Weak constraint 4D-Var at ECMWF

How to deal with model error in data assimilation

Patrick Laloyaux - Earth System Assimilation Section

Acknowledgement: Jacky Goddard, Mike Fisher, Yannick Tremolet, Massimo Bonavita, Elias Holm, Martin Leutbecher, Simon Lang, Sarah-Jane Lock

4D-Var formulations

Bias-blind data assimilation

$$
\begin{aligned}
J(x_0) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}[y_k - \mathcal{H}(x_k)]^T \mathbf{R}_k^{-1}[y_k - \mathcal{H}(x_k)]
\end{aligned}
$$

→ designed to correct the background with observations when errors are **random with zero means**

→ the model is assumed to be perfect

$$
x_k = \mathcal{M}_k(x_{k-1}) \qquad \text{for } k = 1, 2, \cdots, K
$$

Biases in reality

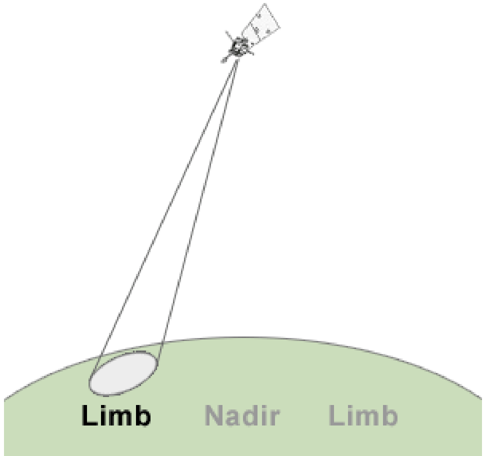
→ Observations (satellite miscalibrations)

→ Observation operator (approximations in radiative transfer calculations)

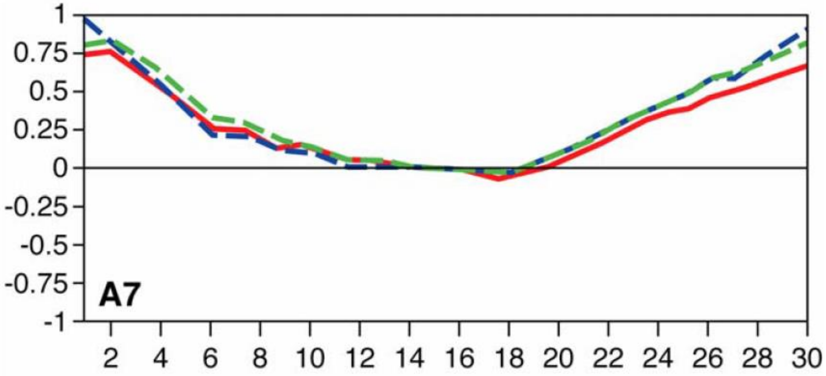
→ Model trajectory (inaccurate surface forcing, simplified representations of physics)

Biases in observations and observation operators

Departure between observations and model varies with the scan position of the satellite instrument

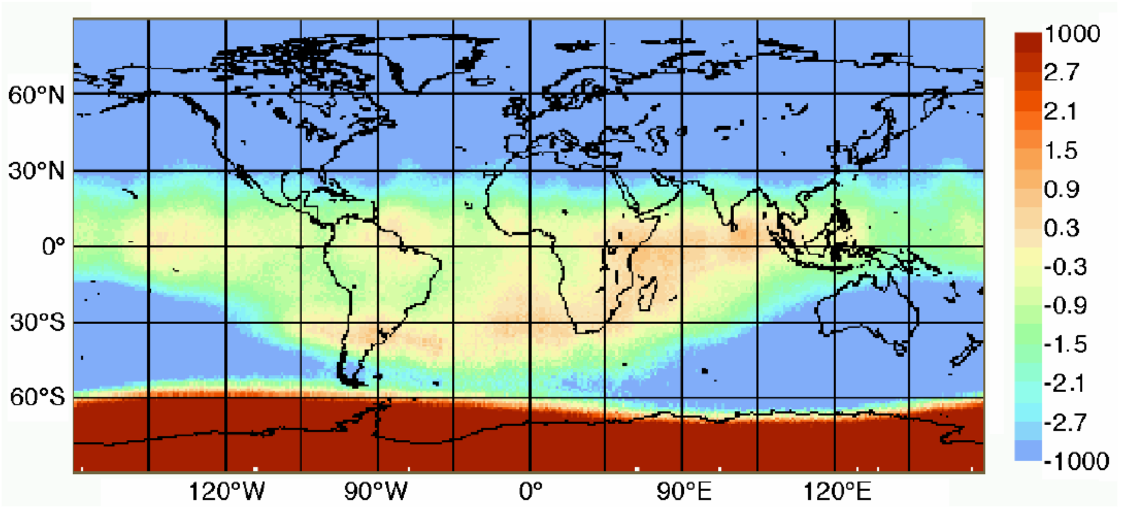


AMSUA channel 7



Limb Nadir Limb

Departure between observations and model are air-mass dependent



AMSUA channel 14

4D-Var formulations

Bias-aware data assimilation (VarBC)

$$
\begin{aligned}
J(x_0,\beta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}[y_k - \mathcal{H}(x_k) - b(x_k,\beta)]^T \mathbf{R}_k^{-1}[y_k - \mathcal{H}(x_k) - b(x_k,\beta)] \\
&+ \frac{1}{2}(\beta - \beta_b)^T \mathbf{B}_\beta{}^{-1}(\beta - \beta_b)
\end{aligned}
$$

→ designed to estimate simultaneously the initial condition and parameters that represent systematic errors in the system

→ the bias model copes with instrument miscalibration (e.g. radiances systematically too warm by 1K) or systematic errors in the observation operator

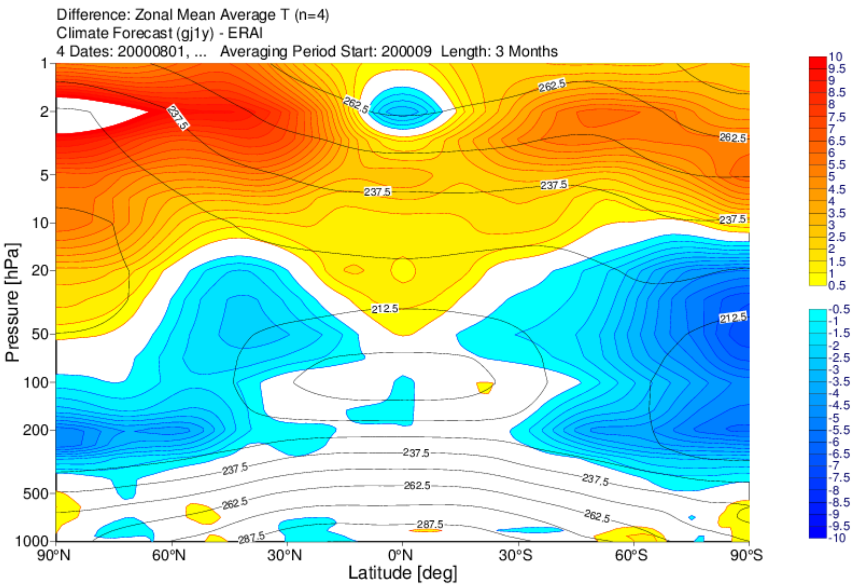
→ the model is a strong constraint $x_k = \mathcal{M}_k(x_{k-1})$

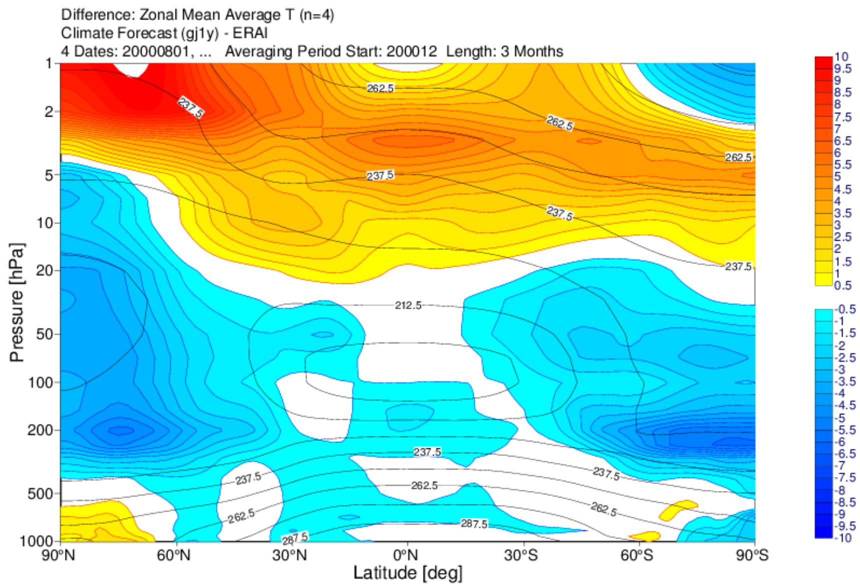
Model biases

A possible method to estimate model biases:

- compute model-free integrations
- use ERA-Interim as a proxy of the truth
- compute the mean difference between the model-free integration and ERA-Interim

Temperature biases for SON (left) and DJF (right)



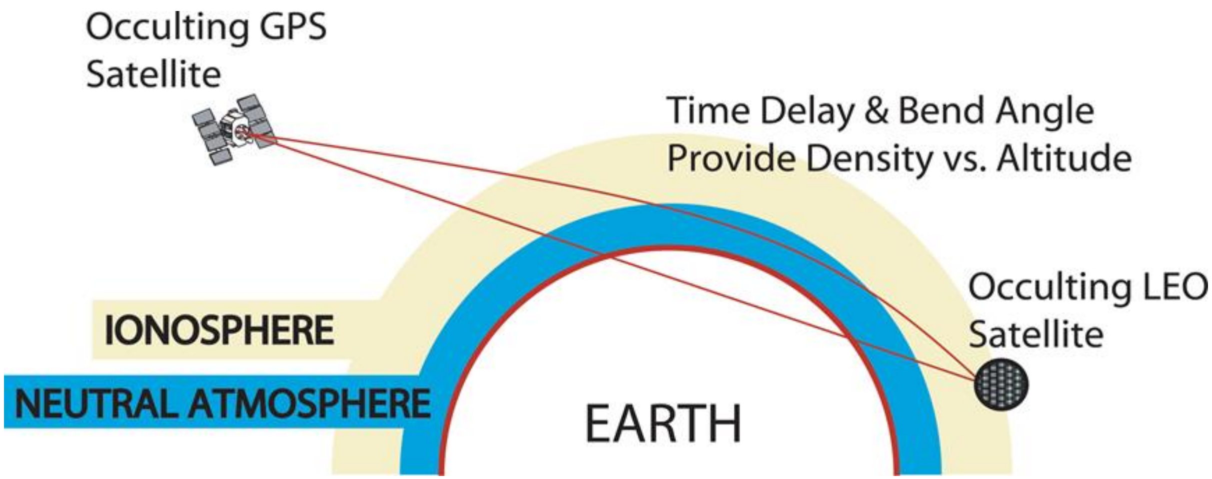


This diagnostic shows a warm bias in the stratosphere and a cold bias in the upper troposphere.

The methodology is far to be perfect especially in the stratosphere where the quality of ERA-Interim is not well assessed

Model biases

GPS-RO is based on analysing the bending caused by the atmosphere along paths between a GPS satellite and a receiver placed on a low-earth-orbiting satellite.



As the LEO moves behind the earth, we obtain a profile of bending angles. Temperature profiles can then be derived (a vertical interval between 10-50 km)

→GPS-RO can be assimilated without bias correction. They are good for highlighting model errors/biases

→GPS RO have good vertical resolution properties

4D-Var formulations

Bias-aware data assimilation (VarBC)

$$
\begin{aligned}
J(x_0, \beta) &= \frac{1}{2}(x_0 - x_b)^T \mathbf{B}^{-1}(x_0 - x_b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}[y_k - \mathcal{H}(x_k) - b(x_k, \beta)]^T \mathbf{R}_k^{-1}[y_k - \mathcal{H}(x_k) - b(x_k, \beta)] \\
&+ \frac{1}{2}(\beta - \beta_b)^T \mathbf{B}_\beta{}^{-1}(\beta - \beta_b)
\end{aligned}
$$

Applying a bias correction to the data to compensate for the NWP model error is the wrong thing to do

→ produce a biased analysis

→ Reenforce the model error

→ degrade the fit to other (good) observations

4D-Var formulations

Bias-aware data assimilation (VarBC + Weak constraint)

$$
\begin{aligned}
J(x_0,\beta,\eta) \quad &= \quad \frac{1}{2}(x_0-x_b)^T\mathbf{B}^{-1}(x_0-x_b) \\
&+ \quad \frac{1}{2}\sum_{k=0}^{K}[y_k-\mathcal{H}(x_k)-b(x_k,\beta)]^T\mathbf{R}_k^{-1}[y_k-\mathcal{H}(x_k)-b(x_k,\beta)] \\
&+ \quad \frac{1}{2}(\beta-\beta_b)^T\mathbf{B}_\beta{}^{-1}(\beta-\beta_b) \\
&+ \quad \frac{1}{2}(\eta-\eta_b)^T\mathbf{Q}^{-1}(\eta-\eta_b)
\end{aligned}
$$

→ Introduce additional controls to target an unbiased analysis and to move the assimilation away from a perfect-model trajectory

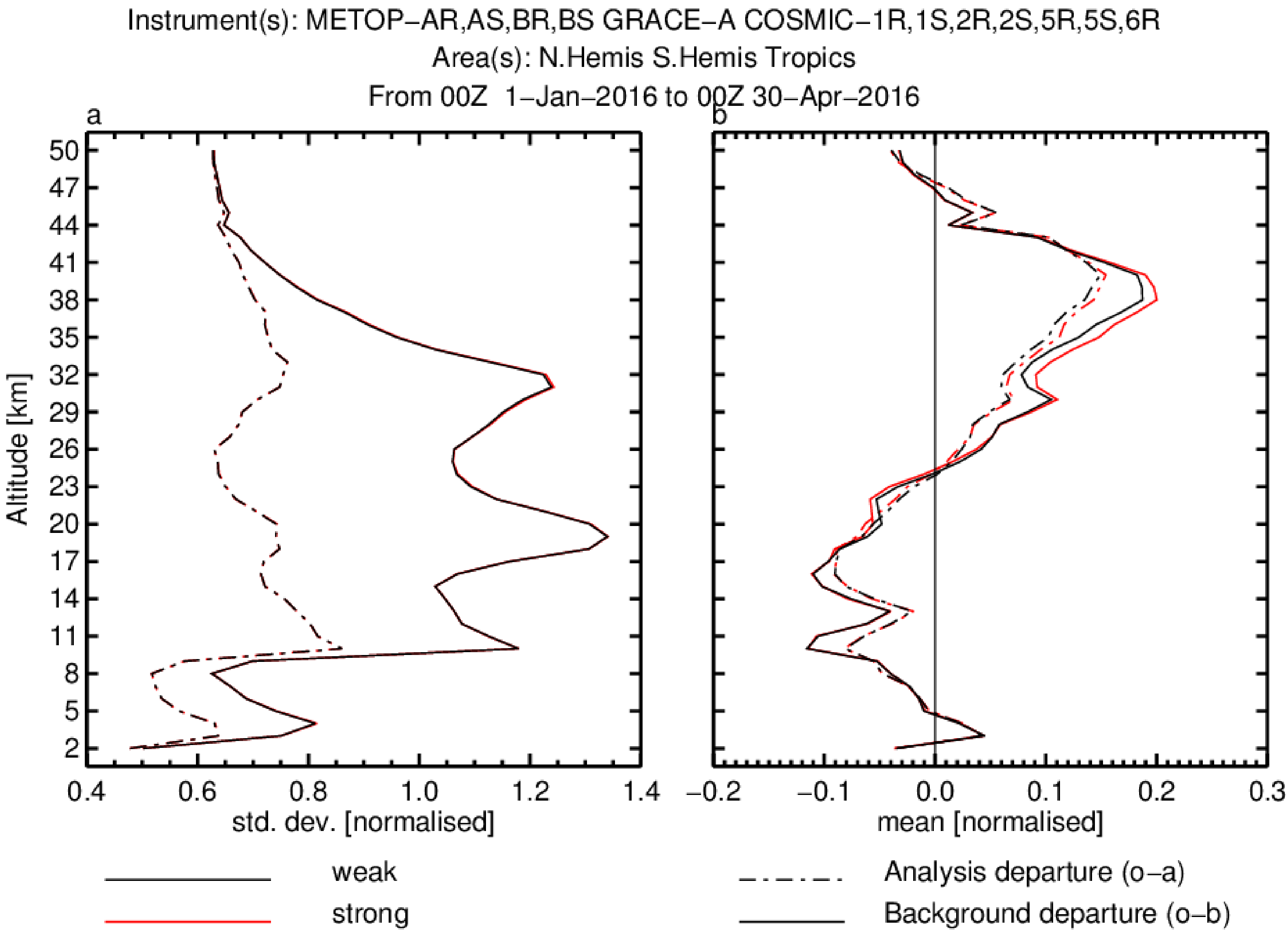
$$x_k = \mathcal{M}_k(x_{k-1})+\eta \quad \text{for } k=1,2,\cdots,K$$

→ The model error represents the systematic error which develops in the model over the assimilation window

→ Size on the control vector increased by ~25%

Impact of weak constraint 4D-Var

Comparison between strong and weak constraint 4D-VAR in the stratosphere.
Verification against GPS-RO (good for highlighting improvements in the stratosphere)

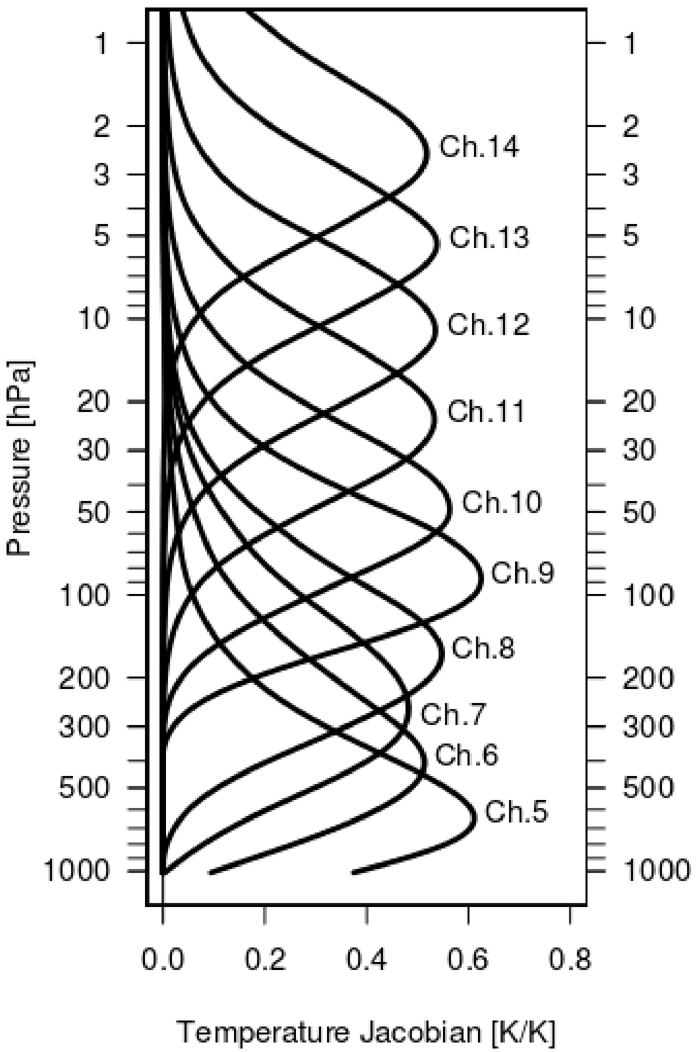


Advanced Microwave Sounding Unit

The Advanced Microwave Sounding Unit (AMSU) is a multi-channel microwave radiometer. The instrument measures radiances that reach the top of the atmosphere.

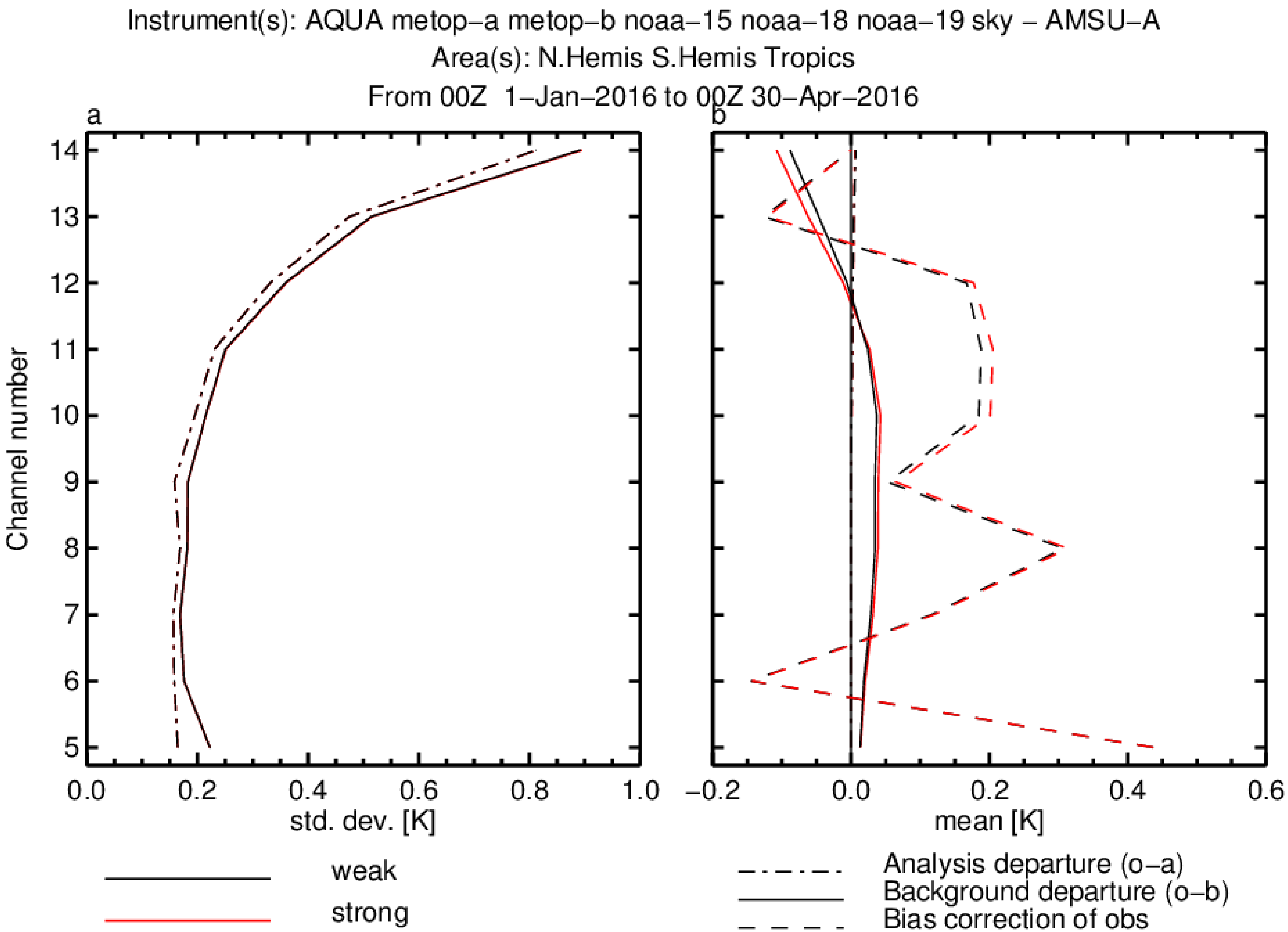
By selecting channels, AMSU can perform atmospheric sounding of temperature and moisture.

From channel 9 to 14 (stratosphere), AMSU provides weighted average of the atmospheric temperature profile.



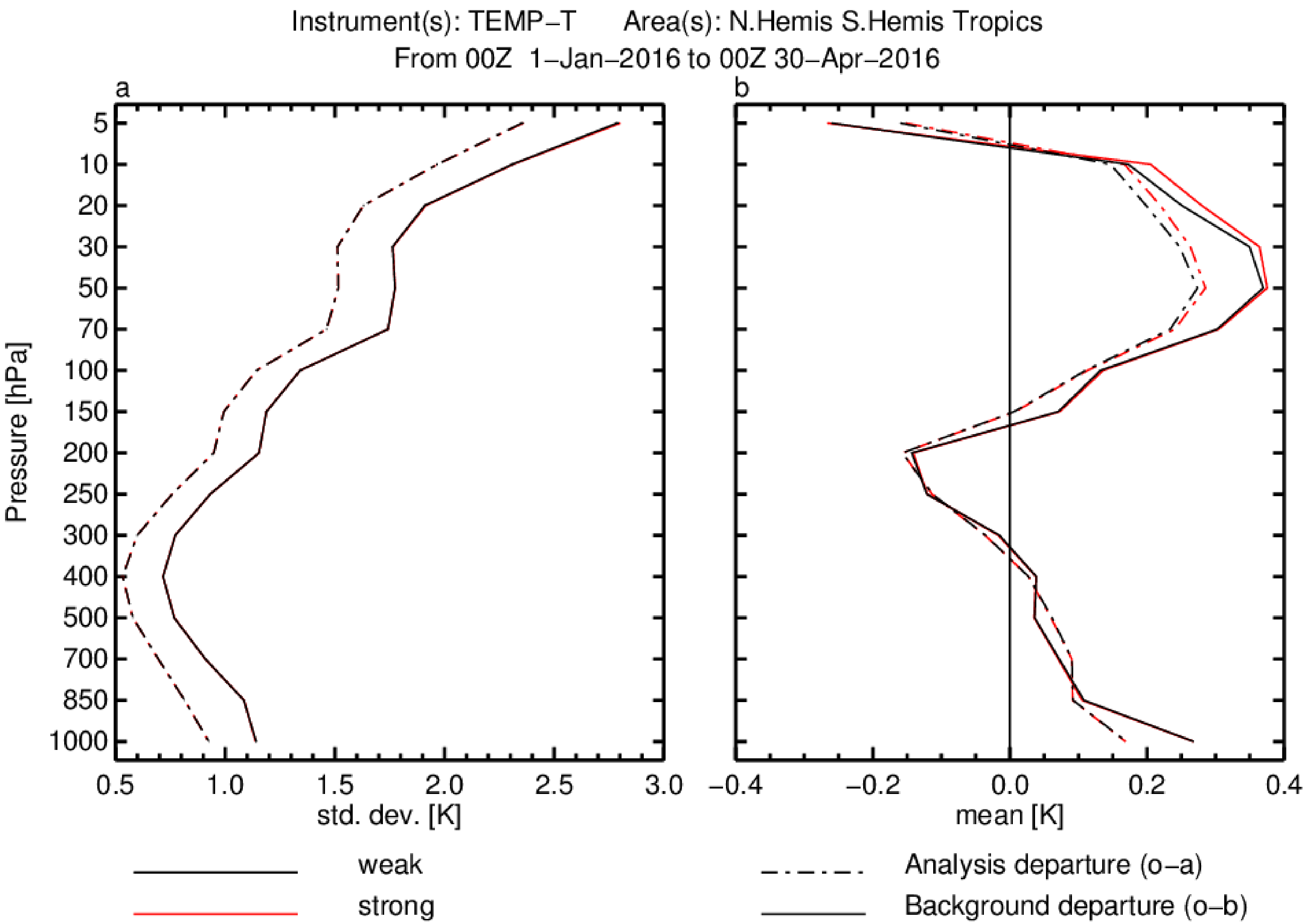
Impact of weak constraint 4D-Var

Comparison between strong and weak constraint 4D-VAR in the stratosphere.
Verification against AMSU-A



Impact of weak constraint 4D-Var

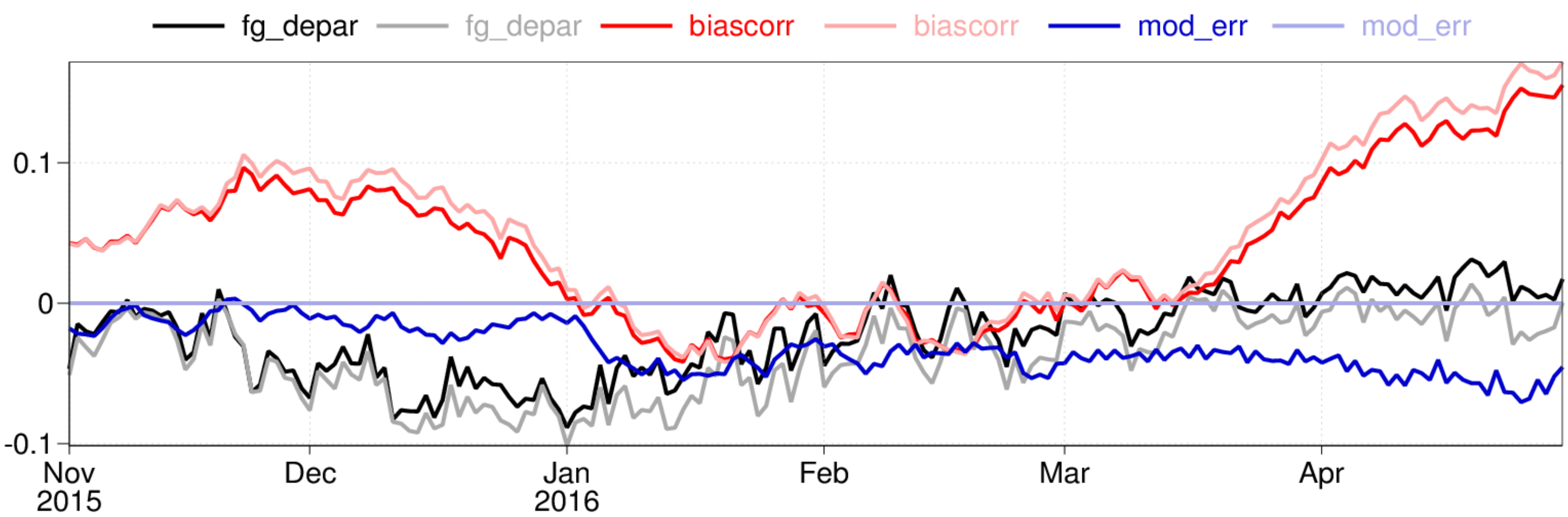
Comparison between strong and weak constraint 4D-VAR in the stratosphere.
Verification against radiosondes.



Impact of weak constraint 4D-Var

Observation feedback from AMSU-A (channel 13) and model error (level 22)

Strong constraint in light colours
Weak constraint in dark colours



→ First-guess departures from strong/weak constraint 4D-Var show an initial warm bias

→ The model error in the weak constraint 4D-Var corrects the bias

→ First-guess departures from weak constraint 4D-Var are improved and bias correction is reduced

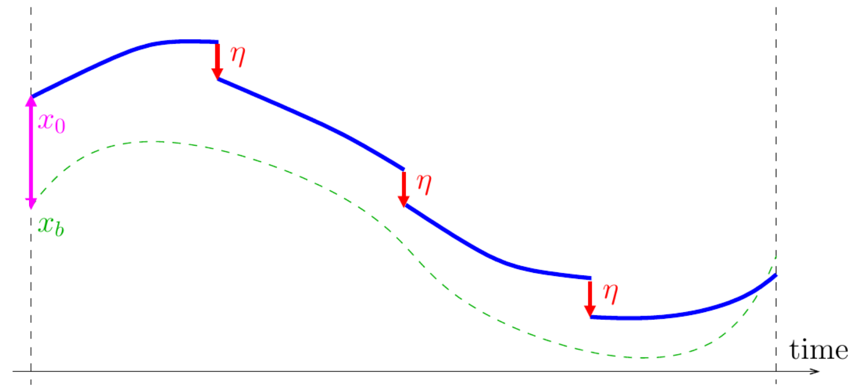
Implemented in operations (22 Nov 2016, CY43R1)

Weak constraint 4D-Var has been implemented in operations

→ constant model forcing applied every hour

$$x_k = \mathcal{M}_k(x_{k-1}) + \eta \quad \text{for } k = 1, 2, \cdots, K$$

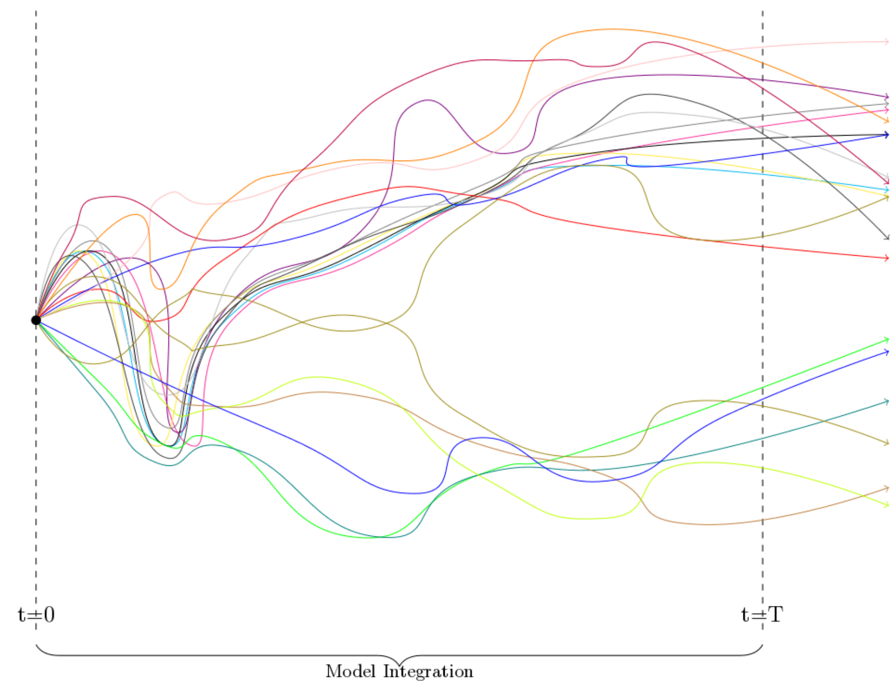
→ only active above 40 hPa (4D-Var is likely to misinterpret error from aircraft observation as model errors)



Estimate the model error covariance matrix (Q)

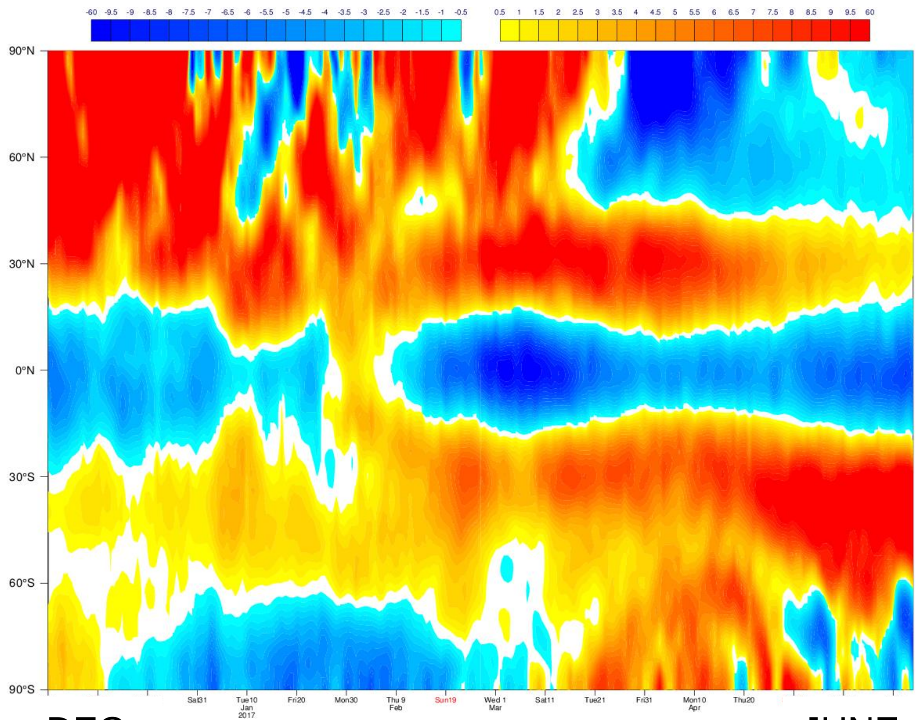
→ run the ensemble forecasting system (ENS) with SPPT and SKEB (51 members with the same initial condition for 20 days)

→ differences after 12 hours are used to compute Q



How does it perform in operations?

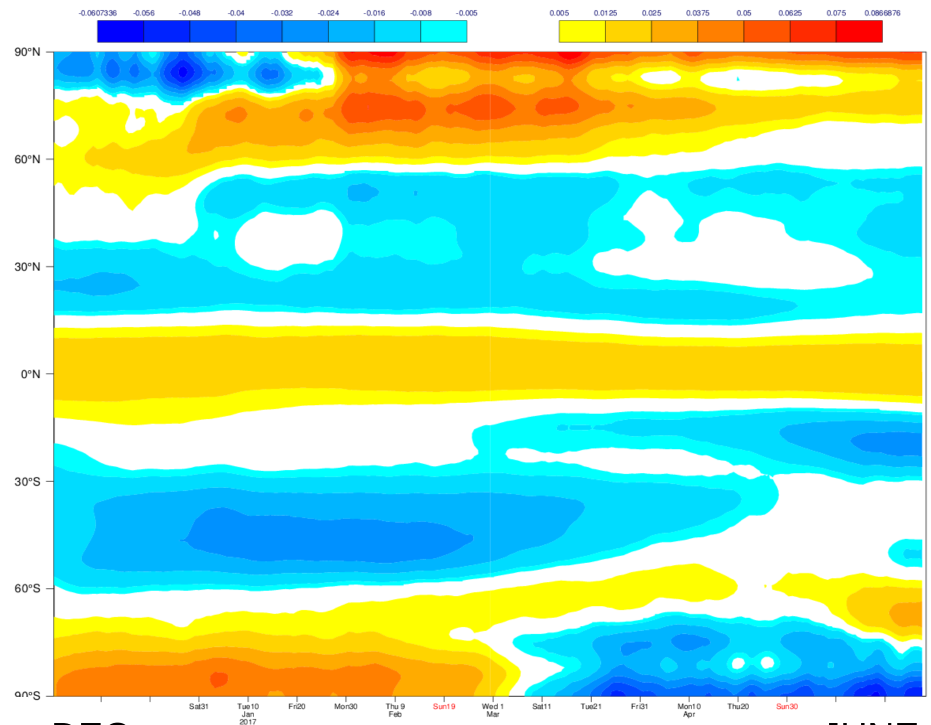
Temperature bias at 1hPa



DEC 2016

JUNE 2017

Temperature model error at 1hPa



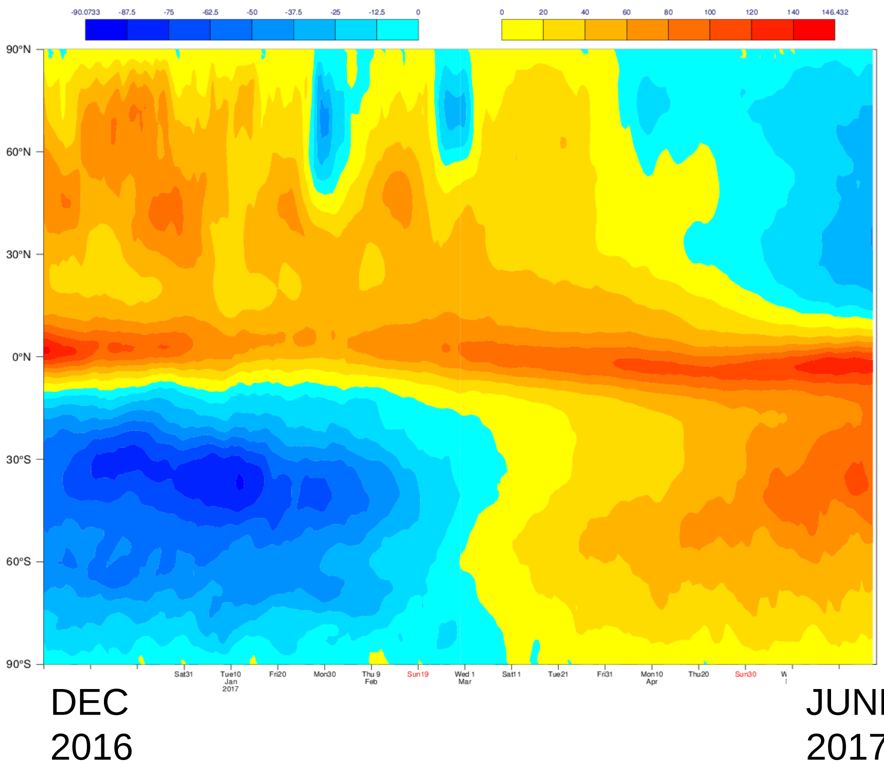
DEC 2016

JUNE 2017

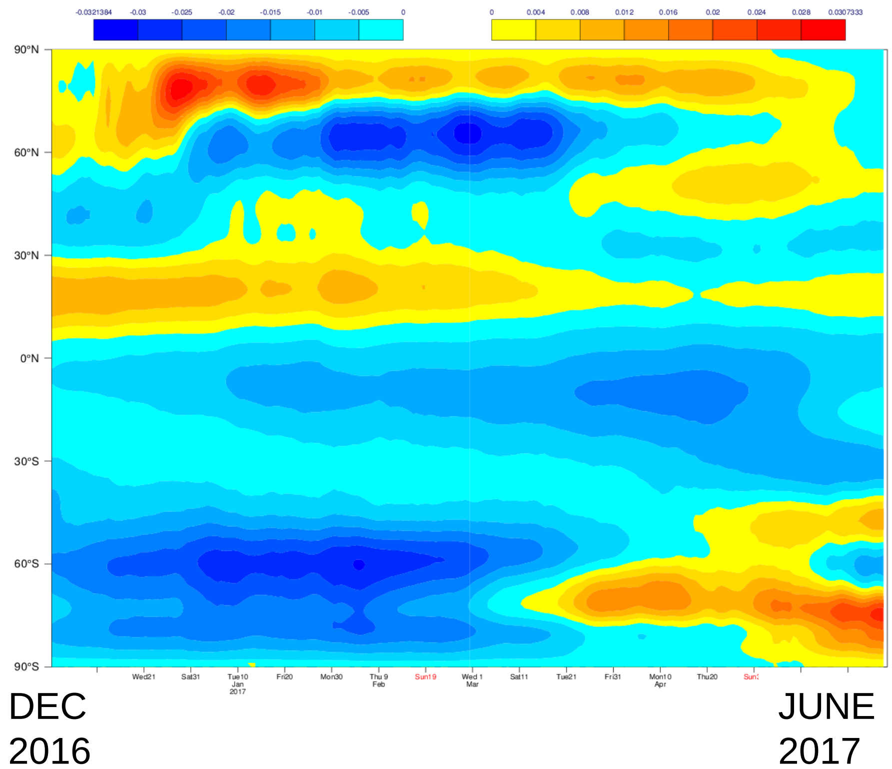
Model bias is estimated by the difference between a free model run and ERA-Interim.
Quality of ERA-Interim at 1hPa?
Temperature model error corrects temperature biases, especially in SH.

How does it perform in operations?

Zonal wind analysis at 0.1 hPa



Zonal wind model error at 0.1 hPa

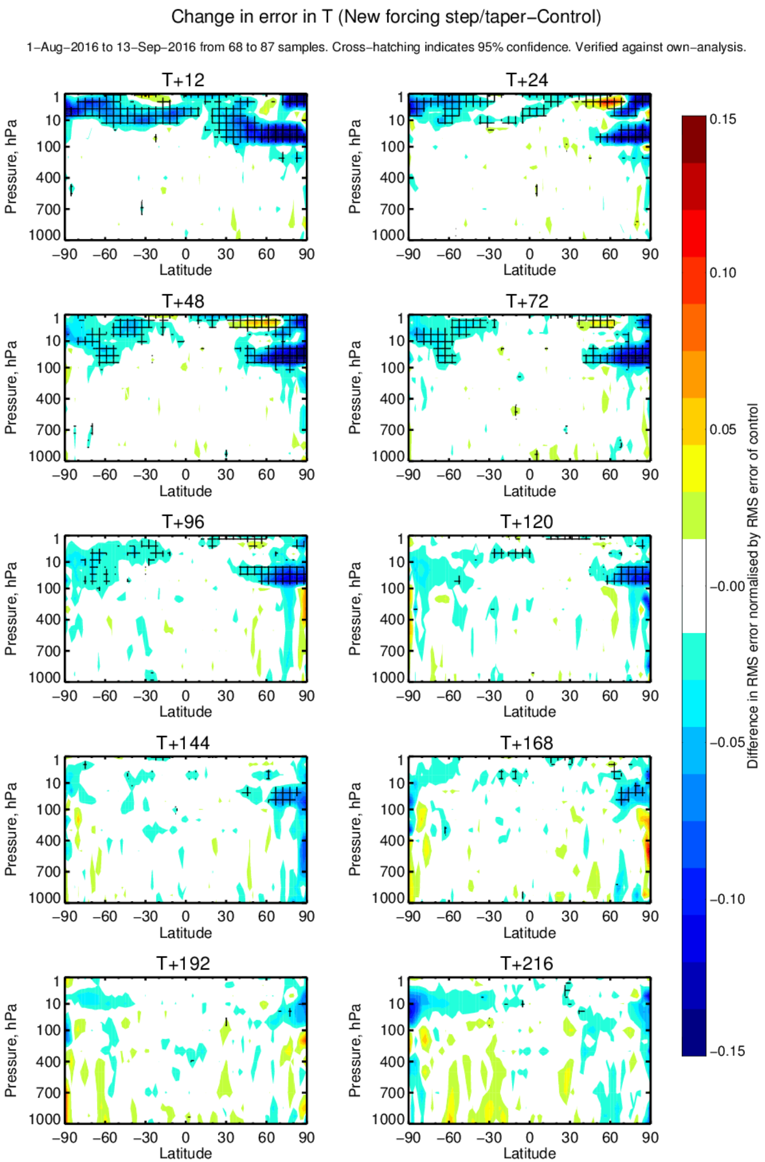


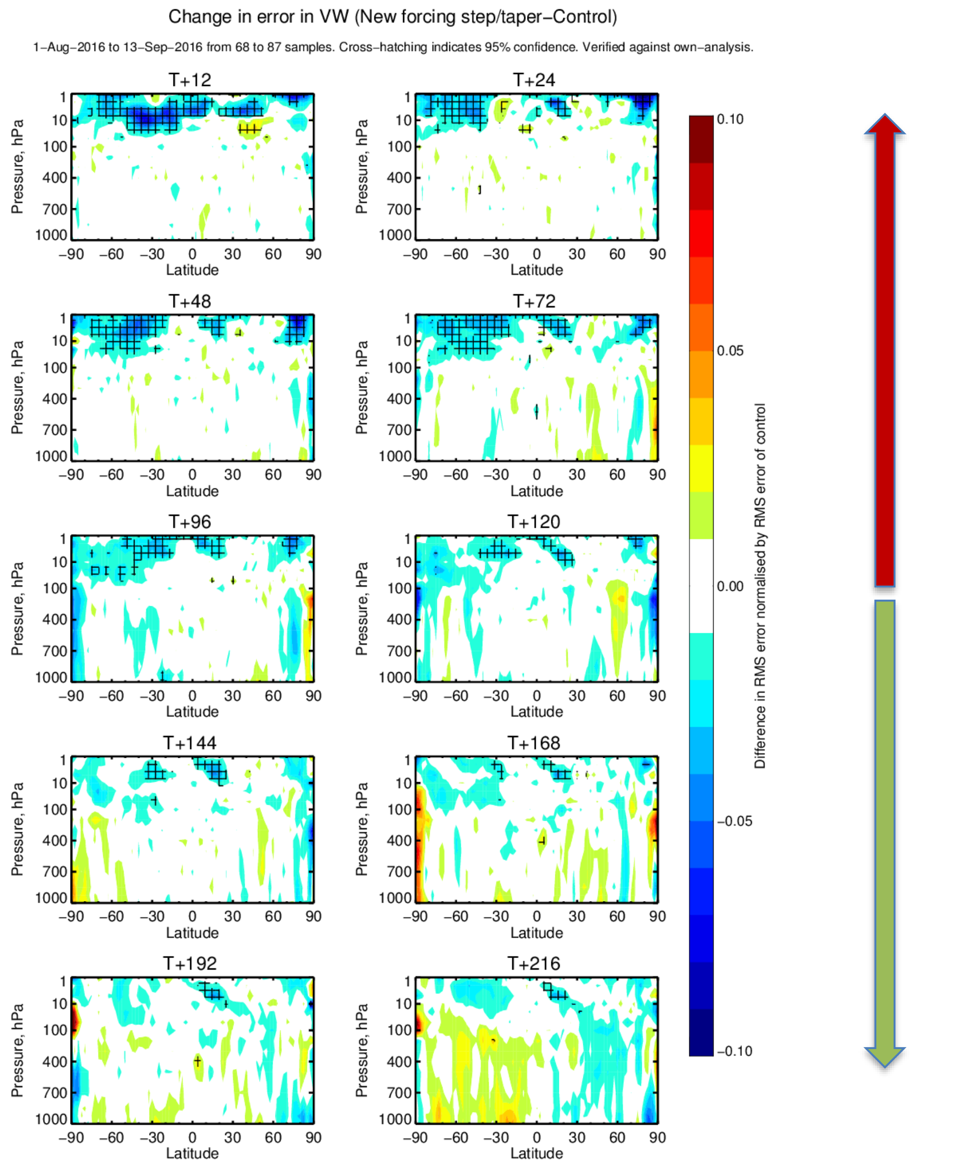
Model error does not correct the too strong jet in the Tropics
It might be more tricky for 4D-VAR to correct a model error in wind from radiances observations

Contribution to the next operational cycle

→Apply the model error every time step to avoid possible shocks

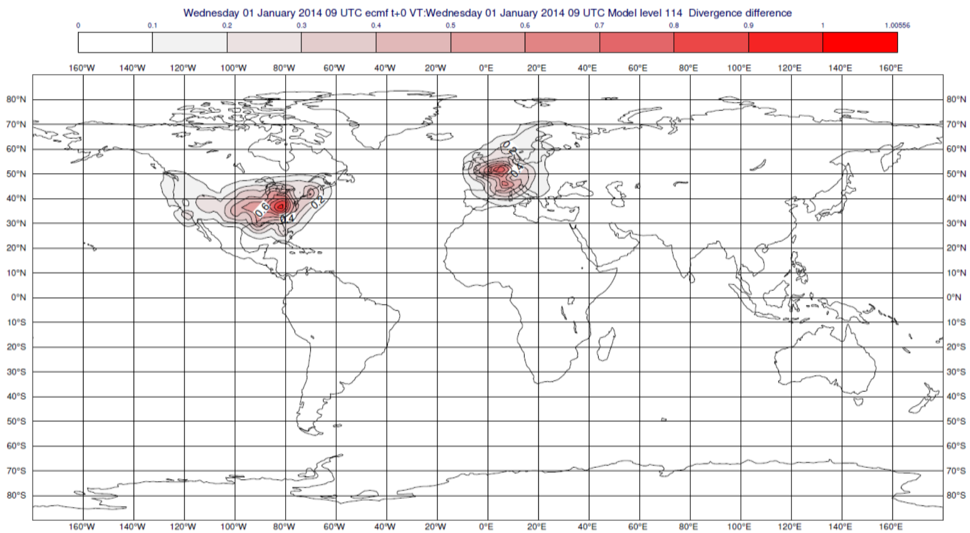
→Apply the model error at the beginning of the 10-day fc (taper off from h15 to h24)



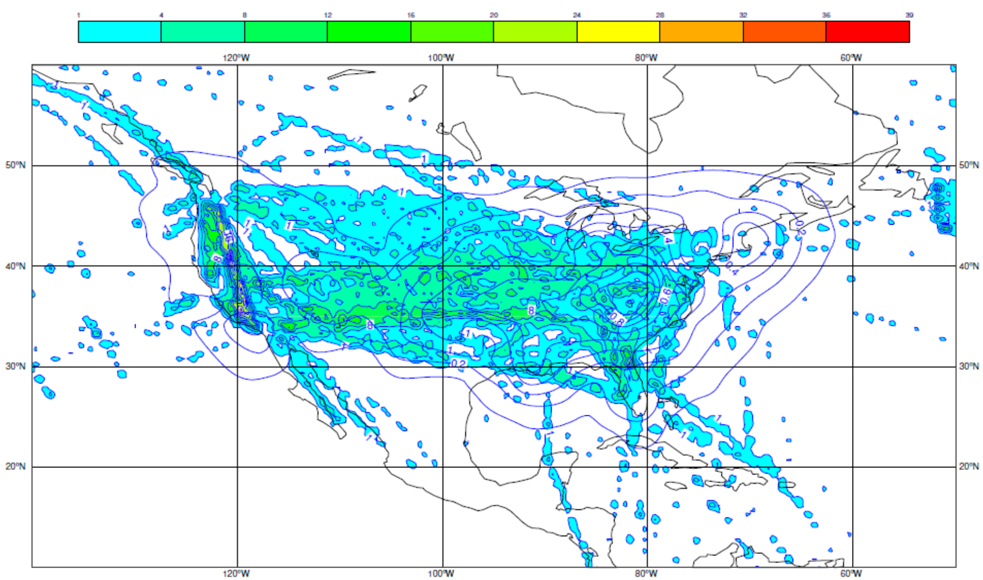


Results of the weak constraint 4D-Var in the troposphere

Statistics from the model error have been computed



Correlation between level 52 (63 hPa) and 114 (850 hPa) for the divergence of model error



This suggests that 4D-Var is misinterpreting aircraft observation errors as model errors

→ weak constraint is activated only above 40 hPa

Courtesy of Jacky Goddard

Weak constraint formulation

$$
\begin{aligned}
J(x_0,\beta,\eta) &= \frac{1}{2}(x_0-x_b)^T\mathbf{B}^{-1}(x_0-x_b)\\
&+ \frac{1}{2}\sum_{k=0}^{K}[y_k-\mathcal{H}(x_k)-b(x_k,\beta)]^T\mathbf{R}_k^{-1}[y_k-\mathcal{H}(x_k)-b(x_k,\beta)]\\
&+ \frac{1}{2}(\beta-\beta_b)^T\mathbf{B}_\beta{}^{-1}(\beta-\beta_b)\\
&+ \frac{1}{2}(\eta-\eta_b)^T\mathbf{Q}^{-1}(\eta-\eta_b)
\end{aligned}
$$

Biased observations can be fitted by the VarBC method (correct) or by specifying a spurious model error. We think that the latter happens with aircraft data.

Future developments

To be tested in the next cycle: Activate the model error in the troposphere

→ New VarBC scheme for aircraft measurements
New predictors take into account the phase of the flight (asc., cruising, des.)
New blacklist for measurements close to airports

→ Work on the Q matrix
From the current small scales to larger scales?

Weak Constraint 4D-Var allows the perfect model assumption to be removed and the use of longer assimilation windows

→ How much benefit can we expect from long window 4D-Var?

Within the OOPS framework, ECMWF is interested by

→ 4D-Var parallelization (subwindows), saddle point algorithm and preconditioning