A NEW HEURISTIC FOR THE MULTIDIMENSIONAL KNAPSACK PROBLEM

M. ELKIHEL, D. ELBAZ AND V. BOYER
email: elkihel@laas.fr ; elbaz@laas.fr; vboyer@laas.fr

LAAS-CNRS, 7 avenue du Colonel Roche 31077 Toulouse Cedex 4 France

We present a new heuristic based upon the surrogate relaxation for the multidimensional knapsack problem.
The NP-Hard, multidimensional knapsack problem (MKP), arises in several practical problems such as the capital budgeting problem, cargo loading, cutting stock problem, and processors allocation in huge distributed systems. It can be formulated as:

$$(MKP)\begin{cases} \max \sum_{j\in N} c_j.x_j = v(MKP), \\ \text{subject to} \sum_{j\in N} a_{i,j}.x_j \leq b_i \text{ , i} \in M, \\ x_j \in \{0,1\}, j \in N, \end{cases}$$

where N=$\{1,2,...,n\}$, M=$\{1,2,...,m\}$, $c_j \geq 0$ for all $j \in N$, $a_{i,j} \geq 0$, for all $i \in M$, $j \in N$.

A specific case of the MKP is the classical knapsack problem (m=1), the Unique Knapsack Problem (UKP), which has been given much attention in the litterature though it is not, in fact, as difficult as MKP, more pricisely, it can be solved in a pseudo-polynomial time. Due to the intrinsec difficulty (NP-hardness) of the MKP, which leads to intractable computation time for larger instances, we tried to transform the original MKP into an UKP. In this purpose, we used a relaxation technic, that is to say, the surrogate relaxation.
The surrogate relaxation of MKP can be defined as:

$$(S(\mu))\begin{cases} \max \sum_{j\in N} c_j.x_j = v(S(\mu)), \\ \text{subject to} \sum_{j\in N} (\sum_{i\in M} \mu_i.a_{i,j}).x_j \leq \sum_{i\in M} \mu_i.b_i, \\ x_j \in \{0,1\}, j \in N, \end{cases}$$

where $\mu = \{\mu_1,...,\mu_m\} \geq 0$ is the surrogate multiplier. As $S(\mu)$ is a relaxation of MKP, $v(S(\mu)) \geq v(MKP)$. So the optimal multiplier, μ^*, is defined as:

$$v(S(\mu^*)) = \min_{\mu}\{v(S(\mu))\}.$$

Several heuristics exist to find the surrogate multipliers.

Our heuristic tries to find a feasible solution of the MKP using its surrogate relaxation. In this purpose, we used a modified dynamic programming algorithm and efficient preprocessing technics given in the litterature [8]. We have tested our heuristic on several established and ramdomly generated test sets and the solution values were compared to both optimal values and the results of other heuristics ([10], [11] & [14]). The first computationnal experiences showed that it seemed to give better results than the existing heuristics, and so a lower gap to the optimal solution.

References

[1] M. Elkihel - Programmation dynamique et rotation de contraintes pour les problèmes d'optimisation entière - PHD thesis, Université des Sciences et Techniques de Lille (1984).

[2] D. El Baz & M. Elkihel - Load balancing methods and parallel dynamic programming algorithm using dominance technique applied to 0-1 knapsack problem - Journal of Parallel and Distributed Computing 65 (2005) 74-84.

[3] M. Elkihel, G. Authié & F. Viader - Methodes mixtes pour la résolution du problème du sac à dos en variables 0-1 - LAAS Report 02021 (2002).

[4] M. Elkihel, G. Authié & F. Viader - A mixed approach to 0-1 problems - 1996 Advanced Summer Institute (ASI'96), Toulouse(France) - LAAS Report 96319 (1996).

[5] M. Elkihel, G. Authié & F. Viader - An efficient hybrid dynamic algorithm to solve 0-1 knapsack problems - International Symposium on Combinatorial Optimization (CO'2002), Paris(France) - LAAS Report 01029 (2001).

[6] L. Pouch - Méthodes et/ou mixte pour la programmation linéaire en variables 0-1 - DEA Report, LAAS-CNRS Toulouse (2003).

[7] F. Ben Salem - Optimisation combinatoire : programmation linéaire en variables 0-1 - DEA Report, LAAS-CNRS Toulouse (1998).

[8] A. Freville & G. Plateau - An efficient preprocessing procedure for the multidimensional 0-1 knapsack problem - Disecrete Applied Mathematics 49 (1994) 189-212.

[9] G. Plateau - Contribution à la résolution des programmes mathématiques en nombres entiers - PHD thesis, Université des Sciences et Techniques de Lille (1979).

[10] S. Hanafi & A. Freville - An efficient Tabu search approach for the 0-1 multidimensional knapsack problem - European Journal of Operational Research 106 (1998) 659-675.

[11] R. Kohli, R. Krishnamaru, P. Mirchandani - Average performence of greedy heuristics for the integer knapsack problem - European Journal of Operational Research 154 (2004) 36-45.

[12] S. Martello, D. Pisinger, P. Toth - New trends in exact algorithms for the 0-1 knapsack problem - European Journal of Operational Research 123 (2000) 325-332.

[13] B. Garvish & H. Pirkul - Efficient Algorithms for solving multiconstraint 0-1 knapsack problems to optimality - Mathematical Programming 31 (1985) 78-105.

[14] S. Hanafi, A. Freville, A. El Abdellaoui - Comparaison of heuristics for the multidimensional knapsack problem - Meta-heuristics: theory and applications - Kluwer Academic (1996) 449-465.