

Reducing the total bandwidth of a sparse unsymmetric matrix

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For a sparse symmetric matrix, there has been much attention given to algorithms for reducing the bandwidth. As far as we can see, little has been done for the unsymmetric matrix A , which has distinct lower and upper bandwidths l and u . When Gaussian elimination with row interchanges is applied, the lower bandwidth is unaltered while the upper bandwidth becomes $l+u$. With column interchanges, the upper bandwidth is unaltered while the lower bandwidth becomes $l+u$. We therefore seek to reduce $\min(l, u) + l + u$, which we call the **total** bandwidth.

Band solvers are simple, avoid the need for indirect addressing, and can make good use of modern cache and vector hardware. If the matrix can be reordered to have small bandwidths, a band solver is likely to be faster than any other.

We consider applying the reverse Cuthill-McKee algorithm [2] to $A + A^T$, to the row graph of A , and to the bipartite graph of A . In addition, we introduce a variation that may be applied directly to A . We have also adapted the hill-climbing algorithm of Lim, Rodrigues and Xiao [3] for improving a given ordering to the unsymmetric case and have proposed a variant of the node-centroid algorithm of Lim et al [3] for unsymmetric A .

When solving linear systems, if the matrix is preordered to block triangular form, it suffices to apply the band-reducing method to the blocks on the diagonal. We have found that this is very beneficial for many matrices from actual applications.

Numerical results for a range of practical problems are presented and comparisons made with other possibilities, including the recent lexicographical method of Baumann, Fleishmann and Mutzbauer [1].

References

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