A Backtracking Correction Heuristic for Graph Coloring Algorithms

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OUTLINE

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- Role of Backtracking Heuristic
- The Backtracking Heuristic
- Results
- Conclusions

Introduction

- Distance-k Coloring
  - Assignment of colors to each vertex so that no two distance-k neighbors are assigned the same color
    (adjacent vertices are distance 1 neighbors)
  - Minimum color for distance-k coloring is the *k-chromatic* number

- Different Coloring Problems
  - Distance 1 (k=1)
  - Distance 2 (k=2)
  - Partial Distance 2 (k=2 on a subset of vertices)
  - Distance 3/2 (k=1 + every path of length 3 uses at least 3 colors)
  - Acyclic Coloring (k=1 + every cycle uses at least 3 colors)

- Applications
  - Compiler Optimization (scheduling use of registers)
  - Evaluation of Jacobians (detecting structurally orthogonal columns)

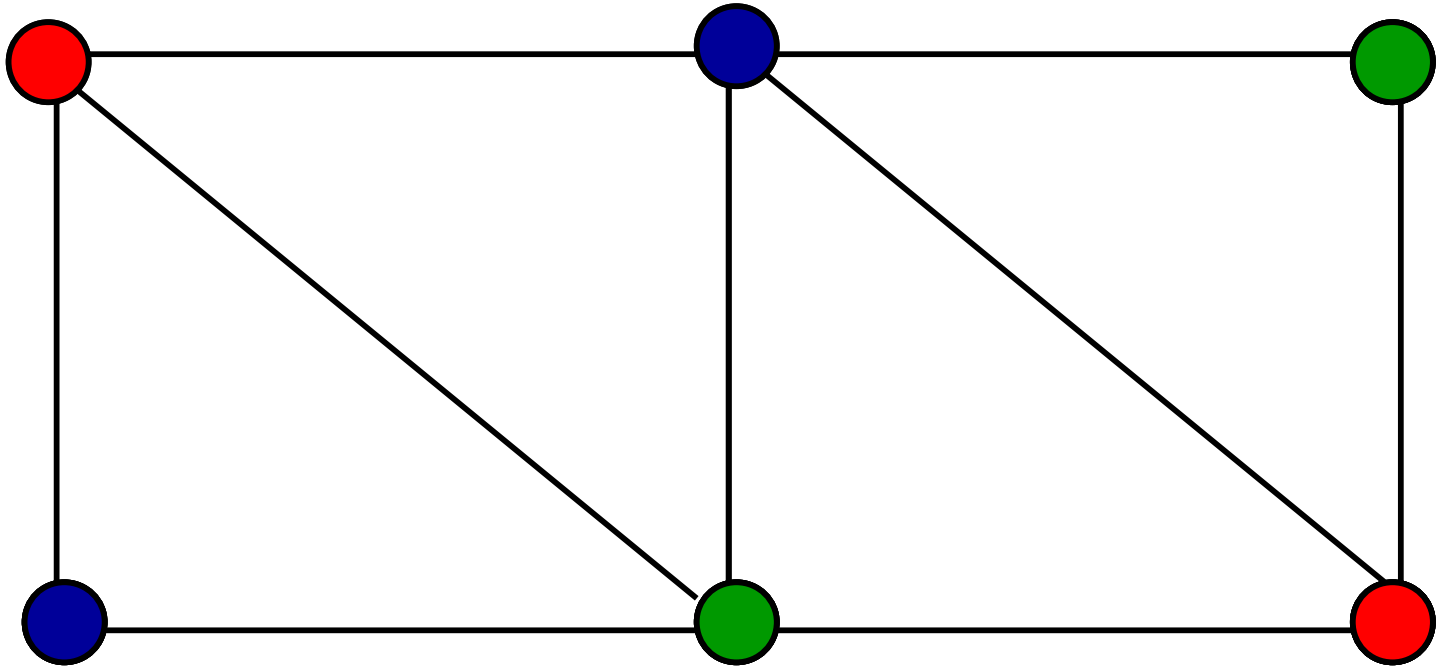
Role of Backtracking Heuristic

- Finding the least number of colors is NP-hard
- Different graph coloring heuristics

  Smallest Last, Largest First, Saturation Degree, Incidence Degree, Depth First, etc…
- Number of colors is often dependent on the order in which the vertices are colored
- Backtracking Heuristic
  - partially changes effects of vertex ordering
  - rearranges the color assignment
  - possibly lead to a lower number of colors

An Example

Minimum Color Required: 3

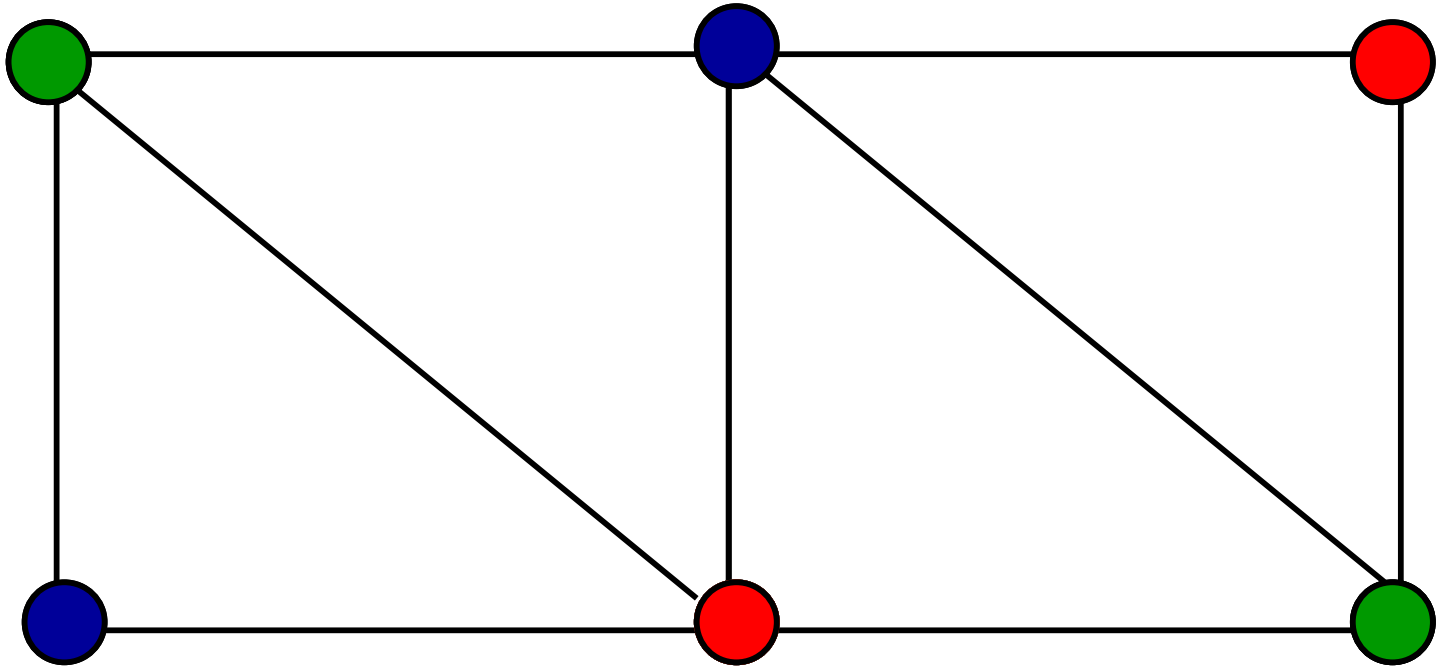


Backtracking Heuristic

- Set a threshold for expected number of colors
- Backtracking heuristic is invoked when number of colors exceed the threshold
- Only one vertex (*last vertex*) is colored higher than the threshold
- Assign a *pseudo-color* to the last vertex within the threshold
- Determine if there exists alternate color assignment to neighboring vertices; If alternate assignment found
  - Implement alternate assignment and last vertex retains pseudo-color
  - Else, threshold is increased by one

Backtracking Heuristic: An Example

Color Threshold: 3



Red Works

Backtracking Heuristic

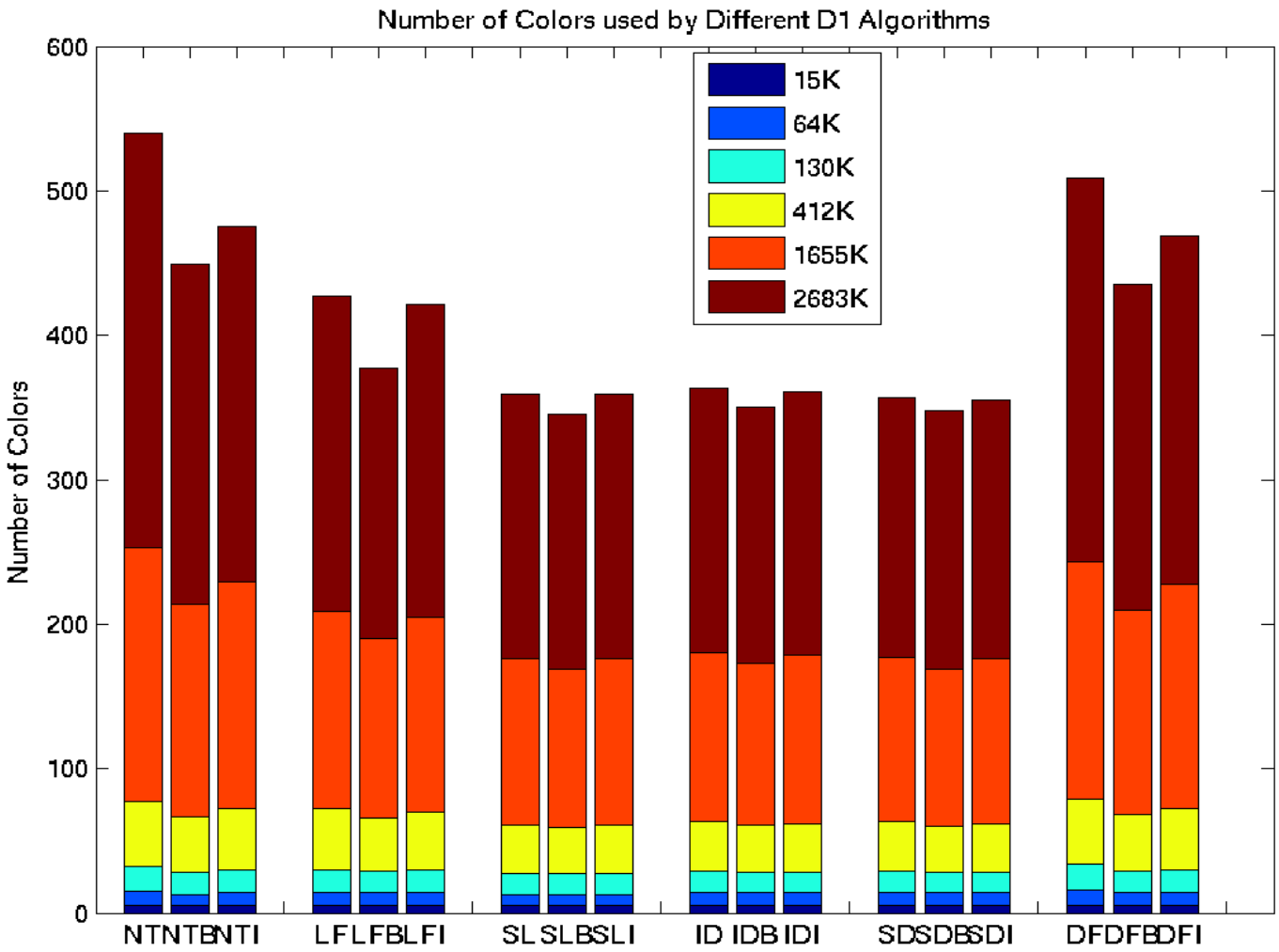
- Advantages
  - Easy to incorporate into other algorithms
  - Can be tuned to user specifications
  - Cost (execution time, memory) of backtracking at each vertex is proportional to its degree
- Disadvantages
  - Performance is limited by the top-level algorithm
  - It is based on a local greedy heuristic

Coloring is as good (often better) than the top level algorithm

Experiments

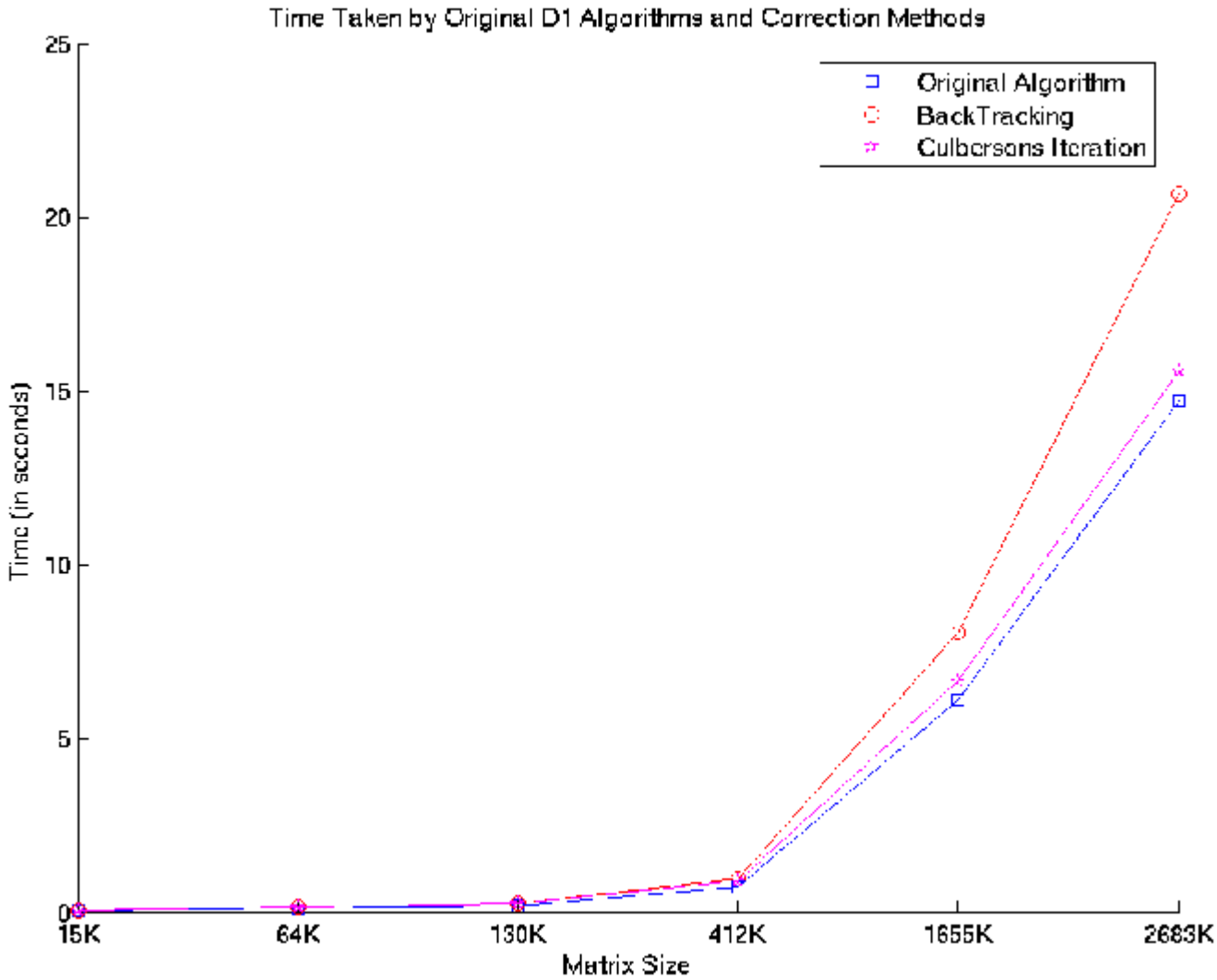
- A set of six matrices from molecular dynamics application
  - Vertices: 11414
  - Edges : [15K-2683K]
- Coloring Problems
  - Distance 1
  - Distance 2
- Algorithms Used
  - Natural, Largest First, Smallest Last, Incidence Degree, Saturation Degree, Depth First
- Correction Algorithms
  - Backtracking
  - Culberson’s Iterative Method
- Color Selection Strategy
  - Smallest First

Number of Colors Used by Different D1 Algorithms



NT: Natural
LF: Largest First
SL: Smallest Last
ID: Incidence Degree
SD: Saturation Degree
DF: Depth First
B: Backtracking
I: Culberson's Iteration

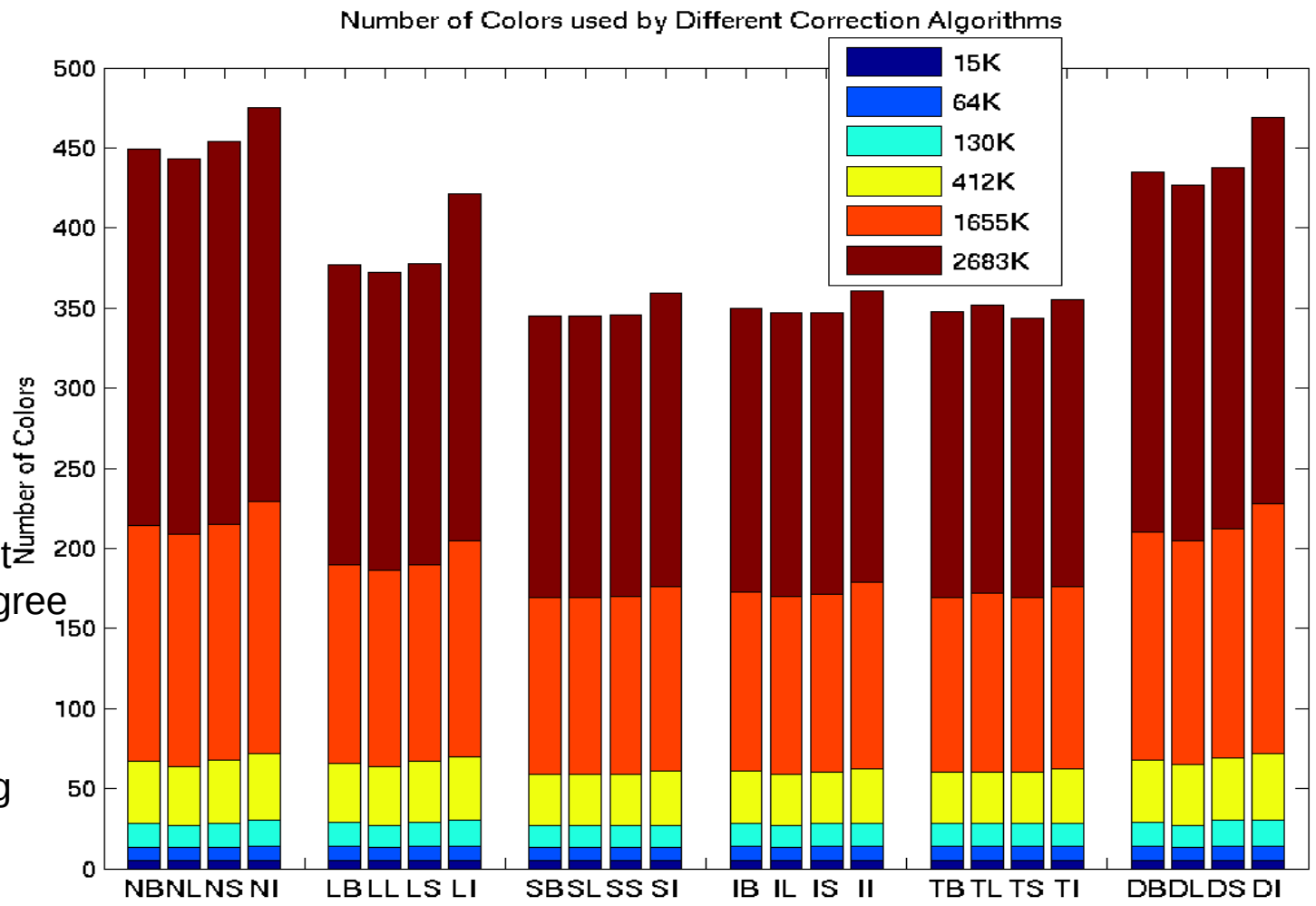
Time Taken by Original Algorithms and Correction Methods



Extensions to Backtracking

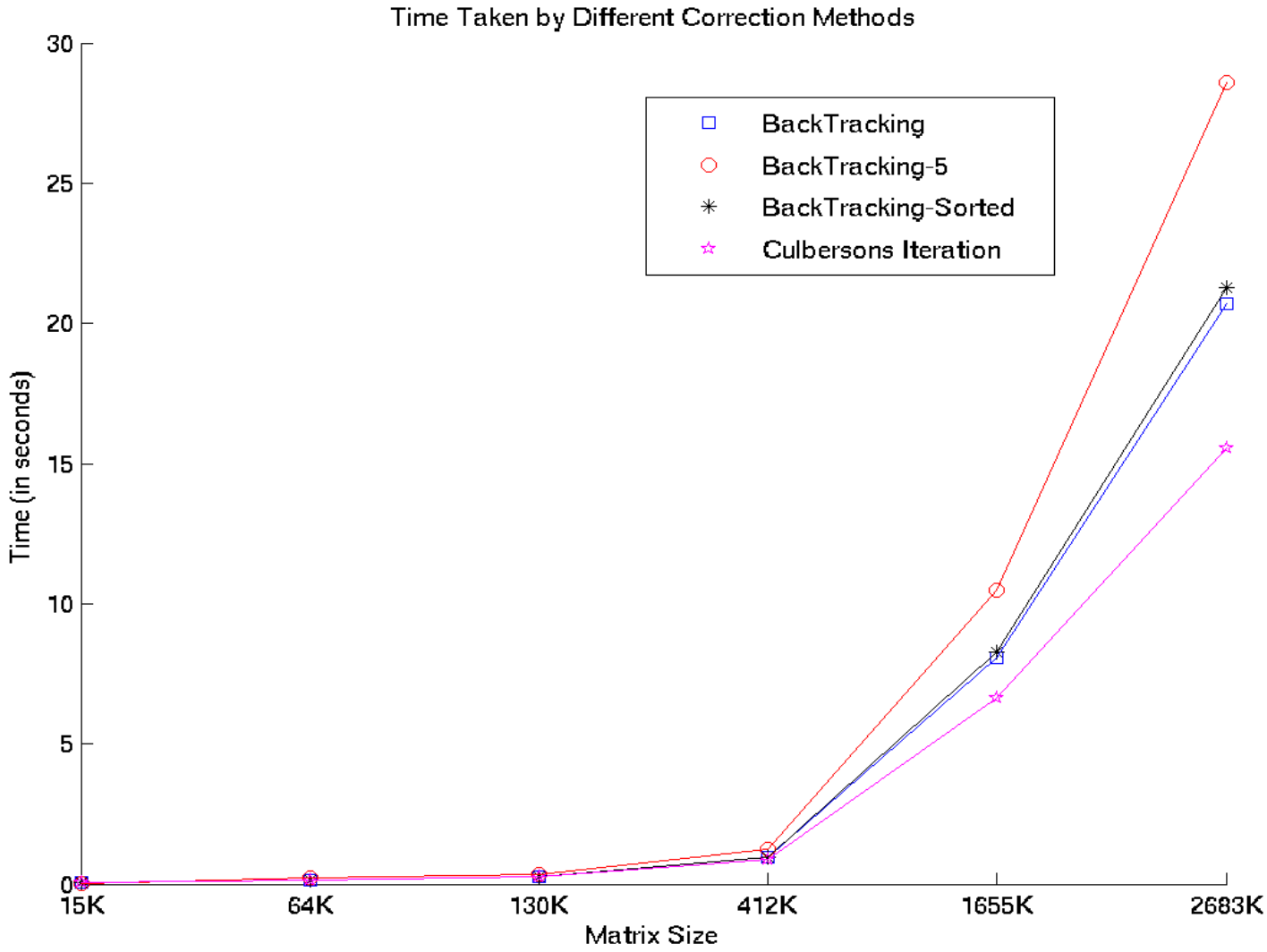
- Backtrack over Multiple Levels
  - Extend search for alternate color assignment to neighbor's of neighbor's and beyond.
    - Possibility of finding a better assignment
    - Time and memory requirement increases with levels
- Sorted Backtracking
  - While assigning pseudo-colors, start with the least used color among neighbors
    - Fewer neighbors with pseudo-color---possibility of fewer conflicts
    - Possibility of quickly finding alternate coloring
    - Extra time required for sorting

Number of Colors Used By Different Correction Algorithms

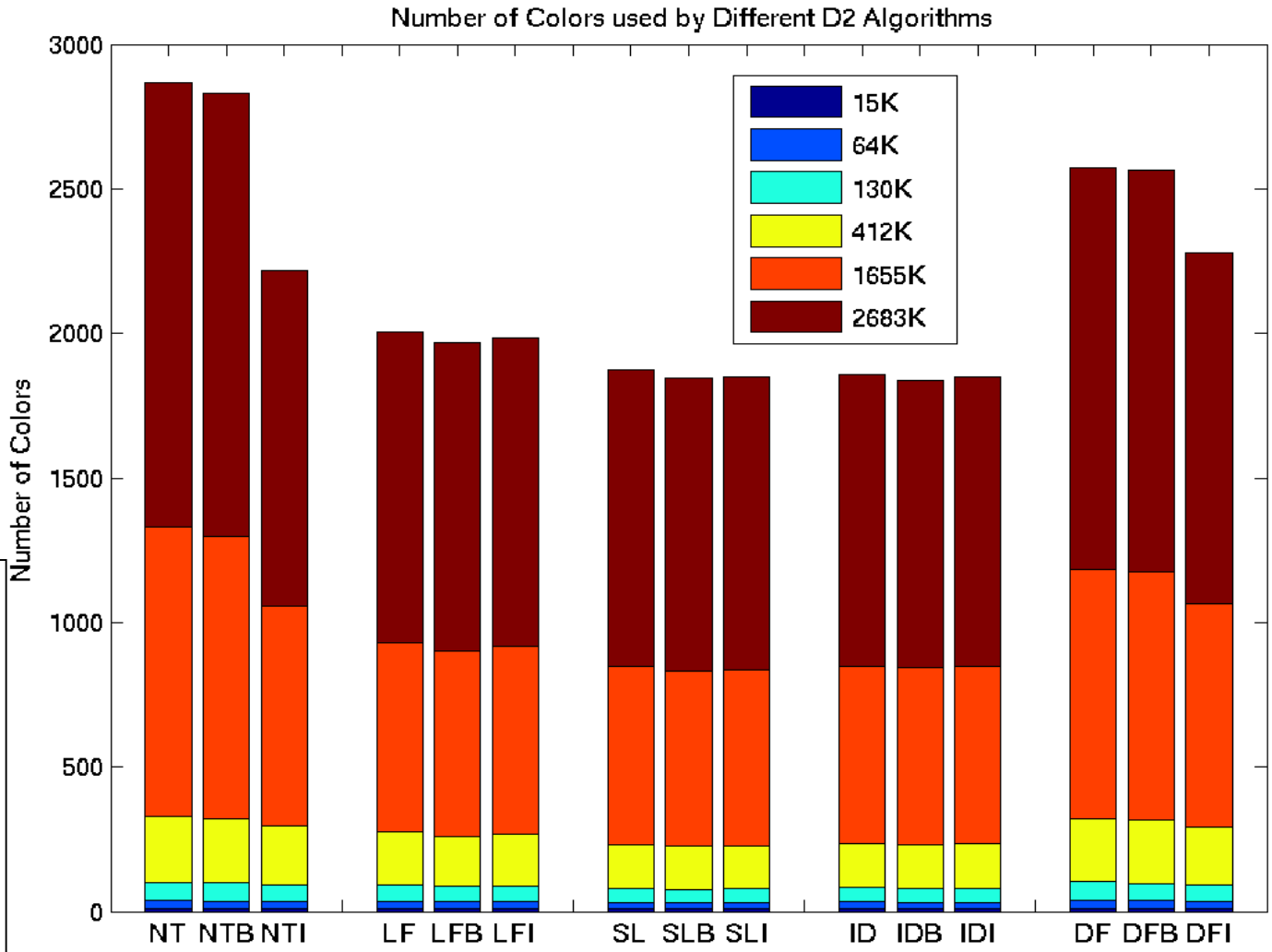


N: Natural
L: Largest First
S: Smallest Last
I: Incidence Degree
T: Saturation Degree
D: Depth First
B: Backtracking
L: Backtracking Level-5
S: Sorted Backtracking
I: Culberson's Iteration

Time Taken by Different Correction Algorithms

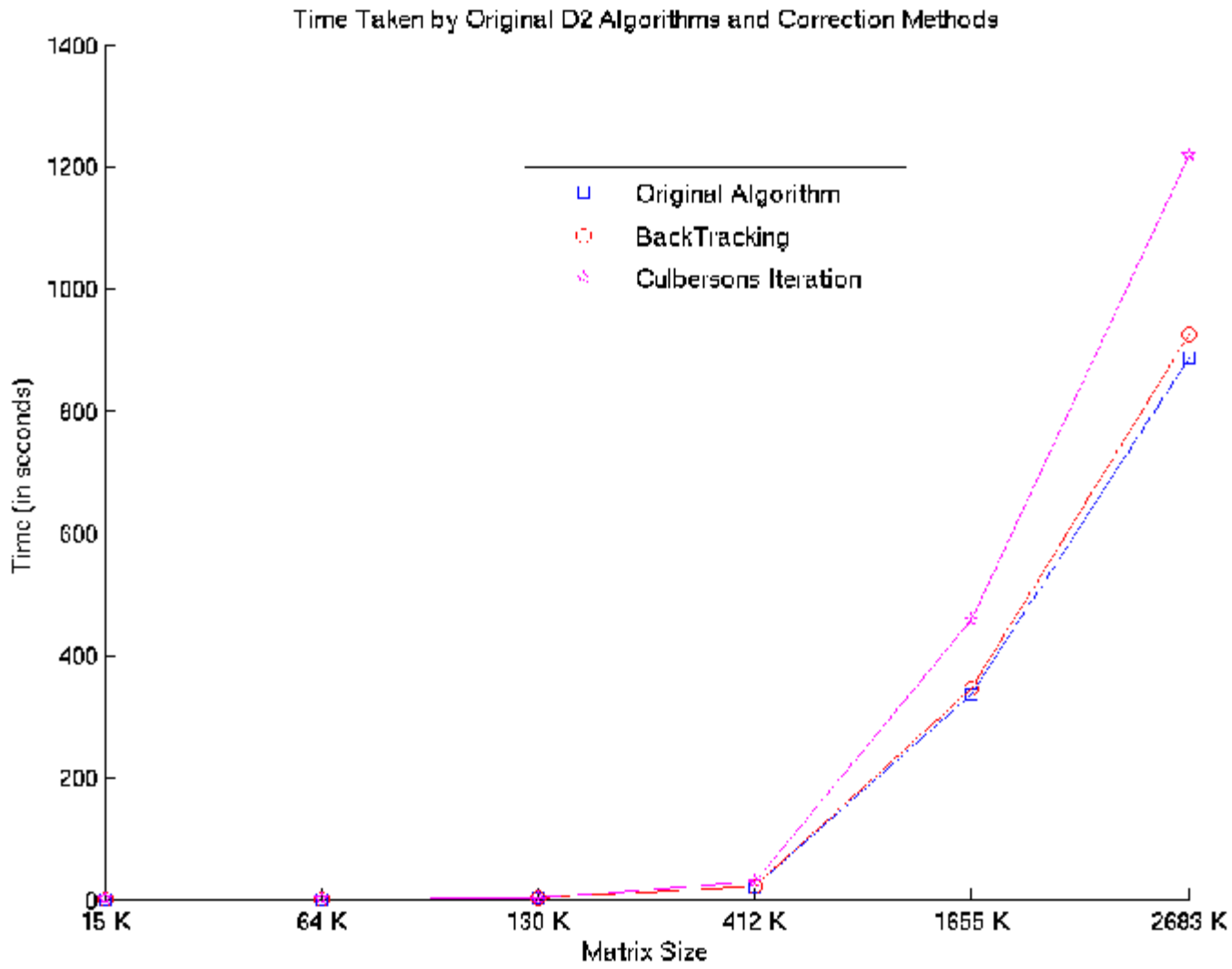


Number of Colors Used by Different D2 Algorithms



NT: Natural
LF: Largest First
SL: Smallest Last
ID: Incidence Degree
SD: Saturation Degree
DF: Depth First
B: Backtracking
I: Culberson's Iteration

Time Taken by Original D2 Algorithms and Correction Methods



Conclusions and Future Work

- Summary
  - Backtracking performs well for D1
  - Performance is not as good for D2-- more constraints to be satisfied
  - Time taken is not significantly high in either case

- Future Research
  - Implementing parallel backtracking
  - A better reordering scheme for D2 (and others?)
  - Analytical results

Acknowledgement

- Backtracking heuristic was inspired by reading a preprint of a review of graph coloring algorithms by Assefaw Grebremdhin, Frederik S. Manne and Alex Pothen

- Thanks to Assefaw Grebremdhin and Rahmi Aksu for use of their graph coloring code. Backtracking heuristic was implemented within this code

- Thanks to DOE for funding this research