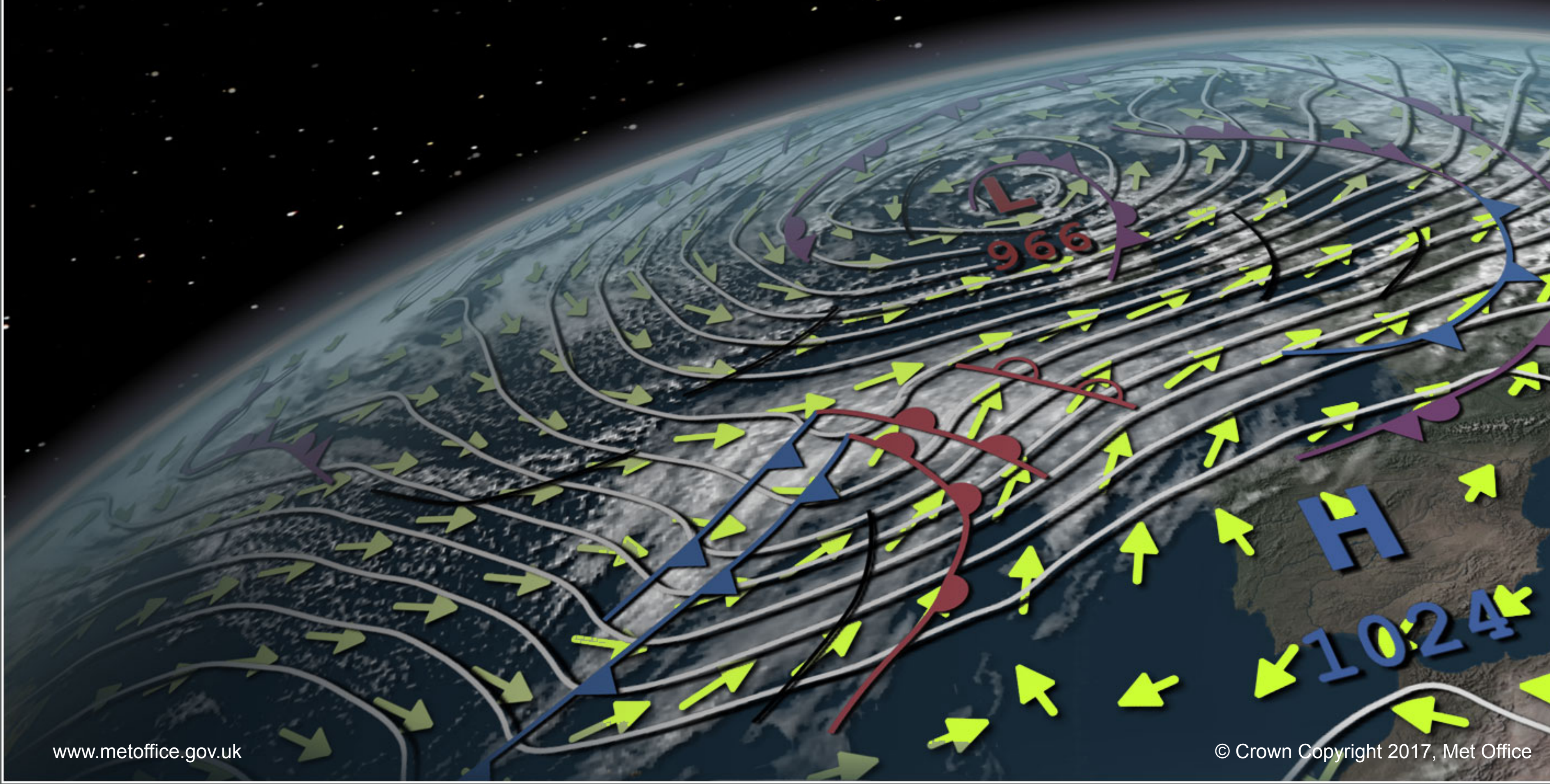
Improving the Ensemble Covariances used in hybrid-4DVar & 4DEnVar.

Andrew Lorenc, *Mohamed Jardak, and others*

CERFACS, Toulouse. June 2017.



Content of talk

- Hybrid-4DVar & 4DEnVar
- Current Met Office strategy ⇒ this work.
- Exploratory toy & hybrid-4DEnVar experiments (Lorenc 2017)
- Choosing settings
  - Ménétrier diagnostics
  - Tuning experiments
- Results from hybrid-4DVar trials currently in progress
- Provisional Summary and Perspectives

Four-Dimensional Variational DA Methods

Ours are **incremental**, with **no outer-loop** and **no model error terms**.

Each gives a 4D best-fit to prior & observations in a 6 hour window.

Use underline to denote 4D variables and operators:

$\underline{\mathbf{x}}^b$	background trajectory
$\underline{\mathbf{P}}$	4D error covariance of $\underline{\mathbf{x}}^b$
$\delta\underline{\mathbf{x}}$	4D analysis increment
$\underline{\mathbf{y}} = \underline{H}\left(\underline{\mathbf{x}}^b + \delta\underline{\mathbf{x}}\right)$	model estimate of obs
$J\left(\delta\underline{\mathbf{x}}\right) = \frac{1}{2}\delta\underline{\mathbf{x}}^T\underline{\mathbf{P}}^{-1}\delta\underline{\mathbf{x}} + \frac{1}{2}\left(\underline{\mathbf{y}} - \underline{\mathbf{y}}^o\right)^T\underline{\mathbf{R}}^{-1}\left(\underline{\mathbf{y}} - \underline{\mathbf{y}}^o\right)$	penalty function

4DVar uses linear model $\delta\underline{\mathbf{x}} = \underline{\mathbf{M}}\delta\mathbf{x}_0$

4DEnVar uses a linear combination of perturbation trajectories $\delta\underline{\mathbf{x}} = \Sigma_{k=1}^{N}\underline{\boldsymbol{\alpha}}_k \circ \underline{\mathbf{x}}'_k$

hybrid $\Rightarrow$ a combination of climatological and localized ensemble covariances.

hybrid-4DVar

4D analysis increment $\delta\underline{\mathbf{x}} = \underline{\mathbf{M}}\left(\beta_c \mathbf{U}\mathbf{v}^c + \beta_e \sum_{k=1}^{N} \mathbf{U}^\alpha \mathbf{v}_k^\alpha \circ \mathbf{x}'_k\right)$

Localised 4D covariance $\underline{\mathbf{P}} = \underline{\mathbf{M}}\left(\beta_c^2 \mathbf{B} + \beta_e^2 \mathbf{C} \circ \mathbf{X}\mathbf{X}^T\right)\underline{\mathbf{M}}^T$

concatenated control vectors $\mathbf{v}^T = \left[\mathbf{v}^{cT}, \mathbf{v}_1^{\alpha T} \cdots \mathbf{v}_N^{\alpha T}\right]$

Gave a 1% improvement in rms errors when implemented at Met Office (Clayton *et al.*, 2013).

Uses extended control variable;
– a non-quadratic J prevents us easily using other minimisation algorithms.
Transforms ensemble $\mathbf{X}$ to variables used in balance transform: Ψ, X, ${}^aP$, μ.

hybrid-4DEnVar

4D analysis increment $\delta\underline{\mathbf{x}} = \beta_c \underline{\mathbf{I}} \delta\mathbf{x}_0 + \beta_e \Sigma_{k=1}^{N} \underline{\boldsymbol{\alpha}}_k \circ \underline{\mathbf{x}}'_k$

Localized 4D covariance $\underline{\mathbf{P}} = \beta_c^2 \underline{\mathbf{I}}\mathbf{B}\underline{\mathbf{I}}^T + \beta_e^2 \underline{\mathbf{C}} \circ \underline{\mathbf{X}\mathbf{X}}^T$

hybrid-4DVar

4D analysis increment $\delta\underline{\mathbf{x}} = \underline{\mathbf{M}} \left(\beta_c \delta\mathbf{x}_0 + \beta_e \Sigma_{k=1}^{N} \boldsymbol{\alpha}_k \circ \mathbf{x}'_k \right)$

Localized 4D covariance $\underline{\mathbf{P}} = \underline{\mathbf{M}} \left(\beta_c^2 \mathbf{B} + \beta_e^2 \mathbf{C} \circ \mathbf{X}\mathbf{X}^T \right) \underline{\mathbf{M}}^T$

Met Office trial of 4DEnVar

Lorenc et al. (2015)

hybrid-4DVar
3.6% better

hybrid-4DEnVar
0.5% better

hybrid-3DVar = hybrid-3DEnVar

- We expected hybrid-3DVar and hybrid-3DEnVar to perform equally.
- Adding the time-dimension in hybrid-4DVar made a big improvement,
- whereas adding it in hybrid-4DEnVar gave much less benefit.

Why?

1. $\mathbf{B}^c$ was given large weight (80%) and treated differently: propagated by model in 4DVar, persisted in hybrid-4DEnVar.
2. Non-propagation of localisation.
3. Initialisation: 4DVar's J_c ensured that its increment was a balanced trajectory of the PF model.
4. Insufficient degrees of freedom: more are needed before increasing weight on $\mathbf{B}^e$, *even more* for $\underline{\mathbf{B}}^e$.
5. Something else?
   Why did hybrid-4DVar beat hybrid-3DVar by so much?

Met Office R&D Strategy for global DA

- Concentrate on improving ensemble covariances
- Implement these first in hybrid-4DVar
  - only later will we revisit 4DEnVar option.

1. Bigger & better ensemble generation using En-4DEnVar (Bowler et al. 2017)
2. Consistent climatological $\mathbf{B}_c$ & related diagnostic studies (Wlasak, *work in progress*)
3. **Improving diagnosed ensemble covariances** (Lorenc 2017)

Exploratory experiments

Poster at ISDA Reading 2016: "Generalised Localisation - Improving ensemble-based covariance estimates for use in hybrid variational assimilation"

Lorenc (2017): Improving ensemble covariances in hybrid-variational data assimilation, without increasing ensemble size. *Q.J.R.Meteorol.Soc.* **143**, 1062-1072, doi:10.1002/qj.2990

- Tested, in a simple toy problem, methods which had been proposed in literature: Horizontal localisation, Spectral Localisation using Wavebands, Scale-dependent localisation, Variance filtering, Hybridisation.
- Tested in hybrid-4DEnVar trials a combination of these, plus use of time-lagged and time-shifted ensemble perturbations.

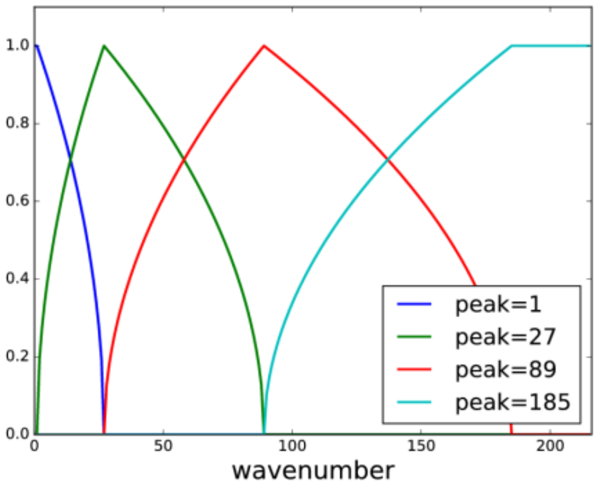
Met Office

Improved processing of the ensemble data

Waveband localisation

- Suggested by Buehner and Charron (2207), Buehner (2012)
  - Correlations decrease with distance between horizontal wavenumbers, so split ensemble error modes into wavebands and assume they are uncorrelated.
  - Apply shorter localisation scales to shorter-scale bands.
  - Performance improvement in hybrid-4DEnVar with 48 members, similar to improvement from doubling the ensemble size to 96 members.Suggested BY

1. Split ensemble perturbations into horizontal spectral wavebands:



2. Localise each band with a suitable spatial localisation scale.

For example:

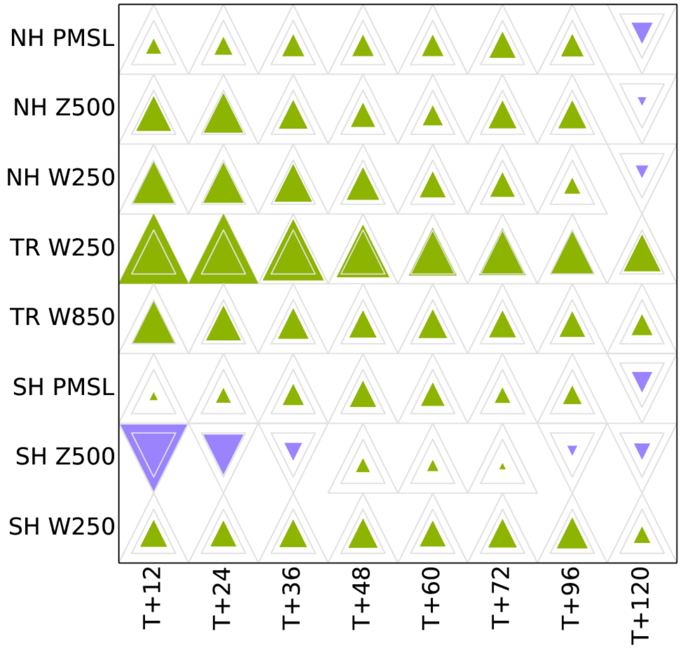
Band: Scale (km)

1: 8115
2: 665
3: 230
4: 120

Improved processing of the ensemble data

Waveband localisation

- Good results in low-res hybrid-4DEnVar experiments.*



% change in RMSE vs. ECMWF analyses

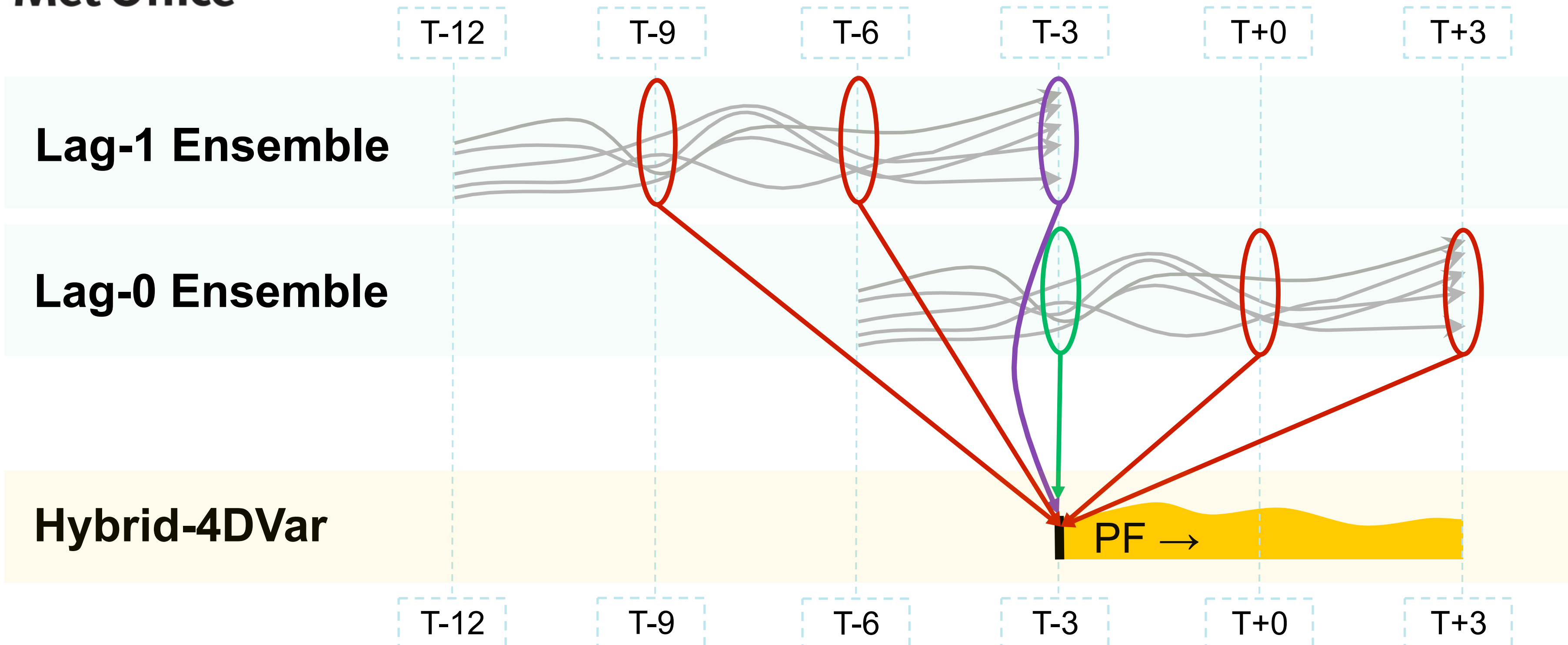
Change ∝ area of triangle

: at least 5% better

* Details in Lorenc (2017).

Improved processing of the ensemble data

Time-lagging and time-shifting



Current standard use of the ensemble.

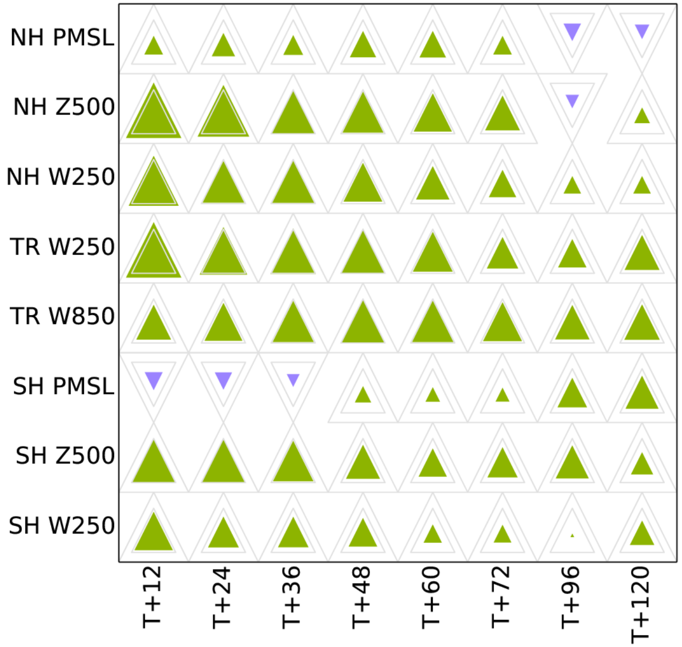
Time-lagging: Add perturbations with longer lead-times, but correct validity times.

Time-shifting: Add perturbations that are displaced in time.
(Equivalent to a smoothing in time.)

Improved processing of the ensemble data

Time-lagging and time-shifting

- Good results in low-res hybrid-4DEnVar experiments.*



% change in RMSE vs. ECMWF analyses

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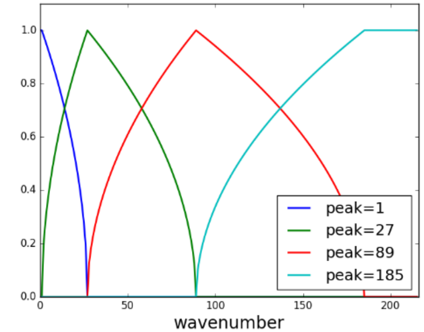
Testing & Parameter tuning of an upgrade to operational hybrid-4DVar

- Number, size and shape of wavebands.
- Number of time-lags & time-shifts to use.
- Horizontal localisation scales for each waveband.
- Truncated eigen-expansion of vertical localisation matrix.
- Hybrid weights in $\mathbf{B} = \beta_c{}^2\mathbf{B}_c + \beta_e{}^2\mathbf{B}_e$

Cost (especially time in critical path) may be an issue.

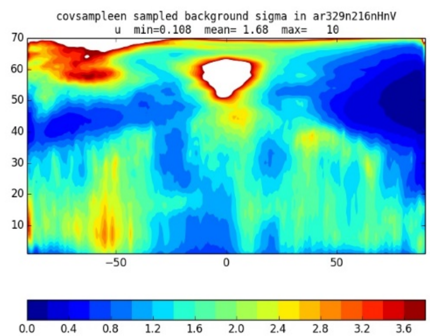
Explore using the hybrid-diag method of Menetrier *et al.* (2015)

Wavebands copied from Lorenc (2017).

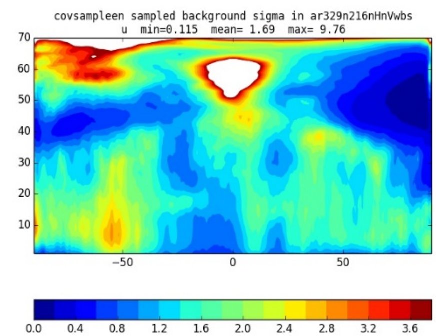


RMS zonal mean X-sections of **u'**

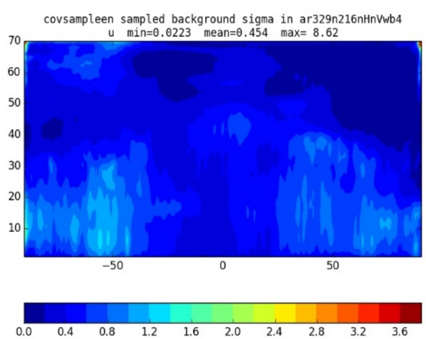
For a randomly chosen date in June



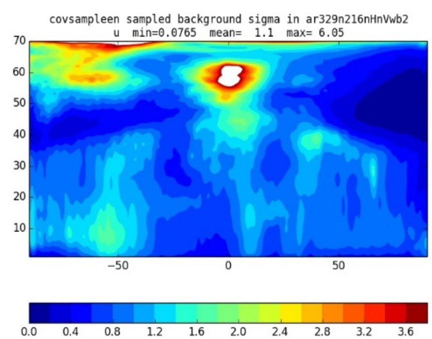
Raw ensemble



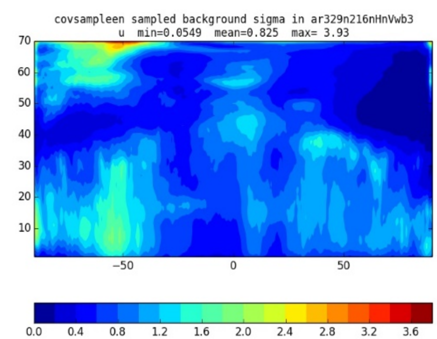
Sum of wavebands



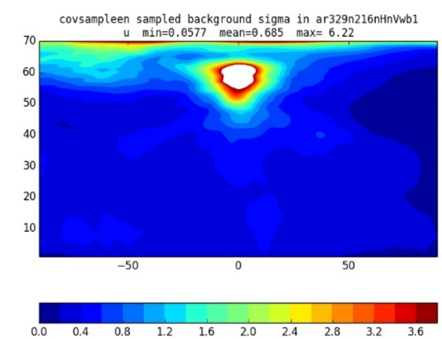
waveband 1



waveband 2

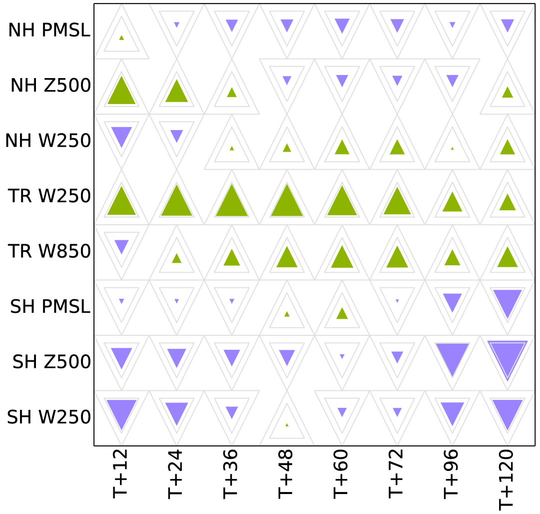


waveband 3

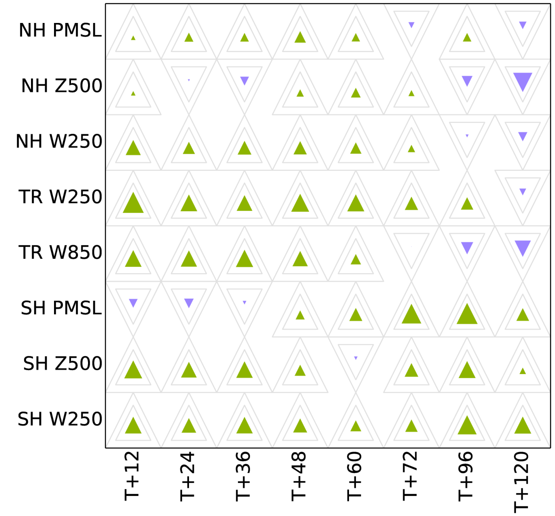


waveband 4

Aside: Wavebands & scale-dependent localisation can help with some tuning!



Changing localisation scale (from 600km to 800km) has mixed benefit when not using wavebands.



Changing all localisation scales (by factor 5/6) has consistent benefit when using wavebands with scale dependent localisation.

⇒ The regional variations in the split between wavebands take care of many regional variations in optimal tuning. (Lorenc 2017)

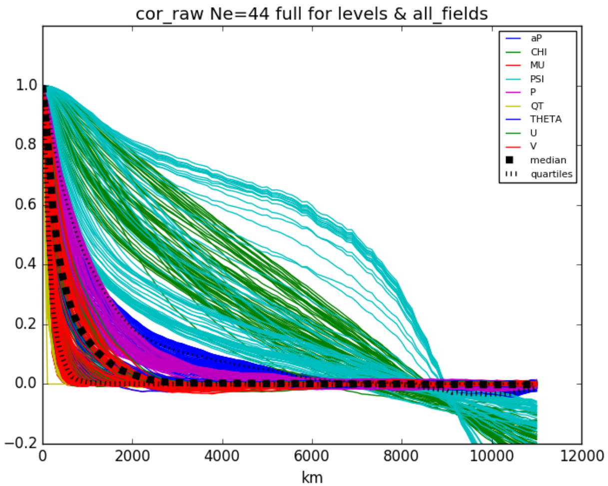
Ménétrier diagnostics

- I used hybrid_diag_20160804 (not now the latest)
  from https://opensource.cnrm-game-meteo.fr/projects/hybrid_diag
- Diagnoses optimal filtering of ensemble covariances, to give best fit (Frobenius norm) to the covariance from a hypothetical infinite ensemble.
  - Frobenius norm "best" may not give best analysis.
    E.g. We localise $\Psi, X,$ but want best analysis of their gradients u, v.
  - Assumes ensemble is perfect;
    often localisation and hybridisation are used to mask deficiencies in the ensemble.
  - Assumes ensemble members are independent;
    does not give correct results using lag-shift ensemble.
- I diagnosed optimal horizontal localisation for each waveband separately, then assumed these apply when they are combined.
  Because of this, I could not use the facility to diagnose hybrid β s.

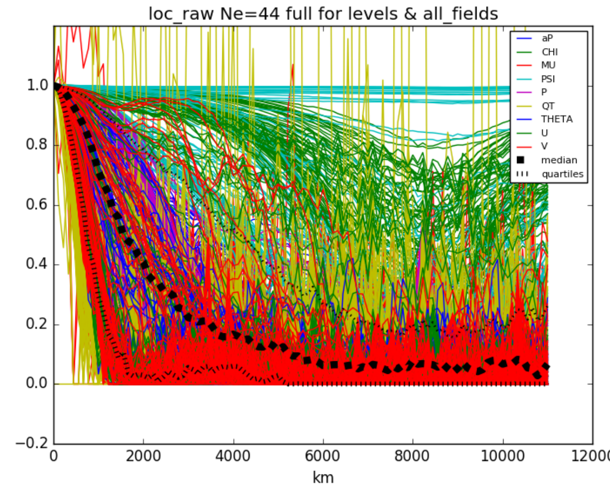
Ménétrier et al. (2015); Ménétrier and Auligné (2015)

Horizontal localisation

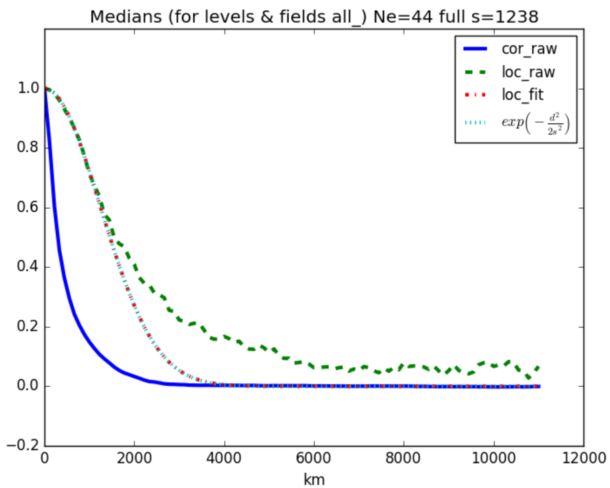
I diagnosed each variable, level independently.



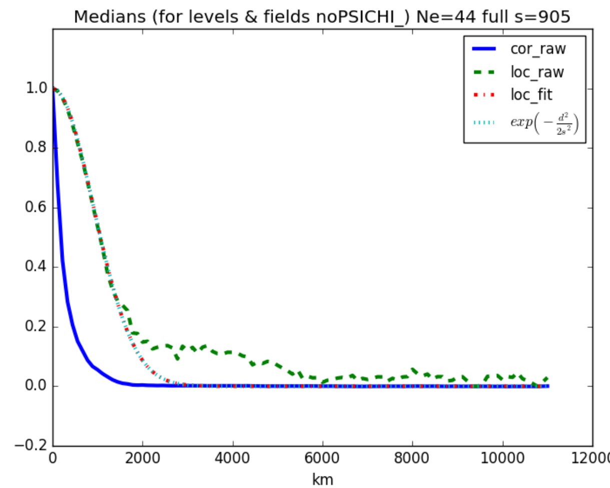
Global mean raw ensemble correlations for each variable and level



Global mean raw diagnosed localisation function for each variable and level

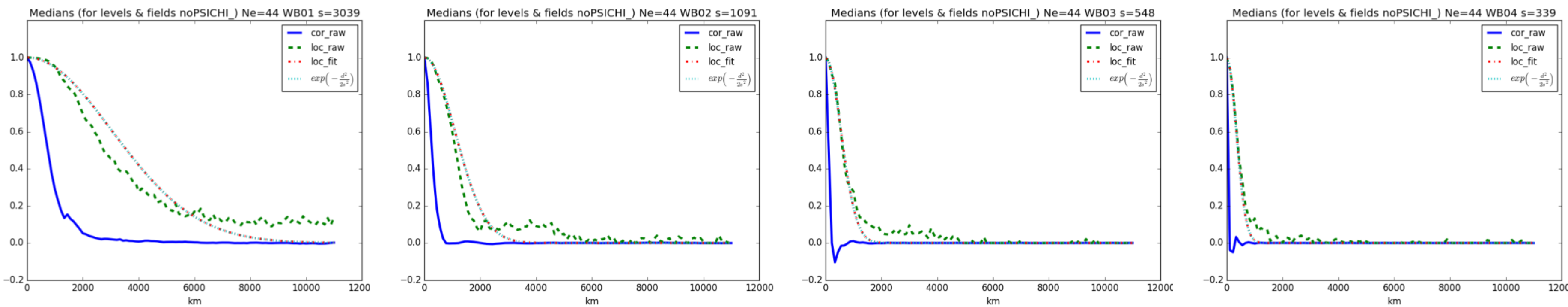


Medians (over all variables and levels) of correlation, diagnosed localisation, fitted localisation function.



Medians as before, omitting streamfunction and velocity potential.

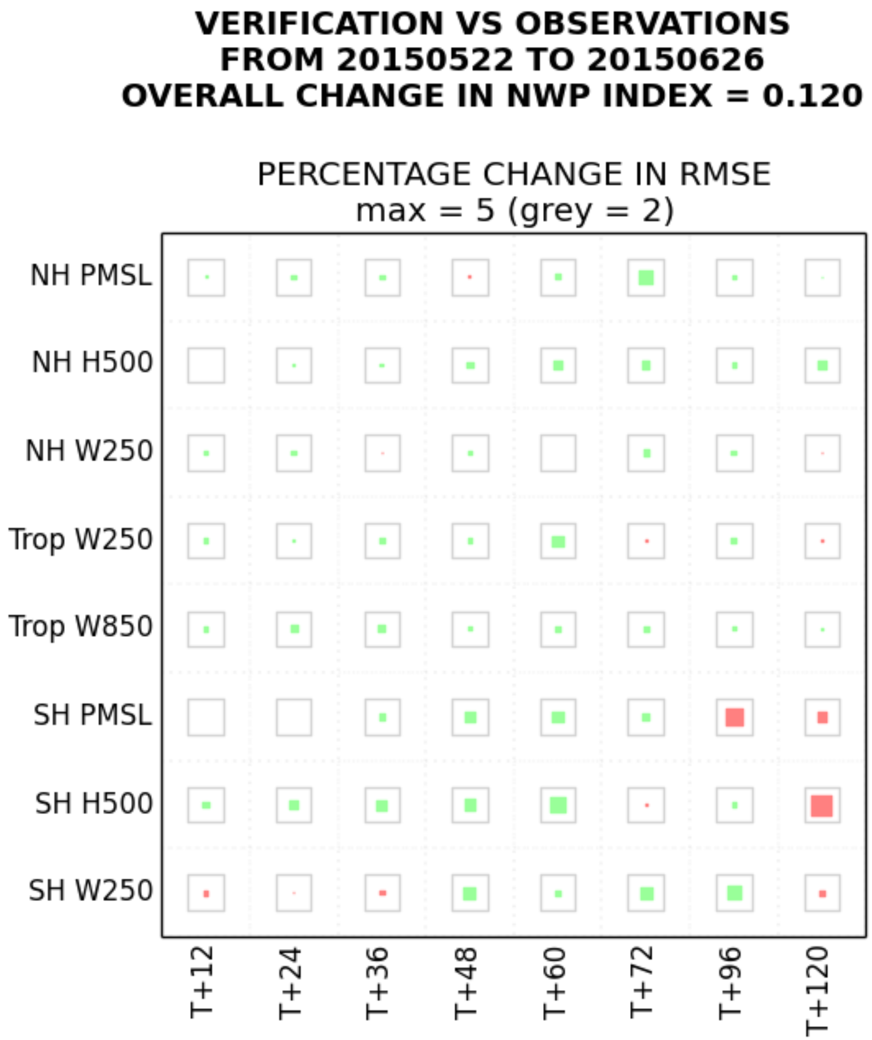
Horizontal localisation for each waveband (excluding Ψ, X)

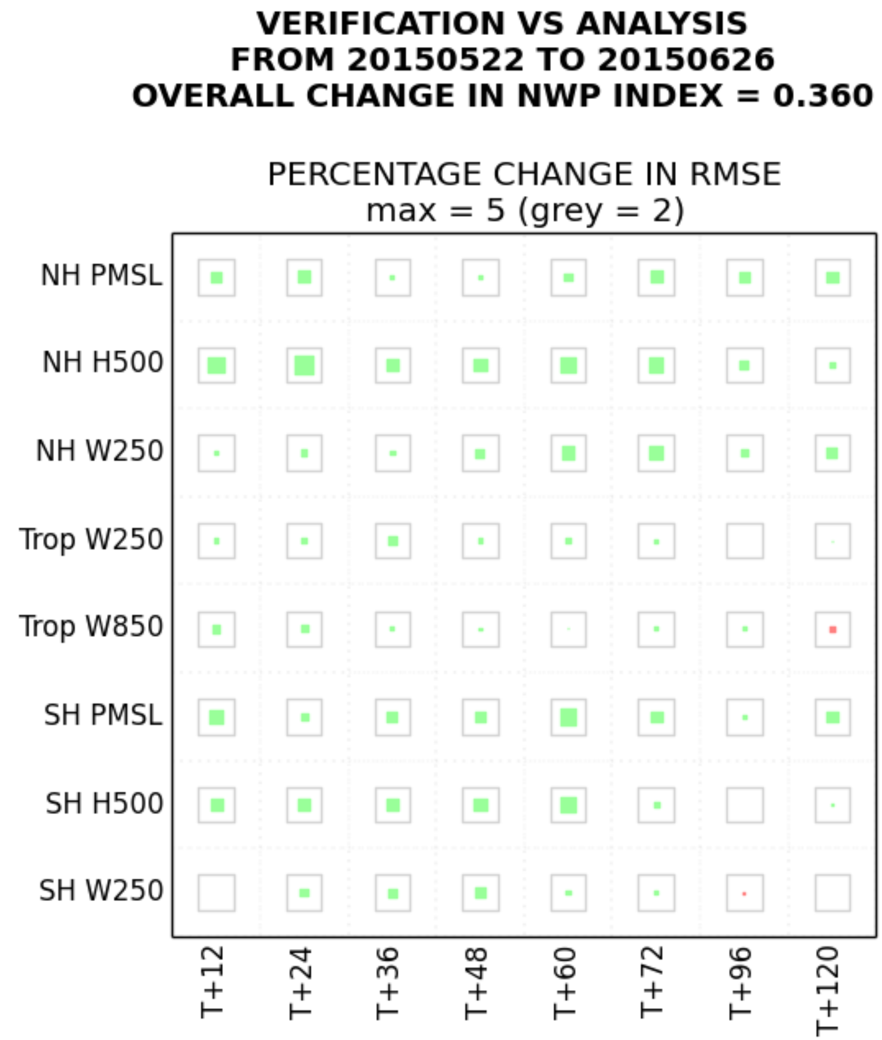


Horizontal localisation scales (km) for each waveband.

	wb1	wb2	wb3	wb4	
Estimated & tuned by Lorenc (2017)	6241	919	389	245	BETTER
Diagnosed by hybrid-diag	3039	1091	548	339	

Effect of horizontal localisation scales: Lorenc (2017) v hybrid-diag diagnosed





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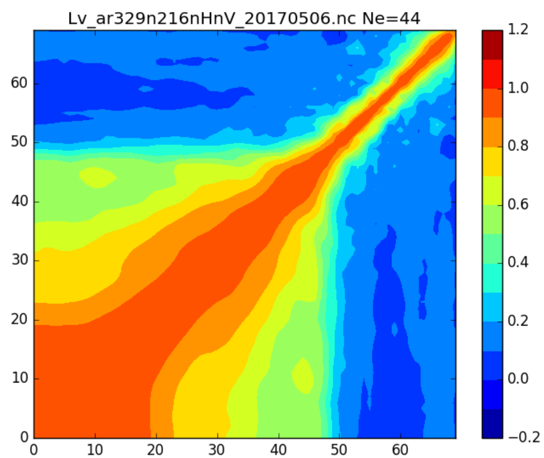
Diagnosing vertical localisation

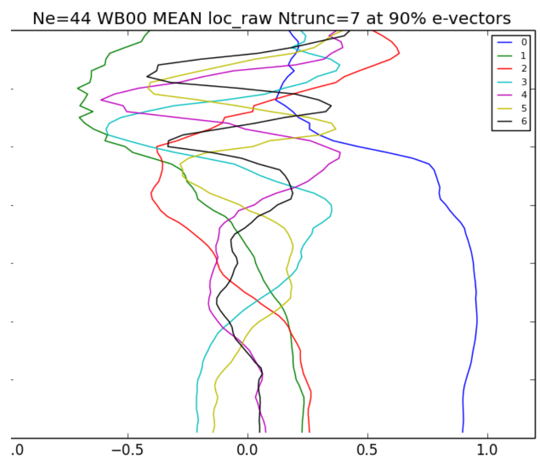
- Only rather small differences between diagnosed localisation for different variables, or different wavebands.

⇒Take mean over all variables, for unfiltered ensemble.

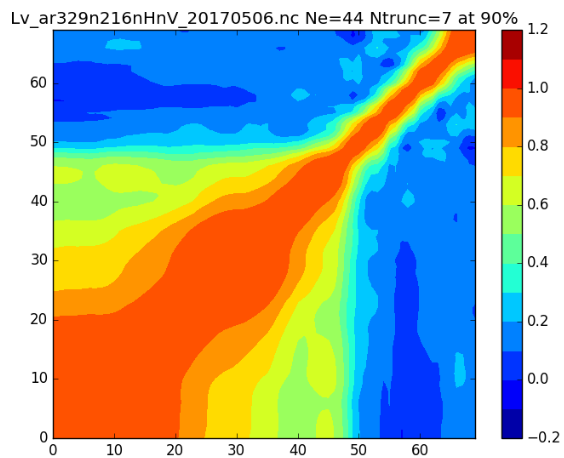
- Truncate to retain 90% variance, then renormalise.



Raw diagnosed localisations

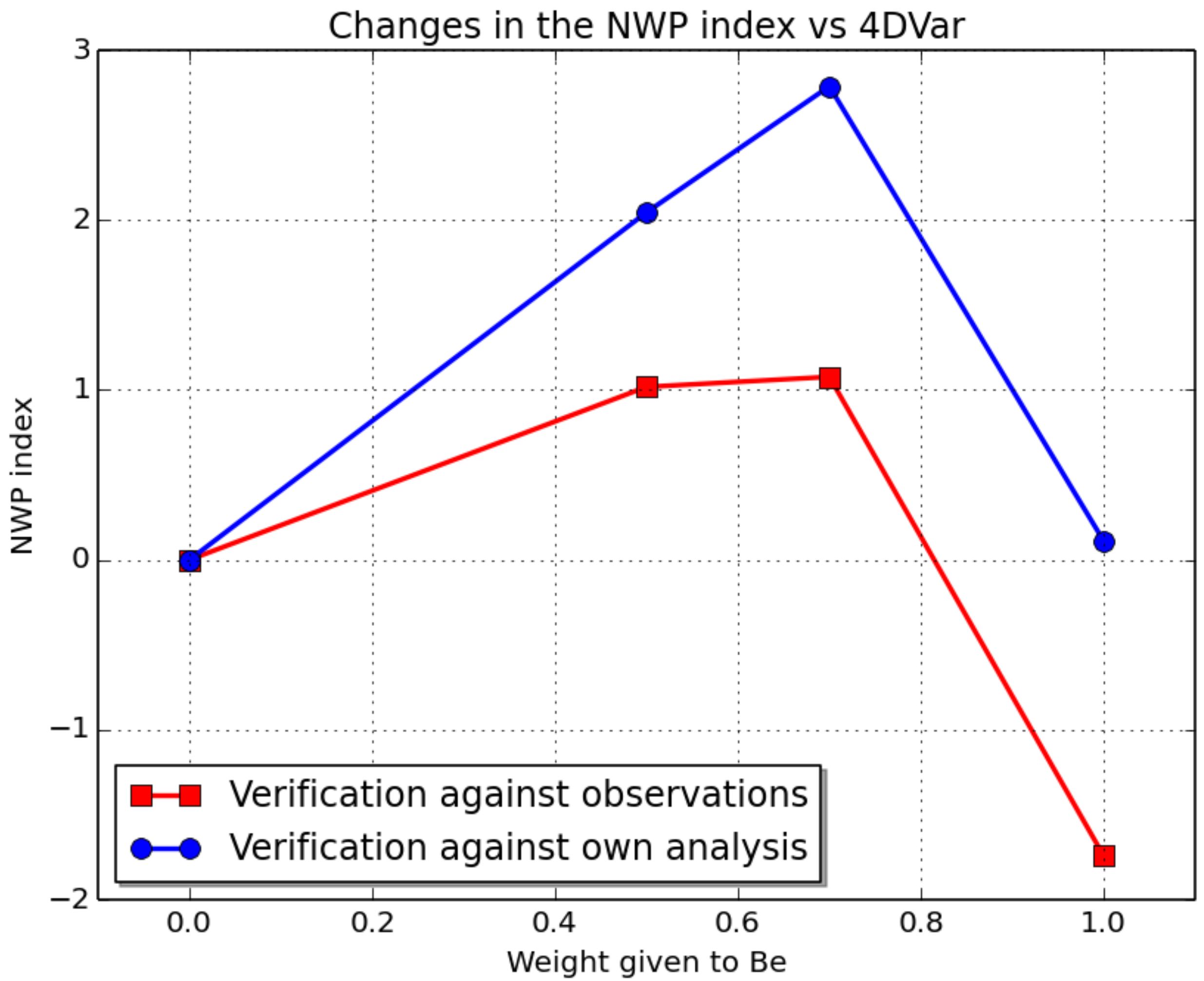


Retained eigenvectors

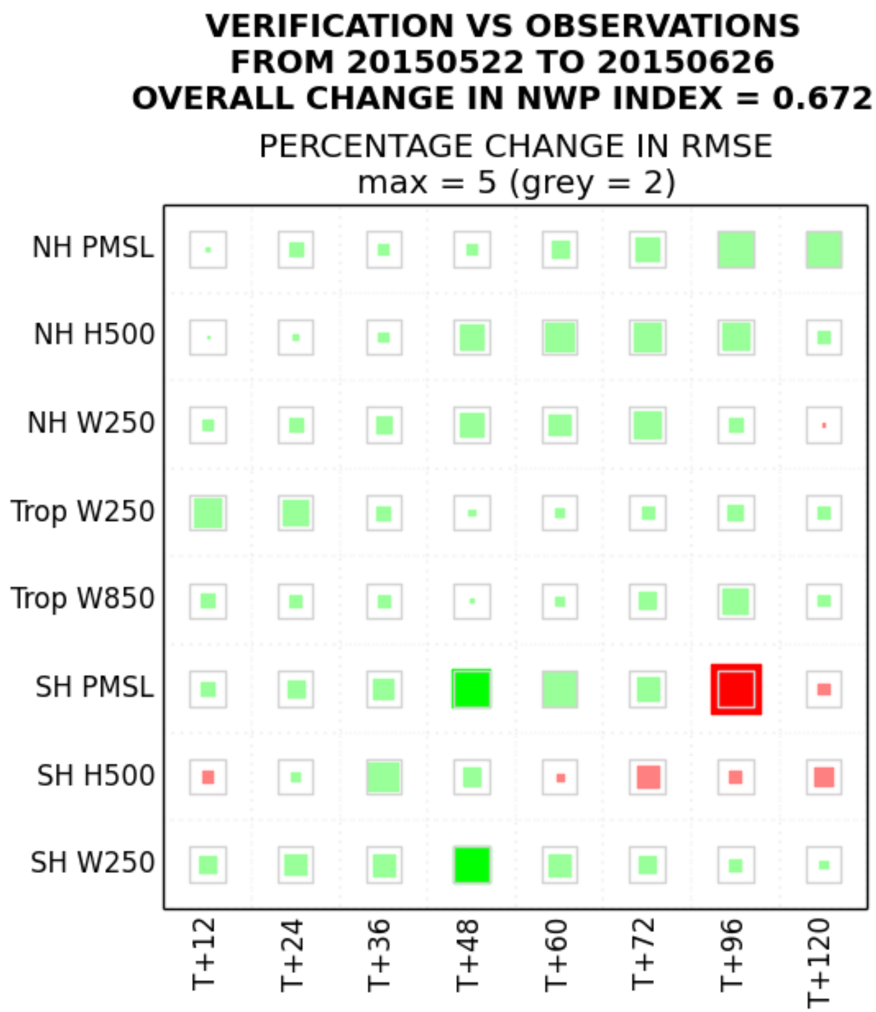


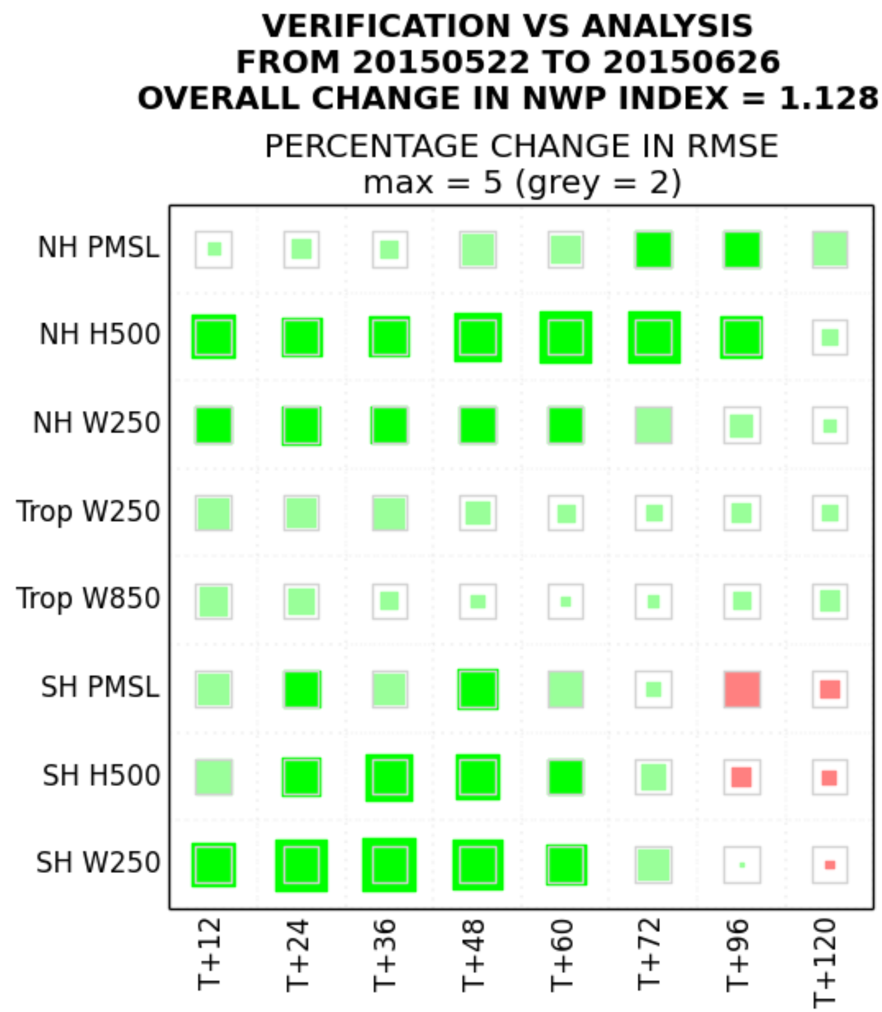
Truncated and renormalised localisations

Effect of hybrid weights in $\mathbf{B} = \beta_c{}^2\mathbf{B}_c + \beta_e{}^2\mathbf{B}_e$



Results from trials compared to operational system (Low resolution, summer only)





Extra Costs dues to wavebands and lag-shift

A. **+6hr** ensemble forecast, & IO to create perturbations
Not on critical path.

B. **×6** input of lag-shift ensembles into hybrid-VAR
Each ensemble is read in parallel, but different lag-shift are not.

C. **×4** (wavebands) **×6** (lag-shift) length of extended c.v.
Distributed over processors.

D. **×4** (wavebands) **×6** (lag-shift) transforms of control variables
Distributed over processors.

High resolution 4DVar costs were dominated by PF and adjoint model forecast each iteration. It relies on low-res preconditioner to need fewer iterations.
This doubles cost B.

The (~16) preconditioning vectors written and read have length **×24** (cost C), and IO is not currently parallel.

Likely Outcome

- Decision on next candidate upgrade (PS40) to be made in July, for final tests before implementation in Autumn.
- Costs of full wavebands + lag-shift package may rule it out. perhaps descope, e.g. wavebands + lag.
- Trials of revised $\mathbf{B}_e$ (Marek Wlasak) have shown benefit (and different optimal hybrid weights). This may be included.
- Coding to reduce critical costs can be done by PS41 in 2018.
- Ensemble of 4DEnVar (Ne=100) also candidate for PS41. This too increases costs.

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My poster at ISDA, Reading 2016