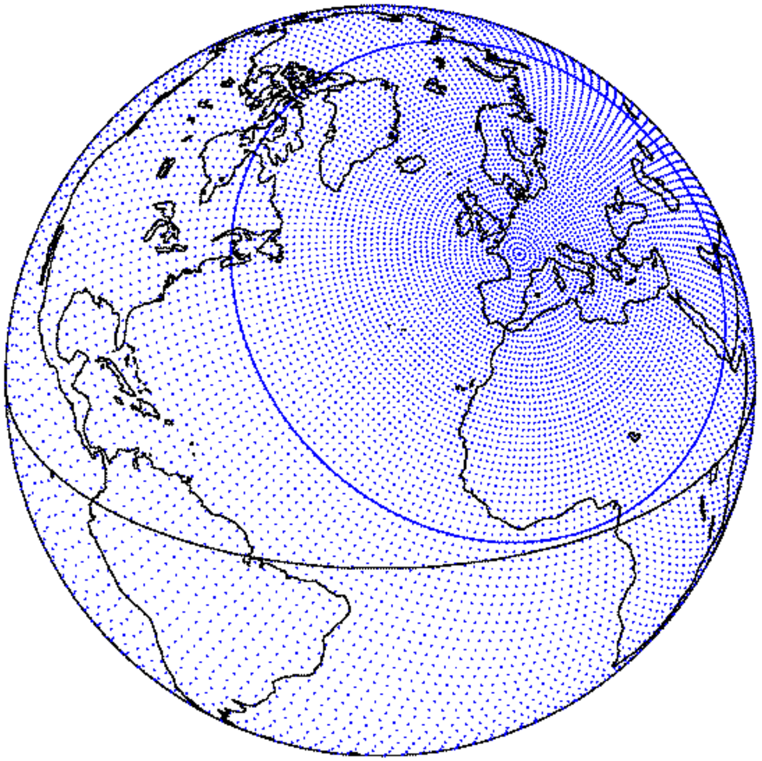
- Introduction
- 4DEnVar formulation
- Optimality-based diagnostics of localisation
- Advection of the localisation
- Hybrid formulation
- En-4DEnVar
- 4D-Var / 4DEnVar comparison
- Conclusion

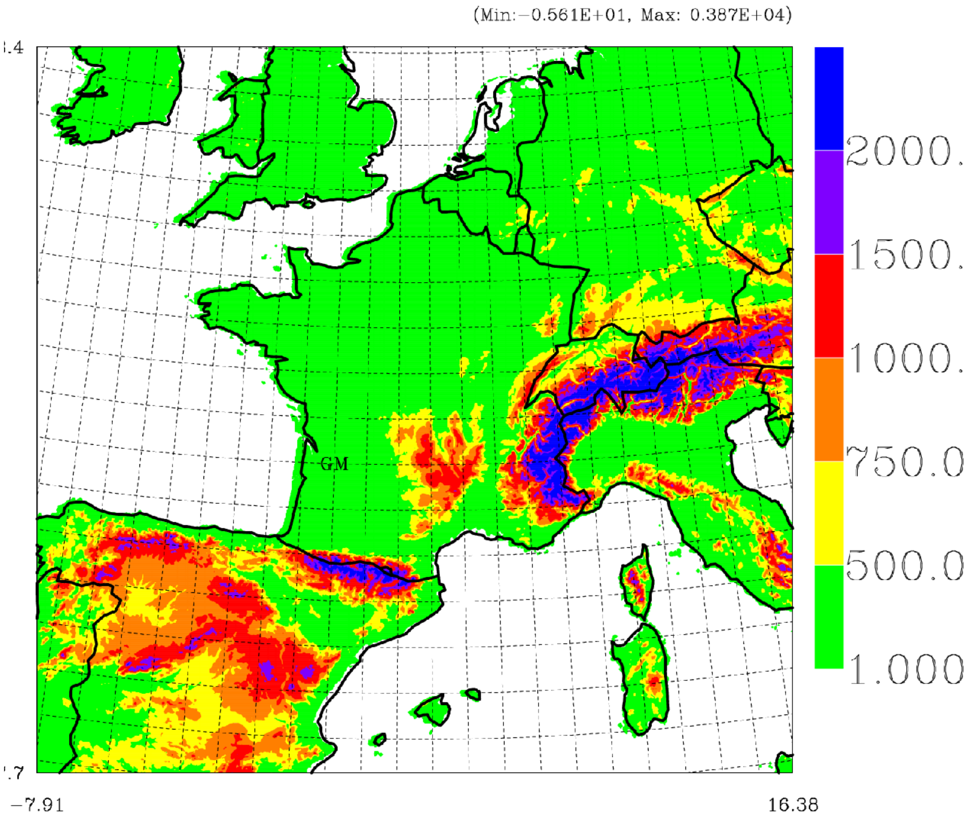
# Numerical Weather Prediction at Météo-France

Global model



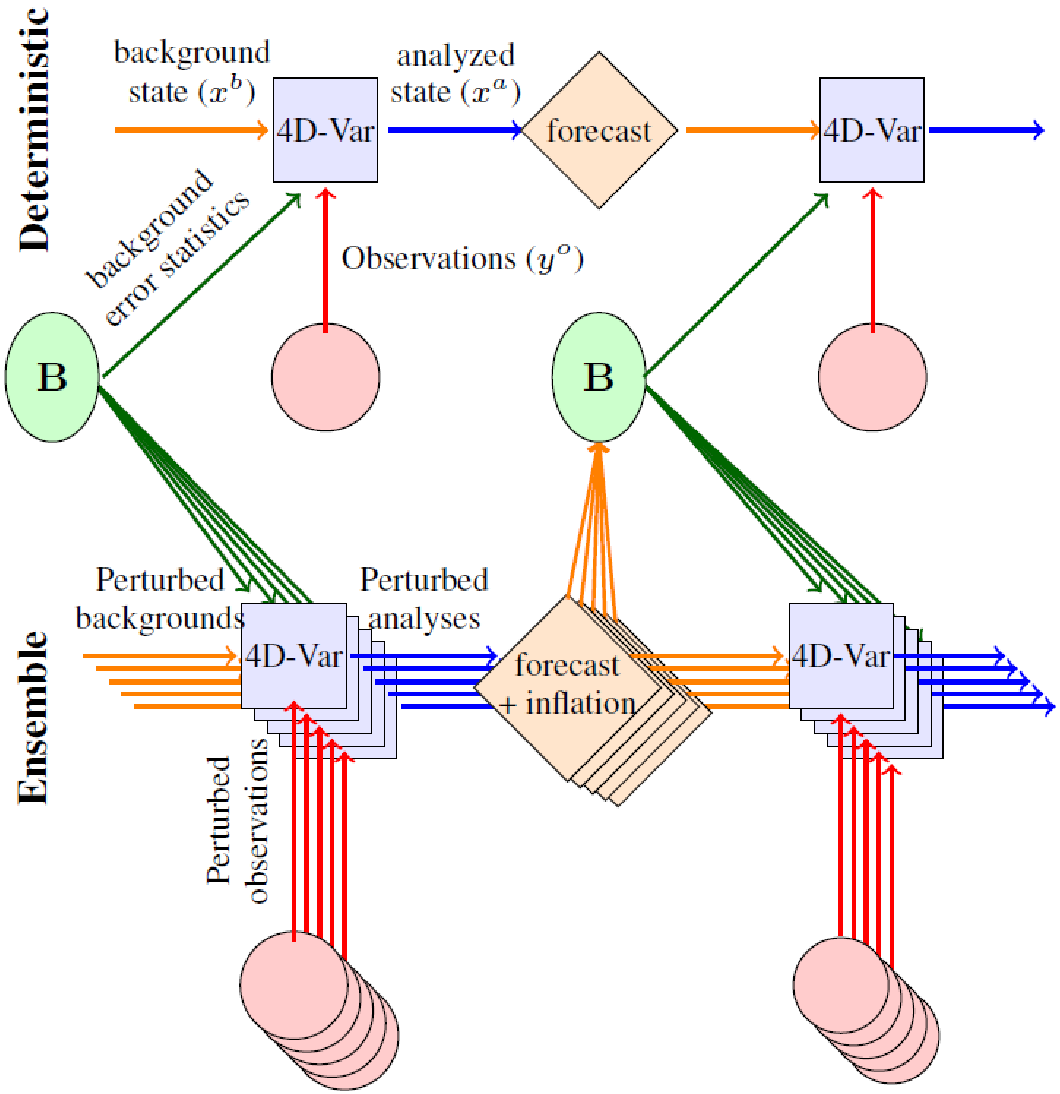
ARPEGE (7 km - 40 km)
4D-Var

High-resolution model




AROME (1.3 km)
3D-Var

# Operational global assimilation




deterministic 4D-Var

25 perturbed 4D-Var

# Operational global assimilation

- Deterministic” 4D-Var

  - 6 hour time window.

  - 2 outer loops

    T1198 C2.2 (7.5 km min) L105 / T149 (~135 km), T399 (~ 50 km).

  - Jc-DFI, VarBC.

  - $\mathbf{B}^{1/2} = \mathbf{K}^b \, \boldsymbol{\Sigma}^b \, \mathbf{C}^{1/2}$, wavelet **C**, $\mathbf{K}^b$ = spectral + non-linear balances.

- Ensemble assimilation

  - 25 perturbed 4D-Vars.

  - 1 outer loop T479 C1.0 (40 km) / T149 C1.0.

  - Multiplicative inflation of 3h forecast perturbations.

- Gives

  - filtered $\boldsymbol{\Sigma}^b$ from last 25 perturbations, updated every 6 h,

  - wavelet **C** from last 6 x 25 perturbations (last 30 h), upd. every 6 h.

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# 4DEnVar formulation
# 4D ensemble covariances

- Minimization of

  $J(\underline{\delta \mathbf{x}}) = 1/2\ \underline{\delta \mathbf{x}}^T \underline{\mathbf{B}}^{-1} \underline{\delta \mathbf{x}} + 1/2\ (\underline{\mathbf{H}}\ \underline{\delta \mathbf{x}} - \underline{\mathbf{d}})^T \underline{\mathbf{R}}^{-1} (\underline{\mathbf{H}}\ \underline{\delta \mathbf{x}} - \underline{\mathbf{d}})$, with

  $$\underline{\mathbf{B}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T},$$

  $\underline{\mathbf{X}}^{b'} = (\underline{\mathbf{x}}^{b'}_1, \ldots, \underline{\mathbf{x}}^{b'}_{Ne})$, and $\underline{\mathbf{x}}^{b'}_n = \underline{\mathbf{x}}^{b}_n - \langle \underline{\mathbf{x}}^b \rangle / (N^e-1)^{1/2}$, $n = 1, N^e$.

  $\underline{\mathbf{x}}^{b'}$ of dimension K+1 (times) x M (3D variables) x N (3D dimension).

  (*Liu et al, 2008, 2009; Buehner et al, 2010; Lorenc, 2012,*

  *Desroziers et al 2014, QJRMS*).

- Expression of $\underline{\mathbf{B}}$

$$\underline{\mathbf{B}} = \begin{pmatrix} \mathbf{B}_{0,0} & \mathbf{B}_{0,1} & & \mathbf{B}_{0,K} \\ & & & \\ & & & \\ \mathbf{B}_{K,0} & & & \mathbf{B}_{K,K} \end{pmatrix}.$$

# 4DEnVar formulation
# Particular implementation at Météo-France

- $\underline{\delta\mathbf{x}}$ as control variable *(Desroziers et al 2014)*.

- Use of a double Conjugate Gradient *(Derber et Rosati 1989)*
  (with $\mathbf{h} = \mathbf{B}\ \mathbf{g}$ computations in the minimization).

- Equivalent to $\boldsymbol{\alpha}$ control variables, related to $(\underline{\mathbf{X}}^{b'}\underline{\mathbf{X}}^{b'T} \circ \mathbf{L})^{1/2}$,
  but (slightly) different use of memory space.

- Also possible in dual space
  (with $\mathbf{h}^y = \mathbf{H}\ \mathbf{B}\ \mathbf{H}^T\ \mathbf{g}^y$ computations in the minimization).

# 4DEnVar formulation
# Particular implementation at Météo-France

Double CG : $\delta\underline{\mathbf{x}}$ size K+1 x M x N

$\delta\underline{\mathbf{x}}_0 = \mathbf{0}$

$\underline{\mathbf{g}}_0 = \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{d}}$

$\underline{\mathbf{h}}_0 = \underline{\mathbf{B}}\, \underline{\mathbf{g}}_0$

$\underline{\mathbf{d}}_{-1} = \underline{\mathbf{e}}_{-1} = \mathbf{0}$

loop over n

$\underline{\mathbf{d}}_n = \underline{\mathbf{h}}_n + \beta_{n-1} \underline{\mathbf{d}}_{n-1}$

$\underline{\mathbf{e}}_n = \underline{\mathbf{g}}_n + \beta_{n-1} \underline{\mathbf{e}}_{n-1}$

$\underline{\mathbf{f}}_n = \underline{\mathbf{e}}_n + \underline{\mathbf{H}}^T \underline{\mathbf{R}}^{-1} \underline{\mathbf{H}}\, \underline{\mathbf{d}}_n$

$\alpha_n = \underline{\mathbf{g}}_n^T \underline{\mathbf{h}}_n / \underline{\mathbf{d}}_n^T \underline{\mathbf{f}}_n$

$\delta\underline{\mathbf{x}}_{n+1} = \delta\underline{\mathbf{x}}_n + \alpha_n \underline{\mathbf{d}}_n$

$\underline{\mathbf{g}}_{n+1} = \underline{\mathbf{g}}_n - \alpha_n \underline{\mathbf{f}}_n$

$\underline{\mathbf{h}}_{n+1} = \underline{\mathbf{B}}\, \underline{\mathbf{g}}_{n+1}$

$\beta_n = \underline{\mathbf{g}}_{n+1}^T \underline{\mathbf{h}}_{n+1} / \underline{\mathbf{g}}_n^T \underline{\mathbf{h}}_n$

end loop

RPCG : $\delta\underline{\mathbf{y}}$ size P (number of observations)

$\delta\underline{\mathbf{y}}_0 = \mathbf{0}$

$\underline{\mathbf{g}}_0 = \underline{\mathbf{R}}^{-1} \underline{\mathbf{d}}$

$\underline{\mathbf{h}}_0 = \underline{\mathbf{H}}\, \underline{\mathbf{B}}\, \underline{\mathbf{H}}^T \underline{\mathbf{g}}_0$

$\underline{\mathbf{d}}_{-1} = \underline{\mathbf{e}}_{-1} = \mathbf{0}$

loop over n

$\underline{\mathbf{d}}_n = \underline{\mathbf{g}}_n + \beta_{n-1} \underline{\mathbf{d}}_{n-1}$

$\underline{\mathbf{e}}_n = \underline{\mathbf{h}}_n + \beta_{n-1} \underline{\mathbf{e}}_{n-1}$

$\underline{\mathbf{f}}_n = \underline{\mathbf{R}}^{-1} \underline{\mathbf{e}}_n + \underline{\mathbf{d}}_n$

$\alpha_n = \underline{\mathbf{g}}_n^T \underline{\mathbf{h}}_n / \underline{\mathbf{d}}_n^T \underline{\mathbf{f}}_n$

$\delta\underline{\mathbf{y}}_{n+1} = \delta\underline{\mathbf{y}}_n + \alpha_n \underline{\mathbf{d}}_n$

$\underline{\mathbf{g}}_{n+1} = \underline{\mathbf{g}}_n - \alpha_n \underline{\mathbf{f}}_n$

$\underline{\mathbf{h}}_{n+1} = \underline{\mathbf{H}}\, \underline{\mathbf{B}}\, \underline{\mathbf{H}}^T \underline{\mathbf{g}}_{n+1}$

$\beta_n = \underline{\mathbf{g}}_{n+1}^T \underline{\mathbf{h}}_{n+1} / \underline{\mathbf{g}}_n^T \underline{\mathbf{h}}_n$

end loop

$\delta\underline{\mathbf{x}} = \underline{\mathbf{B}}\, \underline{\mathbf{H}}^T \delta\underline{\mathbf{y}}$

# 4DEnVar formulation
# 4D ensemble covariances

- Simplification of the localisation

  Same **L** for all covariances between variables and times: $\underline{\mathbf{B}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{L}}$, with

$$\underline{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & \mathbf{L} & & \mathbf{L} \\ \mathbf{L} & & & \\ & & & \\ \mathbf{L} & & & \mathbf{L} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \\ \mathbf{I} \end{pmatrix} \mathbf{L} \begin{pmatrix} \mathbf{I} & \mathbf{I} & & \mathbf{I} \end{pmatrix} = \underline{\mathbf{1}}\, \mathbf{L}\, \underline{\mathbf{1}}^{T}.$$

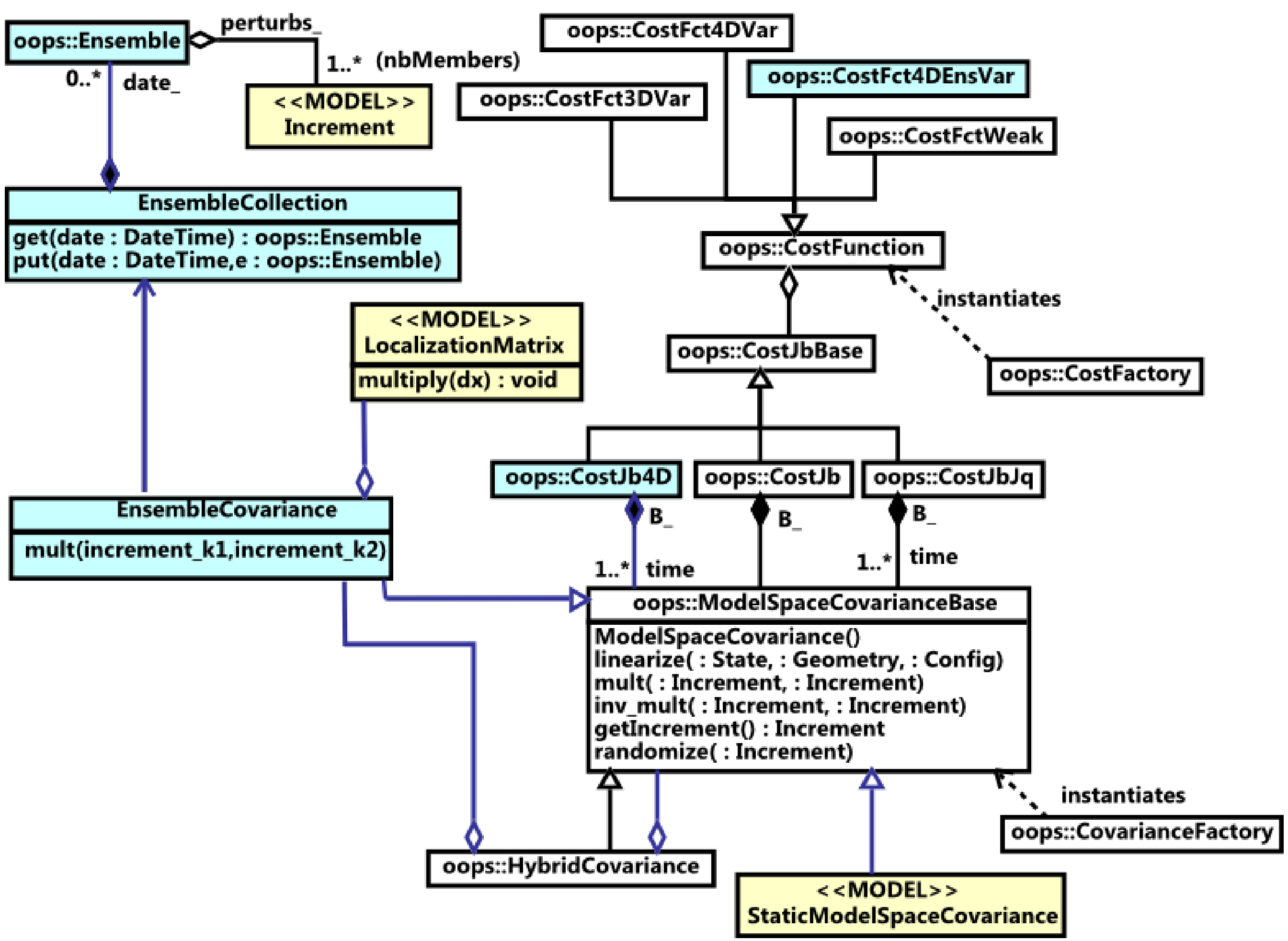
  $\underline{\mathbf{1}}$: K+1 (times) x M (variables) blocks **I**.

  **L**: N x N correlation matrix.

- Application of $\underline{\mathbf{B}}$ to a 4D gradient $\underline{\mathbf{g}}$ in the minimization

$$\begin{aligned} \underline{\mathbf{h}} &= \underline{\mathbf{B}}\, \underline{\mathbf{g}} \\ &= (\underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{L}})\, \underline{\mathbf{g}} \\ &= \Sigma_n\, \underline{\mathbf{x}}^{b'}{}_n \circ (\, \underline{\mathbf{1}}\, \mathbf{S}^{-1} \mathbf{L}^{S} \mathbf{S}^{-T} \underline{\mathbf{1}}^{T} (\, \underline{\mathbf{x}}^{b'}{}_n \circ \underline{\mathbf{g}}\, )). \end{aligned}$$

# 4DEnVar formulation
# OOPS : Object Oriented Prediction System



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# Optimality-based diagnostics of localisation
# Derivation

- Optimal localisation depending on ensemble size $N^e$

  **B** = **B** o **L,** where **B** raw ensemble covariance matrix,

  equivalent to $B_{ij} = B_{ij} L_{ij}$.

- Key idea

  minimize $e = E [ \| \mathbf{B}^t - \mathbf{B} \circ \mathbf{L} \|_F ]$.

- $\partial e / \partial L_{ij} = 0$ gives

  $L_{ij} = [Ne - 1] / [(Ne+1) \times (Ne+2)] \times [ (Ne - 1) - 1 / E[(B_{ij})^2] ]$

  (*Ménétrier et al 2015).*

# Optimality-based diagnostics of localisation
# Change of variables

- Transformation of $\psi$, $\chi$, $P_s$: better agreement between variables

$\psi$ ––> $\Delta^{1/2}\psi$

$\chi$ ––> $\Delta^{1/2}\chi$

T

q

$\ln(P_s)$ ––> $\Delta^{1/2} \ln(P_s)$

- Localisation applied to transformed perturbations

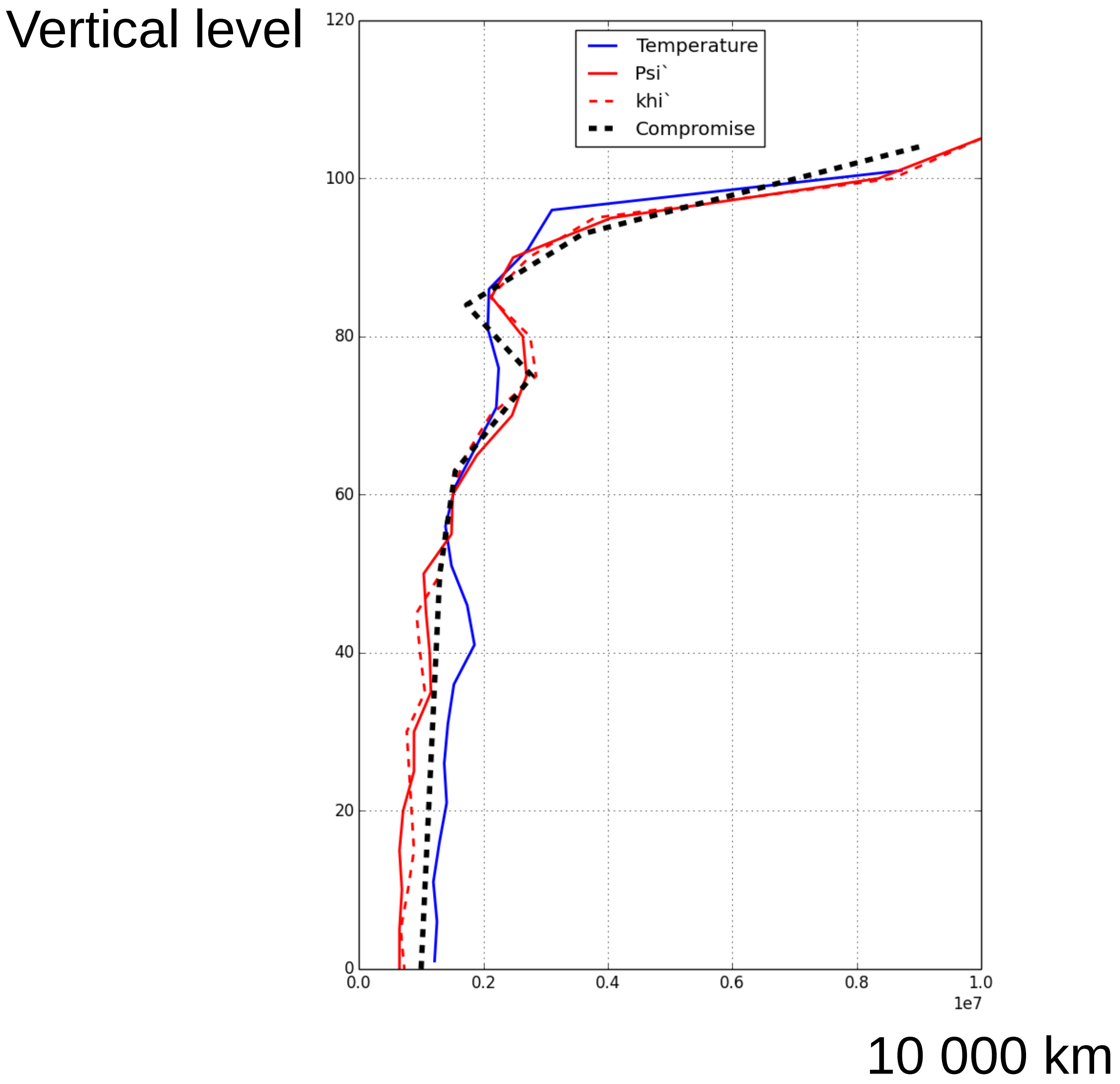
$\underline{\mathbf{B}} = \underline{\mathbf{U}}^{-1} \left( \left( \underline{\mathbf{U}}\, \underline{\mathbf{X}}^{b'} \right) \left( \underline{\mathbf{U}}\, \underline{\mathbf{X}}^{b'} \right)^T \circ \underline{\mathbf{L}} \right) \underline{\mathbf{U}}^{-T}$,

where $\underline{\mathbf{U}}$ is the change of variables.

(*Berre et al 2017*).

# Optimality-based diagnostics of localisation
# Horizontal length-scales

$N^e = 200$



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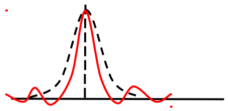
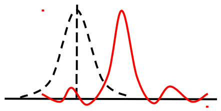
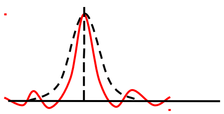
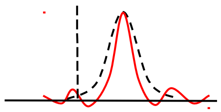
# Advection of the localisation

$$\underline{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & \mathbf{L} & & \mathbf{L} \\ \mathbf{L} & & & \\ & & & \\ \mathbf{L} & & & \mathbf{L} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \\ \mathbf{I} \end{pmatrix} \mathbf{L} \, (\, \mathbf{I} \;\; \mathbf{I} \qquad \mathbf{I} \,) = \underline{\mathbf{1}} \, \mathbf{L} \, \underline{\mathbf{1}}^{T} \qquad \text{Static } \underline{\mathbf{L}}$$

$$\underline{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & \mathbf{L}\mathbf{A}_1^{T} & & \mathbf{L}\mathbf{A}_K^{T} \\ \mathbf{A}_1\mathbf{L} & & & \\ & & & \\ \mathbf{A}_K\mathbf{L} & & & \mathbf{A}_K\mathbf{L}\mathbf{A}_K^{T} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{A}_1 \\ \\ \mathbf{A}_K \end{pmatrix} \mathbf{L} \, (\, \mathbf{I} \;\; \mathbf{A}_1^{T} \qquad \mathbf{A}_K^{T} \,) = \underline{\mathbf{A}} \, \mathbf{L} \, \underline{\mathbf{A}}^{T} \qquad \text{Advected } \underline{\mathbf{L}}$$

Localisation applied to advected perturbations

$\underline{\mathbf{A}} \, ( \, ( \, \underline{\mathbf{A}}^{-1} \, \underline{\mathbf{X}}^{b'} ) \, ( \, \underline{\mathbf{A}}^{-1} \, \underline{\mathbf{X}}^{b'} )^{T} \circ \underline{\mathbf{L}} \, ) \, \underline{\mathbf{A}}^{T}$ (*Desroziers et al 2016*).



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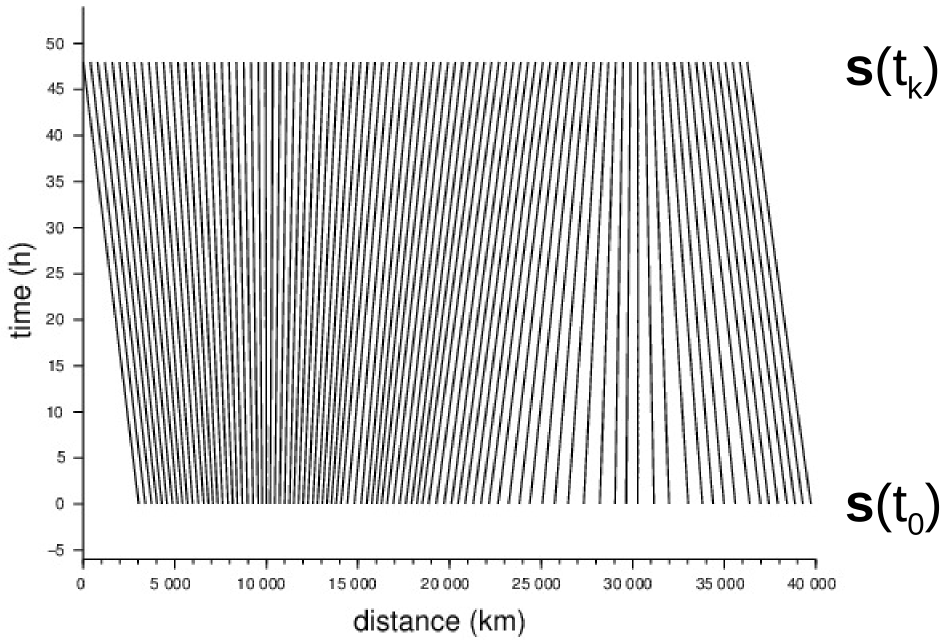
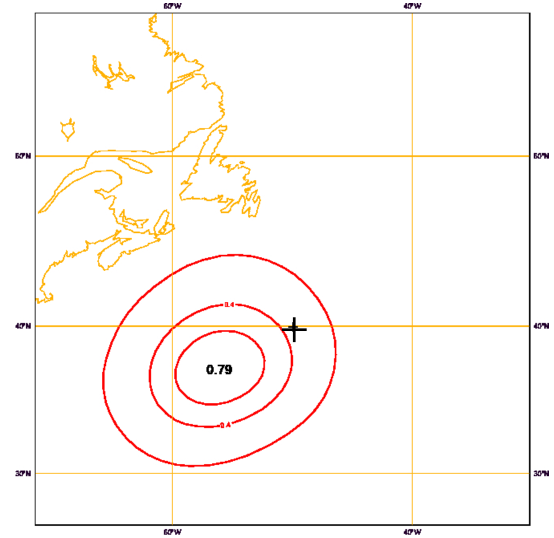
# Advection of the localisation

$$\mathbf{A}_k \alpha(\mathbf{s}, t_0) = \alpha(\mathbf{s}, t_k)$$
$$= \alpha(\mathbf{s}(t_0), t_0)$$
$$= I^{nterpol}(\alpha(\mathbf{s}, t_0), \mathbf{s}(t_0)),$$

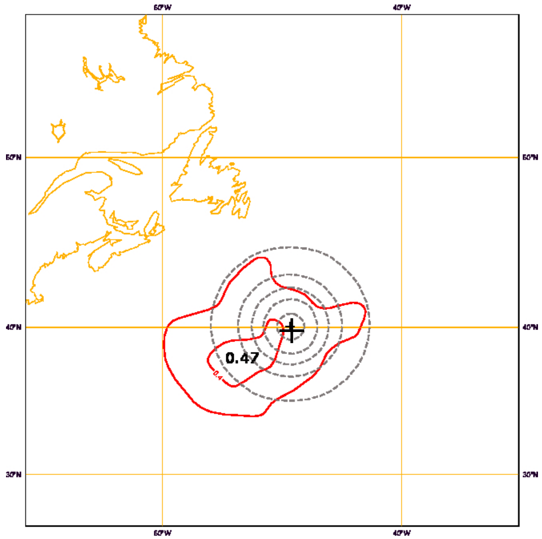
$\mathbf{s}(t_0)$ irregular image at $t_0$ of the regular grid $\mathbf{s}$ at $t_k$.



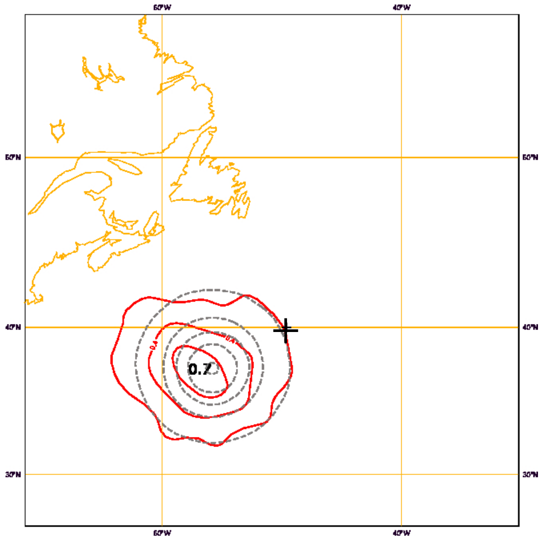
# Advection of the localisation
# T increment at $t_0$ with 1 obs at $t_f$ (@10km)




4D-Var




4DEnVar ($N^e$ = 200)




4DEnVar + advected localisation

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# Hybrid formulation
# Lagrangian advection

- Hybrid matrix: $\underline{\mathbf{B}}^{h} = \gamma^{s2}\, \underline{\mathbf{B}}^{s} + (1-\gamma^{s2})\, \underline{\mathbf{B}}^{e}$.

- Static $\underline{\mathbf{B}}^{s}$

$$\underline{\mathbf{B}}^{s} = \begin{pmatrix} \mathbf{B}^{s} & \mathbf{B}^{s} & & \mathbf{B}^{s} \\ \mathbf{B}^{s} & & & \\ & & & \\ \mathbf{B}^{s} & & & \mathbf{B}^{s} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \\ \mathbf{I} \end{pmatrix} \mathbf{B}^{s} \begin{pmatrix} \mathbf{I} & \mathbf{I} & & \mathbf{I} \end{pmatrix} = \underline{\mathbf{1}}\, \mathbf{B}^{s}\, \underline{\mathbf{1}}^{T} .$$

- Advected $\underline{B}^{s}$

$$\underline{\mathbf{B}}^{s} = \begin{pmatrix} \mathbf{B}^{s} & \mathbf{B}^{s}\mathbf{A}_{1}^{T} & & \mathbf{B}^{s}\mathbf{A}_{K}^{T} \\ \mathbf{A}_{1}\mathbf{B}^{s} & & & \\ & & & \\ \mathbf{A}_{K}\mathbf{B}^{s} & & & \mathbf{A}_{K}\mathbf{B}^{s}\mathbf{A}_{K}^{T} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{A}_{1} \\ \\ \mathbf{A}_{K} \end{pmatrix} \mathbf{B}^{s} \begin{pmatrix} \mathbf{I} & \mathbf{A}_{1}^{T} & & \mathbf{A}_{K}^{T} \end{pmatrix} = \underline{\mathbf{A}}\, \mathbf{B}^{s}\, \underline{\mathbf{A}}^{T} .$$

*(Desroziers et al 2016)*



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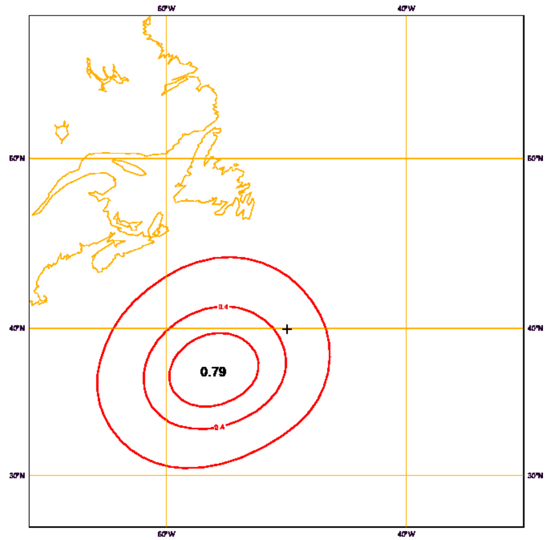
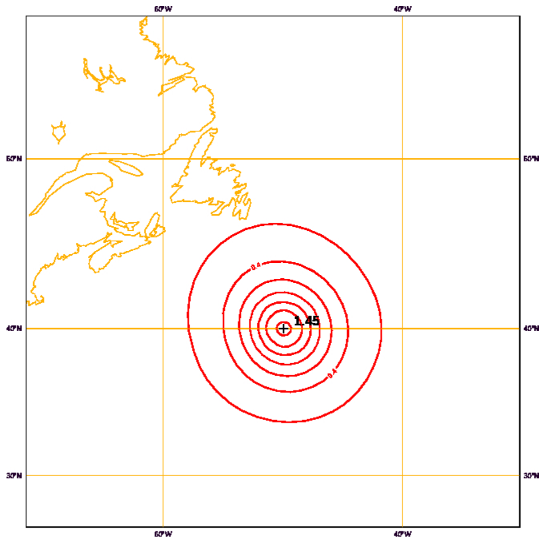
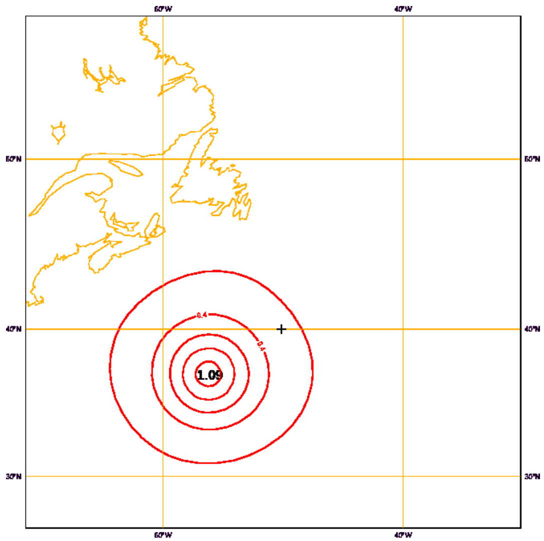
# Hybrid formulation : Lagrangian advection T increment at $t_0$ with 1 obs at $t_f$ (@10km)




4D-Var




4DEnVar with static $B^s$




4DEnVar with advected $B^s$

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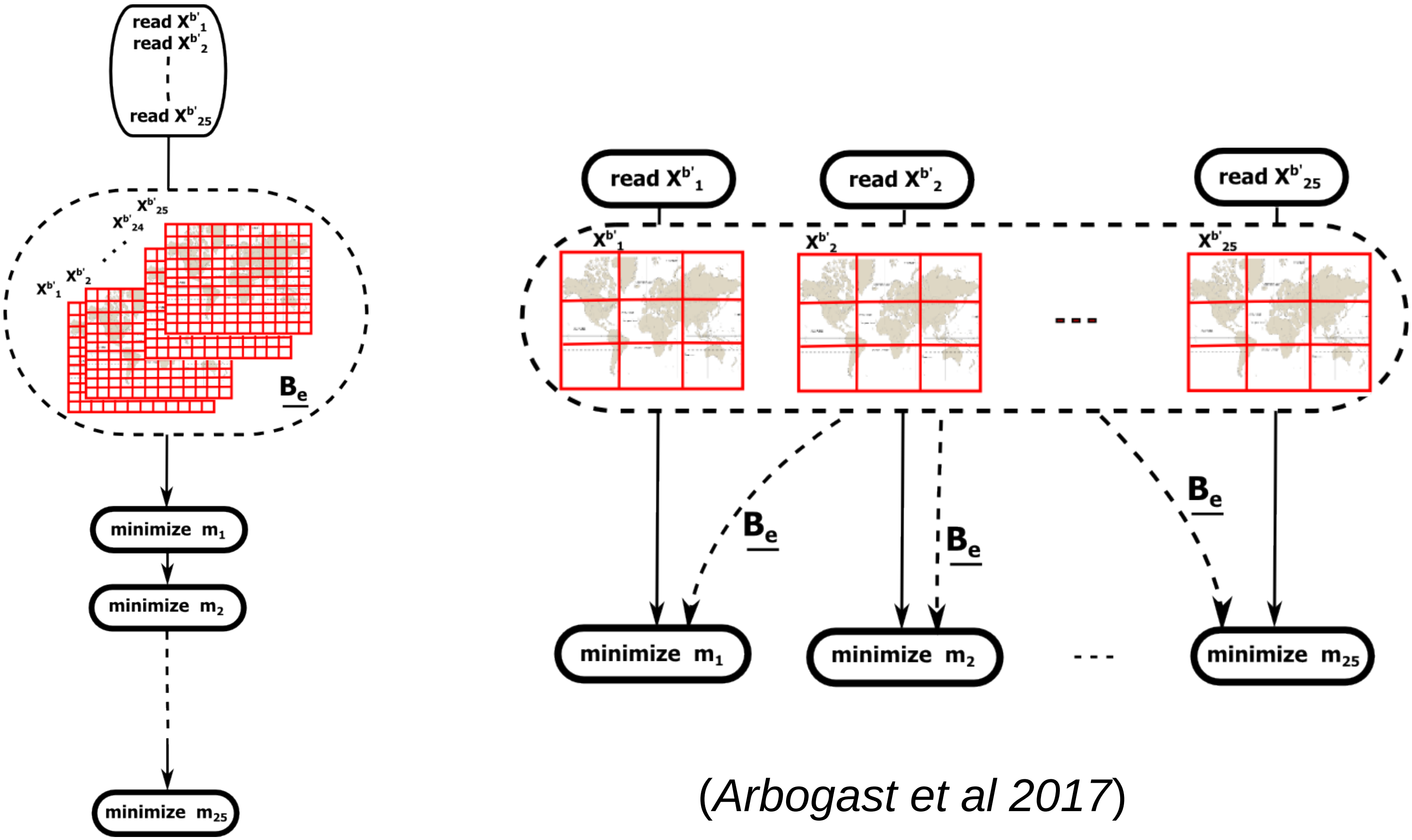
# Ensemble of 4DEnVar

Number of MPI tasks

$N^{MPI}$ | $N^{MPI}/N^{e}$ | $N^{MPI}/N^{e}$ | … | $N^{MPI}/N^{e}$

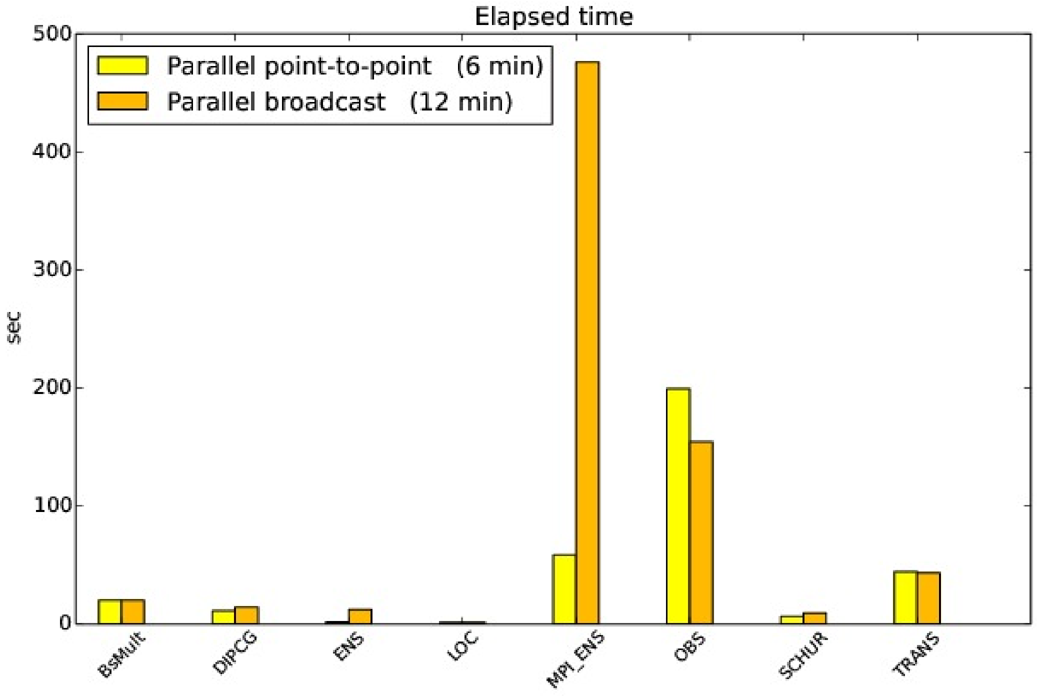



(*Arbogast et al 2017*)

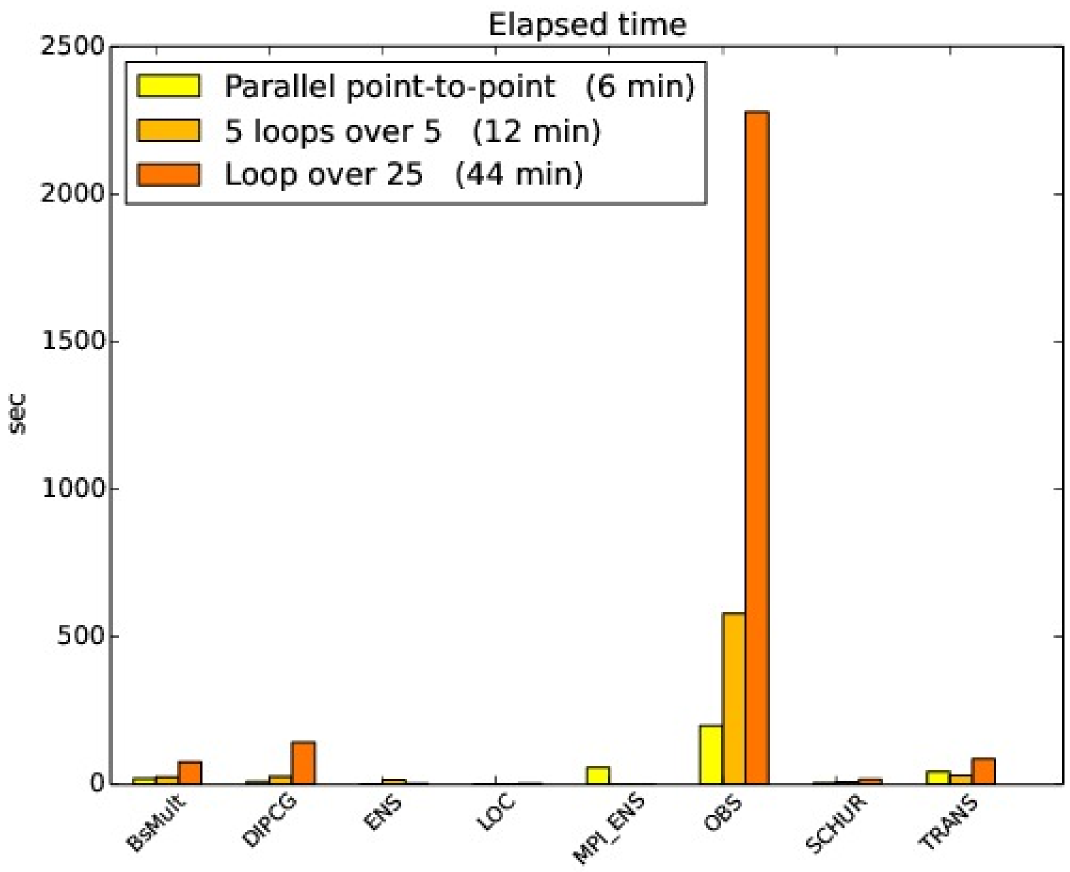
# Ensemble of 4DEnVar Communication strategy

$\underline{\mathbf{h}} = \underline{\mathbf{B}}\ \underline{\mathbf{g}}$ ,
for a given minimization m :

$\underline{\mathbf{h}} = \underline{\mathbf{0}}$
for $n = 1, N^e$
  $m' = [(N^e - m + n) \bmod N^e] + 1$
  if $m' \neq m$
    get $\underline{\mathbf{x}}^{b'}_{m'}$
  end if
  $\underline{\mathbf{h}} = \underline{\mathbf{h}} + \underline{\mathbf{x}}^{b'}_{m'} \circ ( \underline{\mathbf{1}}\, \mathrm{L}\, \underline{\mathbf{1}}^T ( \underline{\mathbf{x}}^{b'}_{m'} \circ \underline{\mathbf{g}} ))$.
end



# Ensemble of 4DEnVar
# Sequential / parallel



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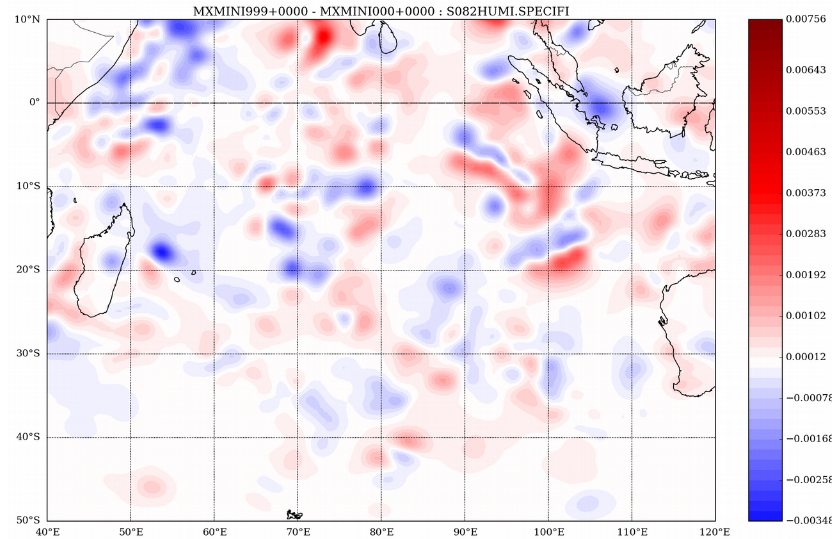
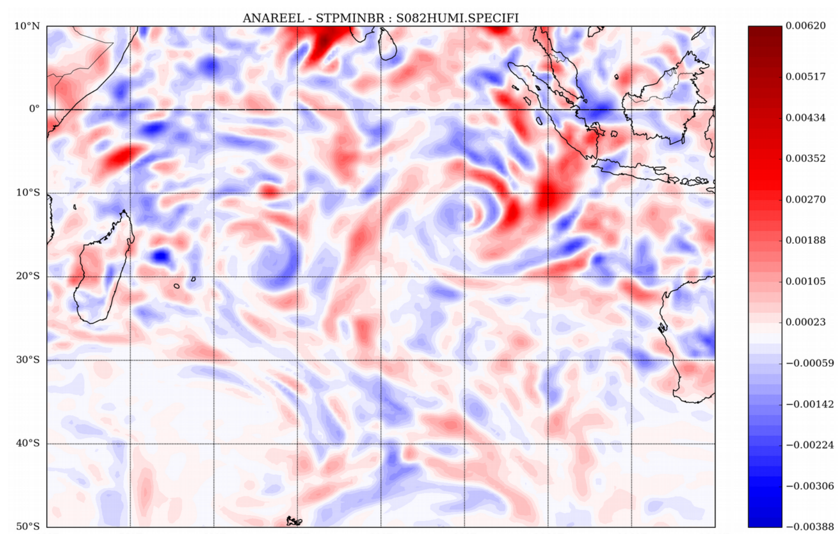
# 4D-Var / 4DEnVar
# Specific humidity increment at $t_0$ (1500m)

4D-Var




4DEnVar



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# Conclusion and plans

- First version of 4DEnVar at Météo-France with innovative options:

  - $\delta$**x** formulation (*Desroziers et al 2014*).

  - Optimality-based localisation and hybridization (*Ménétrier et al 2015, Ménétrier and Auligné 2016*).

  - Change of variable (*Berre et al 2017*).

  - Advected localisation (*Desroziers et al 2016*).

  - Parallelized En-4DEnVar (*Arbogast et al 2017*).

- Other developments

  - Parallel input of perturbations (*Arbogast*).

  - 4D-IAU (*Arbogast*).

  - Scale dependent localisation (*Caron*).

  - Optimal displacement of localisation (*Ménétrier*).

  - Localisation by convolution on a low resolution grid (*Ménétrier*).



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# Conclusion and plans

- Progress of the OOPS project: all observation operators available.

- Test of different configurations of 4DEnVar (target date 2020)
  - Perturbations given, at first, by the ensemble of 4D-Var (50/100/200?).
  - Combine Lagrangian advection and optimal displacement (also for matrix $\mathbf{B}^s$)
  - Change of variables.
  - Optimality-based diagnostics of localisation and hybridization.
  - Scale-dependent localisation and/or time-lagged perturbations.
  - Use of 4D increments: 4D-IAU, outer loop, ...
  - Scalability of 4DEnVar versus 4D-Var.

- Hybrid 4D-Var, as a possible intermediate step, when TL/AD in OOPS
  - Cooperation with ECMWF.

- Parallel development of fine scale AROME 3D/4DEnVar.