Randomized Rounding for Sensor Placement Problems

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The recent emphasis on homeland security in the U. S. has led to a number of new applications involving sensor placement in physical networks. We will describe some of these sensor placement problems including sensor placement in municipal water networks to minimize health effects from accidental or malicious contamination and sensor placement for intruder detection in transportation networks or buildings. We have addressed these problems using parallel integer programming. In this talk we present a parallelizable heuristic method for finding approximate solutions to general sensor placement problems.

An Integer program (IP) is the optimization (maximization or minimization) of a linear function subject to linear constraints and integrality constraints on some or all of the variables. IPs naturally model NP-hard combinatorial optimization problems. Thus integer programming is itself NP-complete, but one can frequently solve instances in practice using branch and bound via commercial or research solvers. The sensor placement problems we consider have n binary decision variables corresponding to the possible sensor locations. The IP must chose at most k sensors (set at most k decision variables to 1). The remainder of the IP sets (integral and/or rational) dependent variables that calculate the objective.

Removing the integrality constraints gives the linear-programming relaxation of an integer program. This is tractible both theoretically and in practice. IP solvers solve this LP relaxation to bound (sub)problems during the search for an optimal solution. One can use this LP solution to find a feasible integer solution. This can provide a fast approximation algorithm (frequently of provable quality) or it can speed an IP search by allowing early pruning of regions that cannot contain an optimal solution.

In this talk, we consider finding heuristic solutions to sensor placement IP problems using randomized rounding. This technique was introduced by Raghavan and Thompson [2]. It's a natural idea in its simplest form. Suppose all integer variables are binary. In the LP relaxation x^* of the IP, for each decision variable x_i, we have $0 \leq x_i^* \leq 1$. Treat each value x_i^* as a probability and round the variable x_i to 1 based with probability x_i. One must then compute the values of the other derived variables, either directly or by resolving the LP with the decision variables fixed.

Randomized rounding has been used to compute provably good approximations for some combinatorial optimization problems[4]. Usually these are for cases where all settings of the variables in the associated IP are feasible and rounding merely harms the objective value. This simple independent rounding is not a good strategy for general integer programs because the computed solution is almost never feasible for the linear constraints. However, the only constraint that might be violated in our sensor placement problems is the limit k on the number of sensors.

We will present a new randomized rounding strategy based on computing a "lucky" rounding. Suppose we were to independently randomly round each of the n decision variables and were lucky enough to select exactly k of them. This would be a feasible solution for the sensor placement problem. This lucky event is unlikely. We present a method to efficiently sample from this lucky distribution. In O(k(n-k)) deterministic serial preprocessing time we compute a data structure. [This can also be parallelized]. Then with a single random number and O(n) additional time, we can select a set of k sensors according to this distribution. When solving an IP in parallel, each processor can independently compute one or more solutions and we can take the best solution found by any processor.

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We will describe the algorithm and compare it to algorithms by Srinivasan [3] and Doerr [1] for randomized rounding with such hard constraints. Their algorithms are also fast, meet cardinality constraints, and have sufficient independence to allow Chernoff bound analysis. Our algorithm has stronger independence and a fundamentally different probability distribution for individual variables.

References

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