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Impact of global warming on air transportation: Sensitivity analysis study on Takeoff distance computation

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MSc in Aerospace Engineering - Aerodynamics and propulsion

Master Thesis Report

Influence of climate change on commercial aviation: Sensitivity analysis study on takeoff distance calculation

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Nomenclature

$ICCA$ Influence of climate change on aviation

$IPCC$ The Intergovernmental Panel on Climate Change

CMIP5 Coupled Model Intercomparison Project 5

dthrust Percentage of thrust decrease

F_N Thrust force

GCMs General circulations models

ISA International standard atmosphere

L Takeoff length

OAT one at a time experiment

QOI Question of interest

RCP Representative Concentration Pathway

SA Sensitivity analysis

TOD Takeoff distance

TOW Takeoff weight

UQ Uncertainty quantification

1 Introduction

Climate change is on the number one global concerns nowadays. There are several aspects that contributes to the temperature rise and the changes in the atmosphere in general. It is predicted that by the end of the century with no changes in the actual human activities, and on the contrary a growth on the industries predicted with business as usual, the mean temperature of the planet can reach +5 or even +7 degrees in the worst cases scenarios. An increase of so many degrees will bring catastrophic consequences for the life on the earth as it is know today. This is why research all around the work are so worried and putting so much effort on quantifying the contributions for every industry that have a negative carbon footprint. Additionally, regulations and new rules are everyday more strict regarding the environmental heath and the prevention of the climate catastrophe. Today it is know that transport industry, including cars, ship and airplanes, are big contributors to greenhouse emissions and contamination. The Aviation sector is responsible for around 3% of total global emissions, that is why is so important to find new technologies and researchers all around the world are working hard on it. However, and what is also important is to evaluate the impact these changes on the atmosphere are going to affect the technologies performance as they are design to operate on actual environments and not on the extreme weather the future may have if nothing changes. This is why some time ago the impact of global warming on aviation started to be a topic of interest. There are some studies already on how climate change will potentially affect the aviation industry in terms of weight restriction days, and thrust and lift generation. However, there is not yet a study that focuses on the uncertainty quantification when studying the this topic. That is why, this report will present an uncertainty study with a simple engine model that leads to a goo methodology to perform this kind of studies with more difficult models.

This project is part of the ICCA program that aim to study the impact of global warming on climate change and will be explained in more detail later in this section. Following the introduction and the presentation of the generalities of the project there will be 6 more sections divided as follow. The second section encloses all the theory needed to understand the question and to solve it. This section will involve basic climate modelling, engine and statistics theory. Third section will present a summary on the state of the art of the subject followed bu all the methodology followed for this project in the Fourth section. Finally, section five include all the results and difficulties and to the sixth on finalices with the conlustions.

1.1 The company

1.1.1 ISAE SUPAERO

The Institut Supérieur de l'Aéronautique et de l'Espace is an establishment Associate Member of the Federal University of Toulouse, whose objective is to training and research in aeronautics and space. The main department involved in this project is the Department of Aerodynamics, Energy and Propulsion (DAEP), which includes 70 people, of which 6 Professors, 15 Associate Professors, about thirty young researchers (doctoral and postdoctoral).

1.1.2 CERFACS

CERFACS is a fundamental and applied research center, specialized in modeling and numerical simulation. Trough its facilities and expertise in High Performance Computing, CERFACS deals with major scientific and technical research problems of public and industrial interest.

CERFACS hosts interdisciplinary researchers such as physicians, applied mathematicians, numerical analysts, software engineers who design and develop innovative methods and software solutions to meet the needs of the aeronautics, space, climate, energy and environmental fields.

CERFACS is involved in major national and international projects and is strongly interacting with its seven shareholders : Airbus Group, Cnes, EDF, Météo France, Onera, Safran et Total. It is also associated with partners like CNRS (Associated Research Unit), Irit (common laboratory), CEA and Inria (cooperation agreements).

1.2 ICCA Project

The scientific objectives of the ICCA project are the development and implementation of methodologies adapted to the study and quantification of impacts related to climate change (CC) on Aviation. One of the main challenges for the aviation industry is the adequacy and possible evolution of the flight conditions and operations envelopes for which different aircraft models are currently manufactured and certified. It is thus necessary to specify the influence of warming and heat waves on aircraft performance (until impossibility take-off), the impact of changes in atmospheric circulation at altitude (medium position and fluctuations of the Jet Stream) on the conditions of shear and turbulence in clear air, the weather the consumption and therefore the releases, the change in the frequency of icing and turbulence, etc. This multidisciplinary field of research is recent; the first results are still very qualitative and partial and many questions remain. The ICCA aims to do a state of the art of existing scientific knowledge and remaining questions before defining a methodological road map to explore more quantitatively the impacts of the CC on aviation and the associated cascade of uncertainties. The program is lead by different collaborators

1.3 Project Motivation and objectives

1.3.1 Motivation

As exposed in the ICCA program section, it is important to study how the already visible changes in climate will affect the aviation as it is known today. On one hand it is well known and documented how Aircraft contribute to global warming through greenhouse emissions, contrails and soot. However, it is not very well known yet how the changes in the atmosphere due to the climate change, will affect the aircraft performance. This changes could bring several negative impacts for the aeronautical industry in economic terms. This is the reason why it is important to investigate the possible effects and generate more knowledge on this topic, so the industry can evaluate their options and be prepared for the future. Additionally, when knowing the risk that exist if the business as usual continues, not only for the future climate, but for the future of the enterprises as they are known today, it is possible to generate awareness and motivation to change. This is to say that if the industry knows as a fact, their methods that contribute to the climate change, will in the future also affect them directly they will make all the effort to become more sustainable and harm less the environment. On the other hand, it terms of academy importance, this project was motivated for the opportunity to apply the uncertainty quantification with an engine tool. Additionally, the collaboration between both entities and the immense opportunity to integrate sporadically climate models with engine modelling. This project is a big opportunity to contribute to the state of the art on this topic, since there is no previews studies on the uncertainty quantification of the impact on global warming on aviation.

1.3.2 Objectives

This project's general objective was to study the impact of global warming on the operability and performance of aircraft engines using uncertainty quantification applied with climate and engine models. This objective was extremely wide and due to different situations that will be explained in this report, the main objective to the project was reformulated and states as folow: **Focus on how changes in atmospheric variables could affect the takeoff distance, using sensitivity analysis indices, under different climate scenarios.**

2 State of the art

2.1 Aviation effects on climate change

The aviation sector represents one of the biggest economic activities of the modern world. However, their activity brings with it an important contribution to the changes in the atmosphere, that are translated into changes in climate. Quantifying the exact impact aviation has on the atmosphere is a really big challenge and the scientific community has been working hard to address this question. The reason of this difficulty is because aircraft generate more impact than just the actual emission of CO_2, an schematic overview of the processes by which aviation emissions and increased cirrus cloudiness affect the climate system is presented below in figure 1. The estimation of these non-CO_2 aviation effects has been particularly challenging. However, from what has been possible to calculate, the primary non-CO_2 effects comes from NO_x emissions, along with water vapor and soot that results on contrail formation [9]. Understanding the actual influence of aviation on climate forcing is very important due to its significant negative effects on the atmosphere.

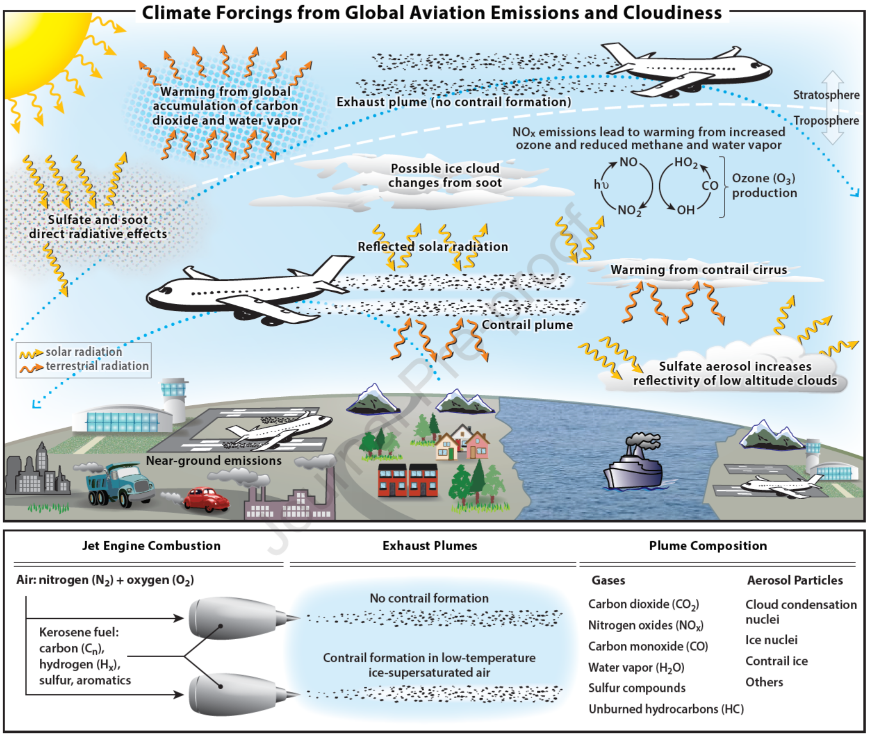


Figure 1: Climate forcing from Global aviation Emissions and Cloudiness [9]

It is important to remark that the knowledge and understanding of these effects has improved in the last decade however it remains incomplete. It is estimated that in 2005 Aviation contributed between 2 and 3% of total annual anthropogenic CO_2 emissions but possibly as much as 4.9% of radiative forcing (energy imbalance imposed on the climate system either externally or by human activities), including cirrus cloud effects [8]. However aviation has grown very fast in the last decade and the future scenario of its emissions for the future years is a big concern, taking into account that aviation impact on climate comes from both CO_2 and non CO_2 emissions and effects.

The Intergovernmental panel on climate change (IPCC) developed the SRES scenarios related to the aviation's impact on global warming in the future. These scenarios predicts the impact of aircraft's emissions assuming different possible scenarios. There are four main scenarios A1B, A2, B1 and B2, each one of those has a story line told in terms of some sociopolitical factors and their time series are shown in figure 2. These scenarios do not include mitigation policies for climate change. This represents a problem because while emissions from other industries are more and more regulated every year, the aviation non CO_2 forcing remain outside emission reduction targets. That will cause that while aviation if not currently one of the main drivers of global warming, emissions from aviation would become far greater contributors over the coming decades.

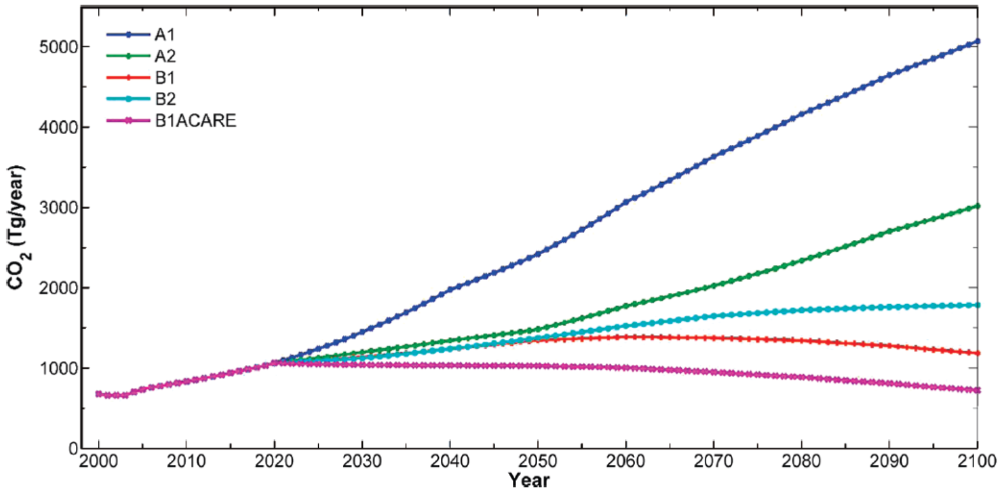


Figure 2: Time series of SRES aviation CO_2 emission scenarios (with outlook to 2100)[10]

2.2 Atmospheric changes and their impact on aviation

Changes in the atmospheric conditions imply alteration of aircraft usual performance. As will be explained in the flight envelope section, there is a range of altitudes, temperature and pressures for which an aircraft is allowed to operate, and within it will have the optimum and expected performance. These alterations can go from weight restriction to engine failures and in this section it will be presented a resume of some of the studies already found in the literature regarding this issue. Climate change has been studied for decades already, as summarized in section 2.1 and it is well known the effects the aviation emissions have on the atmosphere. However, now it is a concern for aircraft designers how these changes may affect the performance of the machines.

As stated before, the atmosphere's state is defined by the pressure, the temperature and the humidity, and the engines optimum point is build for specific atmospheric conditions. By changing the pressure,the temperature an the altitude, the density of the air will change and this will have an impact on the lift and the drag of the aircraft. Both aerodynamic features are function of the density, as explained in previous sections, which means they will be affected in some portion if the density varies too drastically. In the last decade the interest on studying this topic has raised and some research has been performed on the field. The influence of the atmospheric changes on aircraft performance as well as on the potential economic impact those changes may bring for companies are the biggest questions. The studies go from understanding the variations in thrust, lift and drag with the ambient conditions change, to estimations on fuel consumption and maximum takeoff weight for the future.

Balicki et al. [2] studied the effect of the atmosphere on the performance of aviation turbine engines. In this study they aim to answer the question "How pressure, temperature and humidity are affecting the performance of the aircraft turbine engines?". To do that the climate and the airport altitude influence on the engine performance in terms of lift and drag were studied and at the same time the humidity was taken into account. To answer those questions, the researchers perform the study using the software GasTurb. In order to investigate the effect of the altitude as well as the latitude, the international standard atmosphere, along with the hot, cold and tropical weather definitions were used depending on the airport to be studied as shown in Figure: 3.

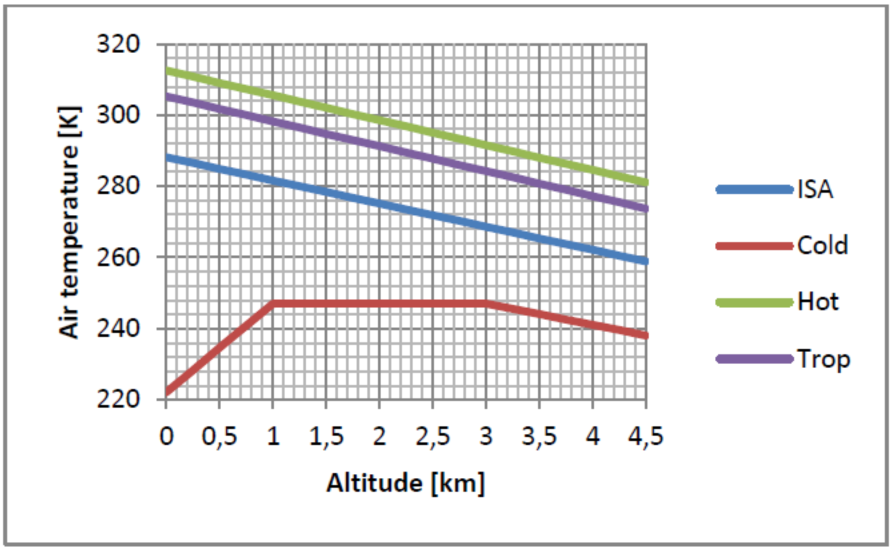


Figure 3: Changes in air Temperature depending on the altitude and latitude [2]

The main Balicki et al. [2] findings are summarize hereafter. It was found that when temperature and humidity increases, the lift force and the engine thrust decrease. This can be proven with the definitions of lift, drag and thrust presented in section 3. These parameters have a linear relationship with respect to the density, that varies with temperature, humidity and pressure. It was demonstrated that the humidity has certain influence on the thrust, however to be able to see that impact on simulations, it is important to take into account the different climate types already mentioned. On the other hand, the engine thrust decreases with the increased of altitude and temperature.

Ren and Leslie [11] perform a study to estimate the changes in fuel efficiency of a particular aircraft, using 26 different global circulation models. Here the different models were used to predict atmospheric conditions for different periods of time, all the models were taken from different and independent modelling centers to evaluate also their accuracy. A line by line adding method was used to estimate changes in fuel consumption integrating along the flying trajectory, in order to do so, one aircraft was selected and its aerodynamics was fixed.Additionally, airlines operation details of all the flights in a year used. Calculations over the mechanical and overall efficiencies and pressure drag were also computed.

The main findings of this study is that pressure drain trends to be proportionally affected by air temperature and density changes. Additionally, by the year 2100, the changes in mechanical efficiency are expected to be about 0.35% and the net reduction in overall engine efficiency is 0.7% with no changes in technology with respect to the actual one. Finally, it was found that the future fluctuations in tropopause elevation might involve changing the cruising altitude of commercial aircraft [11].

With the objective of determine the economic impact that climate change will bring to the aviation sector Coffel et al. [2] performed some research to quantify the effect of higher temperatures on the maximum takeoff weight restrictions in the future. For this study four different airports in the united states of America were consider. The recorded observations of maximum temperature and the weight restrictions trends in time were examined. Additionally, along with the recorded information, seventeen circulation models from the CMIP5 multi modem ensemble were used. These models proportionate the projected future temperature for the different airport locations. On the other hand,the number of weight restriction days per year was calculated using the observed temperatures, bias-corrected historical CIMP5 data.

It was found that the increased weight restriction is a potential climate change's impact on aviation. As an example of this conclusion, the number of weight restriction days per year in the past and future for the one of the airports is presented in the figure 4. The black line is calculated from observations and the red line is the observed trend. The gray error bars represent the standard deviation of the number of weight restriction days per year between the 17 CMIP5 models

Following the same path as Coffel et al [3], and on the frame of the ICCA program in 2019 a project was started with the objective of evaluate the weight restrictions days in the

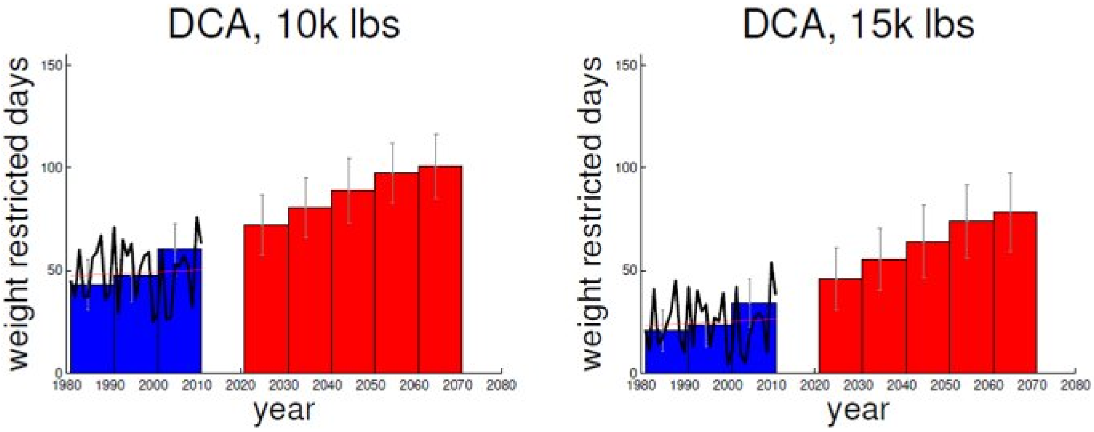


Figure 4: Number of weight restriction days per year in the past and future for the Ronald Reagan Washington National Airport[3]

future with the latest available data from climate models. Victoria Gallardo, researcher from CERFACS, developed her master thesis with the objective of quantify the possible impact of future climate change on the aircraft's takeoff based on extreme heat events. Similar to what was found for Coffel et al [3], Gallardo found that the percentage of days with weigh restriction will increase each decade due to the temperature rise. The predicted increasing rate of this percentage will depend, of course, on the climate scenario [5].

3 Theoretical background

3.1 Atmosphere and climate models

3.1.1 Atmosphere definition

The atmosphere is the accumulation of gases surrounding the planet. Withing this gases, the nitrogen and the oxygen are the most abundant ones. There are different criteria to stratify the Earth's atmosphere, one of the most common ones is based on temperate distribution [NASA REF]. Based on that, there are different layer that are the troposphere, the stratosphere, the mesosphere and the thermosphere. As these layers are defined by the temperature, their longitude will varies depending on the region. It is the troposphere which is of major interest for aeronautics since most of the aircraft operate within this layer, as shown in figure: 5. The state of the atmosphere is defined by its temperature, pressure and its humidity, and extreme variations of those variables lead to changes in the atmosphere as it is known today, leading to changes in the troposphere level.

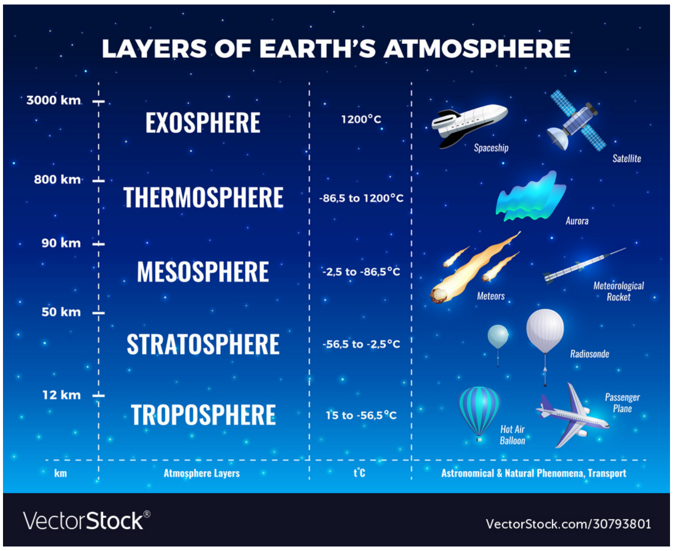


Figure 5: Atmosphere layers

3.1.2 International standard atmosphere ISA

The real atmosphere does not remain constant at any time and place. However, for aviation purposes it is imperative to have a reference to calibrate the temperature, the pressure, the humidity and the speed of sound distributions. An hypothetical model was constructed assuming the air free from dust, moisture, water vapor, winds and turbulence, leading to a steady air with respect to the earth. The international civil aviation organization ICAO accepts the standard atmosphere since 1962 and its values at sea level are presented in table:1

Pressure	P_0	101325	N/m^2
Density	ρ_0	1.225	kg/m^3
Temperature	T_0	288.15	K
Speed of sound	a_0	340.294	m/s
Acceleration of gravity	g_0	9.80665	m/s^2

Table 1: Standard atmosphere definition (ISA) at sea level

As mention before, the air is consider a perfect gas in the ISA model and it is said that the temperature remains constant from the tropopause up to 20000m and starts decreasing linearly with respect to the altitude at a constant rate of -6.5 as shown in figure

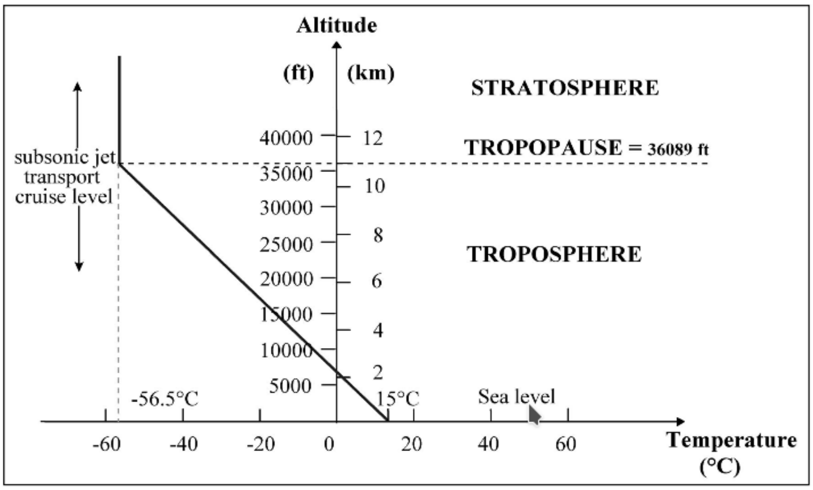


Figure 6: Temperature variation ISA

$$T = T_0 - \frac{6.5 * h(m)}{1000} \tag{1}$$

As this model is used to compare the real atmospheric conditions to the corresponding engine performance, the temperature, and in general the atmospheric conditions, at a given flight level will be express as ISA $\pm \triangle ISA$

3.1.3 Climate models

Climate models are build to help understanding complex systems. They simulate the transfer of energy and materials through the climate systems and are also known as general circulation models GCMs. These are numerical models which try to simulate the atmosphere, ocean and sea ice behaviour. Thank to these models that today it is possible to predict the weather in the next week and in hundreds of years, it is also possible to calculate how the climate was in the past, centuries ago. These models also allow the calculation of the requirements in carbon dioxide emission diminution needed to reach climate targets. To perform these simulations, climate models divide the earth's surface into a three-dimensional grid of cells as shown in figure 7. There are different model resolution and they are defined by the grid cell size.

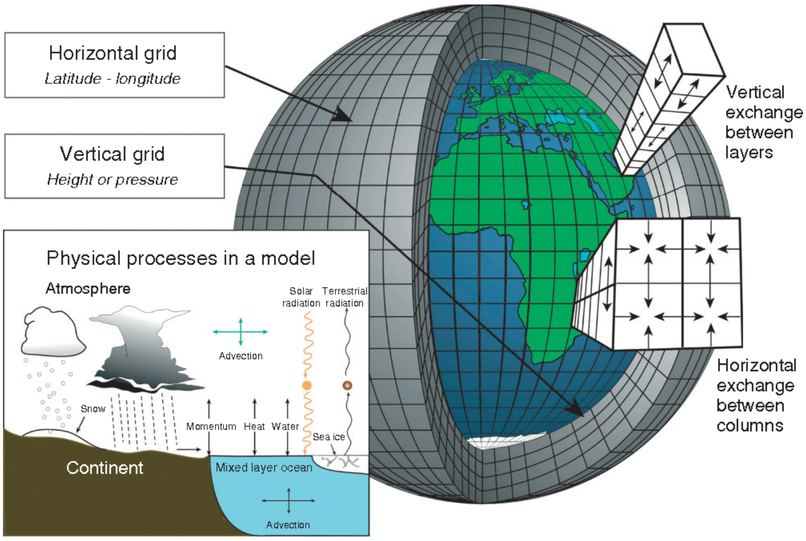


Figure 7: Climate model scheme [4]

There are different types of models that scientist use to study the earth's climate behaviour. The list goes from very simple energy balance models to very complex Earth system models (ESMs). The standard climate models assessed in the last intergovernmental panel of climate change (IPCC), were the Atmosphere-Ocean General circulation models (AOGCMs). The objective of these numerical models is to understand the dynamics of the physical components of the atmosphere, the ocean, the land and the sea ice and make projection depending on different greenhouse gas and aerosol forcing. Apart from the AOGCMs and the ESMs, there are the Regional climate models (RCMs), they are used typically to evaluate some particular geographical region to provide more detailed information. The resolution of the models depends on their complexity, and what is used to simulate the atmosphere and the climate phenomena. The resolution is defined in horizontal resolution,typically 1 to 2 degrees and vertical layers, typically around 30 to 80. This typical regional climate model resolution improves each time and it has increased from 50km to 25km, being 10km the smallest cell size ever run for a regional climate model[6].

3.1.4 Uncertainty in climate change predictions

Climate change predictions will depend not just on the model resolution, but also in several aspects that may cause the prediction to be underestimated and uncertain. The uncertainty in climate predictions derives from 3 main sources [18]:

1. **Forcing:** This represents the incomplete knowledge in external factors as:

i Future trajectories of anthropogenic emissions of greenhouse gases

ii Stratospheric ozone concentrations

iii Land-use changes

2. **Model response:** Basically, different models have different responses to the same external forcing, one cannot expect that one multimodel ensemble will represent the full spectrum of model uncertainty. Usually, the techniques that try to include measures of model uncertainty only sample an ensemble (e,g CMIP5), which may not be representative of all the population of all physically plausible models.

3. **Internal Variability:**The internal variability is the neutral variability of the climate system in the absence of external forcing. It has been found to be the dominant source of uncertainty in the simulated climate response at middle and high latitudes, and it is associated with the annular modes of circulation variability. It has also been found to play a large role in uncertainties of future sea level pressure projections, particularly at higher latitudes.

Temperature predictions and uncertainty:

Several studies (in the first decade of the 2000s) tried to estimate the uncertainty of the temperature predictions based on expert judgment and/or on the range of values found in the Atmosphere-Ocean general circulation model, the IPCC previews reports and different models data. The latest report of the IPCC presents the range of the predicted temperatures for the different scenarios and human activities are estimated to cause +1C of global warming by 2030, with respect to the pre-industrial levels, with a likely range between 0.8C and 1.2C. However, this information is the mean average and does not represent the prediction for a particular place on earth, but it gives an idea of the range in which the prediction might vary.

Pressure and humidity changes and predictions and uncertainty details:

Climate models also include the air pressure which changes with global warming. Consequently, changes in air pressure could have a big effect on climate. It has shown that there is also a greenhouse effect on air pressure. Air pressure controls the atmosphere's circulation then influences the humidity. That it to say, that regarding the climate aspects, everything is co related somehow.It has been found that the change in air pressure has been underestimated in the last years and it is known that uncertainty is usually larger for sea level pressure and for air temperature.

3.1.5 Climate scenarios

Predicting how the climate will evolve in the next century is not an easy question, and it depends on a countless number of parameters and variables that interact on the atmosphere and that evolve constantly. To answer this question it is mandatory to define particular scenarios depending on how much human societies develop and how much resources are used in this developing process. The IPCC created future scenarios predicting the greenhouse gas emissions associated with human developments. When reporting this scenarios, scientist affirm that even without climate policies, future emissions depend very much in people decisions. Following this, a set of Shared Socioeconomic Pathways SSPs were defined and developed. These pathways are used as a reference in climate modeling, and they describe possible trends in the evolution of ecosystems and society. The goal of these scenario framework is to explore the uncertainty in mitigation, adaptation and impacts that comes with climate and socioeconomic changes in the future. This scenarios takes as a reference the Representative concentration pathways RPC, that provide time-dependent projections of atmospheric greenhouse gas concentration and their trajectory [6].

The RCP scenarios are used for the IPCC on its lattes reports to predict climate behaviour in the following century. These scenarios take into account assumptions about economic activity, energy sources, population growth and other socioeconomic aspects. There are four different scenarios bases on different radioactive forcing as follow: RCP8.5 with a rising pathway, RCP6.0 and RCP 4.5 with a stabilization of the pathway without overshoot, and finally RCP2.6 with a peak and decline of the pathway as shown in 8..., where the number indicates the radioactive forcing in Wm2 in 2100.

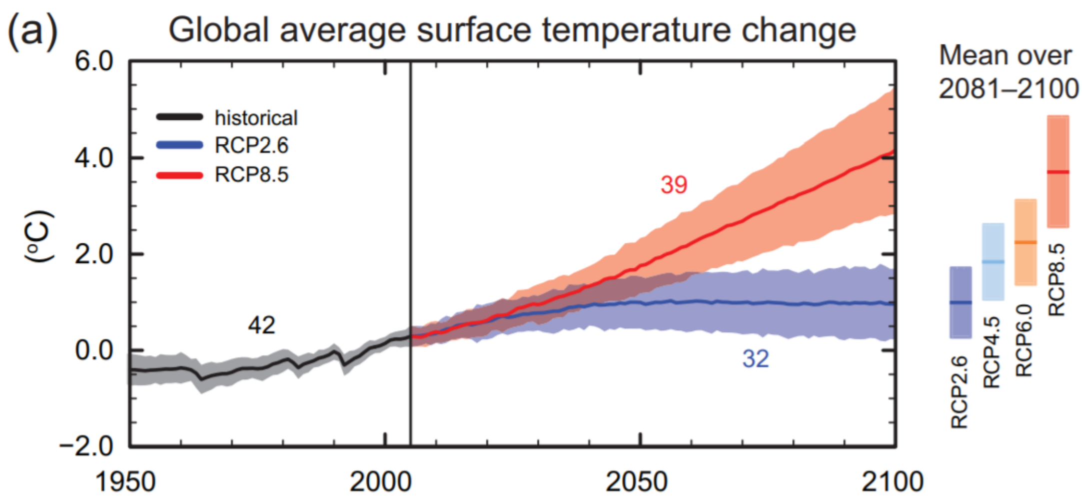


Figure 8: IPCC Representative Concentration Pathways [6]

3.2 Engine generalities

3.2.1 The aircraft environmental flight envelope

The flight envelope of an aircraft is defined by the ranges of static pressure and altitudes within the airplane can operate. It defines the maximums and minimums atmospheric conditions for which the aircraft was designed.

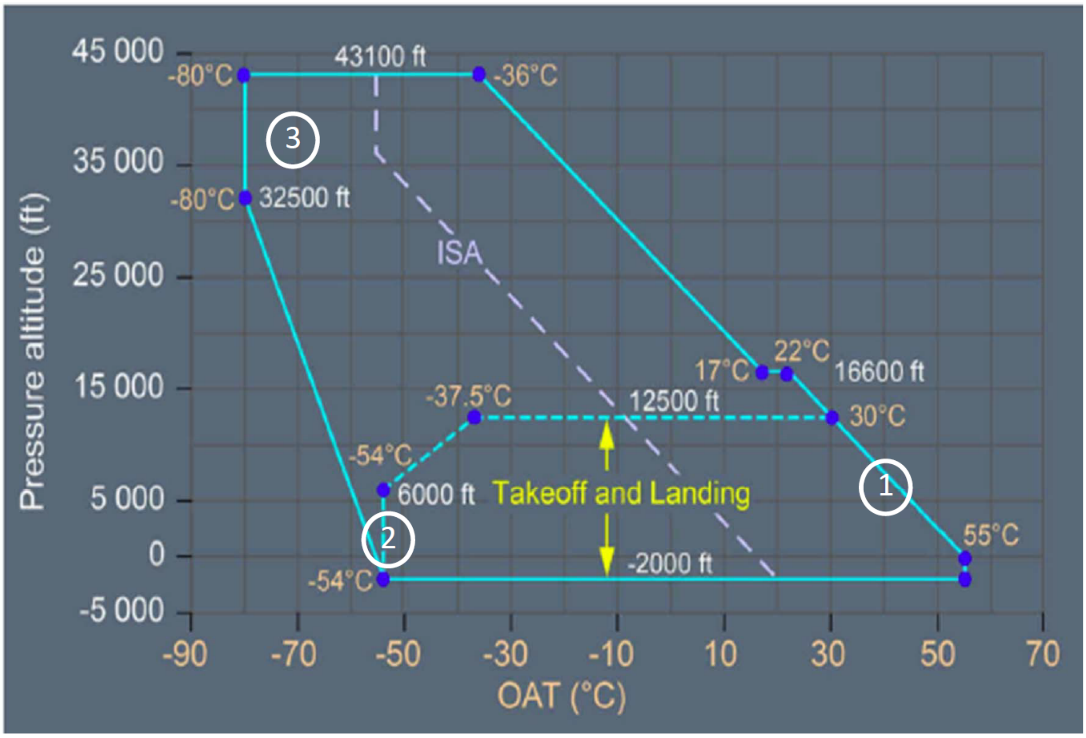


Figure 9: Typical environmental flight envelope*Taken from AIRBUS presentation

3.2.2 Modelling of the Takeoff distance

Takeoff and landing phases are the most dangerous ones during any flight, that is why studying its performance and the changes it could have due to atmospheric changes, is of great importance for aircraft safety. This section aims to describe the generalities that are considered when calculating takeoff distance and show how temperature changes might affect badly the normal conditions when performing takeoff. When talking about takeoff there are several factors that are involved. The most important ones are speed, distance and of course regulations. Regulation is important due to the different possibilities an aircraft may encounter during takeoff. To start with, one or more engines may fail suddenly and it may encounter a variety of situations. There are four situations that should be considered when calculating the takeoff of an aircraft, and are listed below and are shown in figure 10:

- Takeoff with all engines operating (AEO)
- Takeoff with one engine inoperative (OEI)
- Rejected takeoff with all engine operative
- Rejected takeoff with one engine inoperative

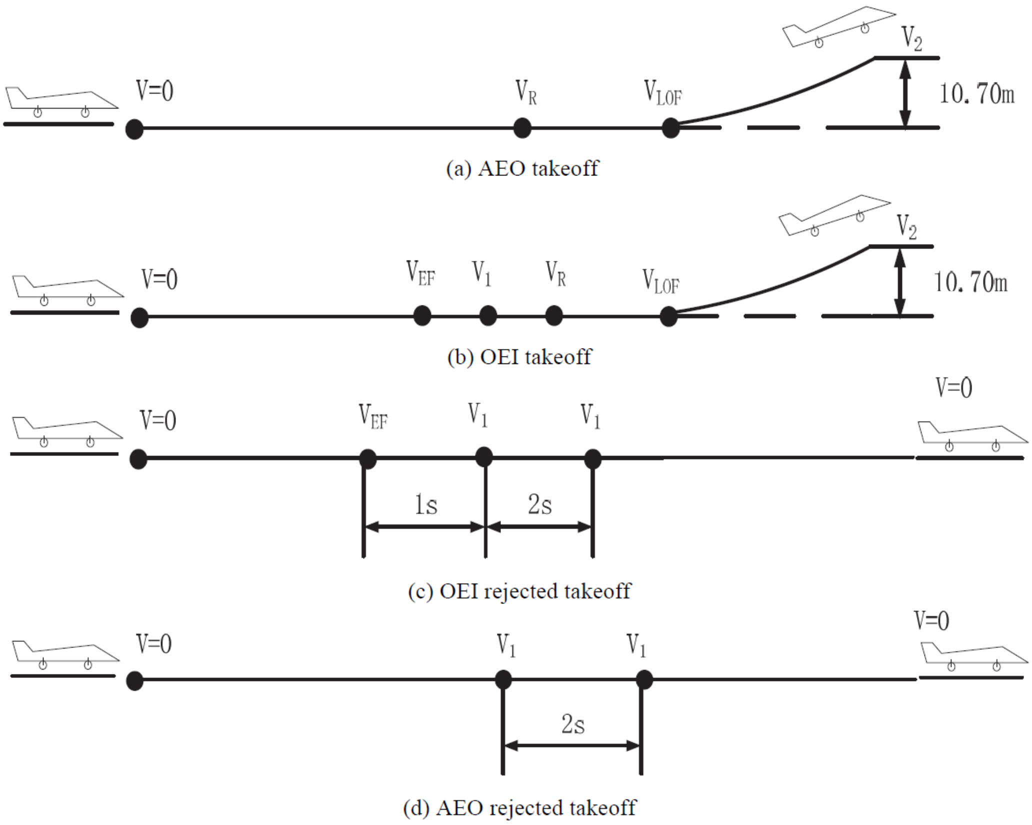


Figure 10: Takeoff scenarios [19]

Figure 10 shows the possible situations for takeoff and additionally marks some important speed that will be explain hereafter. To start with V_{EF} is Engine failure speed, at which the critical engine is assumed to fail. This spedd must be selected during the calculation of the OEI takeoff by the applicant. Same way V_1 is also selected by the applicant; however *"V_1 may not be less than V_{EF} plus the speed gained with critical engine inoperative during the time interval between the instant at which the critical engine is failed, and the instant at which the pilot recognizes and reacts to the engine failure, as indicated by the pilot's initiation of the first action to stop the airplane during accelerate-stop tests"*[19]. V_R is rotation speed, the speed at which pith angle starts to increase and V_{LOF} is lift-off speed at which the airplane loose contact with the ground. V_2 is takeoff climb speed the speed at which the aircraft reaches the 10.3 meters (35ft) or 15ft if the runway is wet of contaminated; it must be maintenaid until at least 400ft. V_{EF}, V_1, V_R, V_{LOf} and V_2 are important takeoff speeds and some limits are prescribed by law and are presented below

[19]:

$$\begin{cases} V_{EF} \geq V_{MCG} \\ V_1 \leq V_R \\ V_R \geq 1.05 V_{MC} \\ V_{LOF} \geq 1.1 V_{MU}, \quad AEO \\ V_{LOF} \geq 1.05 V_{MU}, \quad OEI \\ V_2 \geq 1.1 V_{MC} \\ V_2 \geq 1.2 V_S \end{cases} \tag{2}$$

Where V_{MCG} is the minimum control speed on ground, V_{MU} is the minimum unstick speed, that is the lowest calibrated airspeed at and above which the aircraft can safely lift off from the ground and continue takeoff without facing critical conditions, V_S is stall speed that occurs at the maximum aircraft lift coefficient which is also known and is defined as:

$$V_S = \sqrt{\frac{2(G/S)}{\rho C_{LMAX}}} \tag{3}$$

with G being the aircraft weight, S the aircraft wing area and ρ the atmosphere air density.

Additionally from the already mention, some of the basic regulation principles for take-off distance calculation include:

i Failure of one engine must be envisaged at any time during takeoff

ii It has to take into account all types of runway (dry, wet, contaminated)

iii CG max forward must be considered since it is the most critical term when taking about drag and lift capability

Accordingly with the regulation, the takeoff distance TOD must be calculated for the total number of engines TOD_n and for the takeoff distance with one engine inoperative TOD_{N-1}. This way it is possible to ensure that the airplane will be able to takeoff in any of the case scenarios already mentioned before. To illustrate these distance, figure 11 shows the corresponding distance for both cases.

As already stated before, the takeoff distance will depend on the runway conditions. Here only dry runway will be consider and to resume figure 11 the following formula is proposed:

$$TOD_{dry} = Max(TOD_{N-1}, 1.15 TOD_N) \tag{4}$$

Additionally, it is important to take into account some other limitation called the "Second segment limitation". First, in order to understand what his limitation refers to, it is important to identify all the takeoff flight paths and they are presented in figure 12.

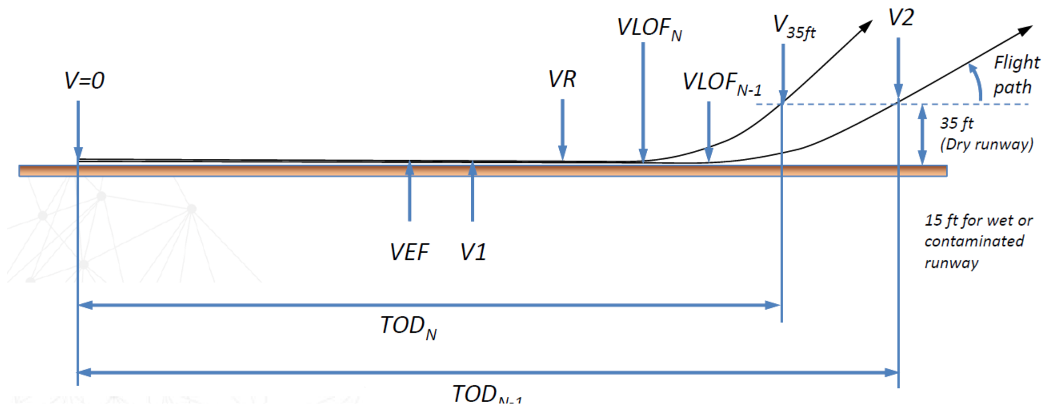


Figure 11: Takeoff distance

This limitation is a constraint for Airbus and Boeing aircraft and the conditions are listed below:

- Landing gear is fully retracted
- Engines are at takeoff thrust
- Flaps/slats are in the takeoff configuration
- Maximum height is 400ft

This segment is important because in most of the cases it is the performance limiting segment of the climb. Each segment has a minimum gradient that in the case of the second segment goes from 2.4 -3.0 percentage depending on the number of engines. In some cases V_2 may have to be increased to meet this requirement and that will depend on the temperature among other factors.

As it is possible to imagine at this point, calculate the takeoff distance while taking into account all the restrictions and requirements is not an easy task. However, using a very simplified approach it is possible to calculate a reasonable order of magnitude of the takeoff distance length against primary parameters as will be exposed hereafter:

$$\frac{d^2L}{dt} = a \tag{5}$$

where L= field length and Knowing that,

$$V = at \tag{6}$$

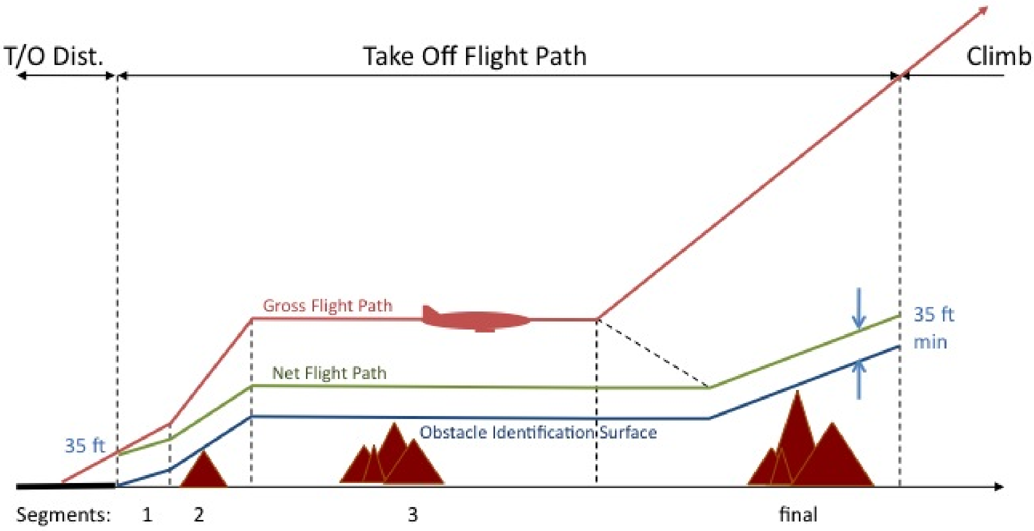


Figure 12: Takeoff flight paths

$$L = \frac{at^2}{2} \tag{7}$$

It is possible to express L like,

$$L = \frac{V^2}{2a} \tag{8}$$

and,

$$ma = F_N + F_{contact} + F_{aero} + ... \tag{9}$$

when neglecting all external forces, it is possible to express the acceleration as:

$$a \approx K\frac{F_N}{m} \tag{10}$$

and,

$$mg = \frac{1}{2}\rho V^2 S \frac{CL_{\max}}{K_{vs1g}^2} \quad \longrightarrow \quad V^2 = \frac{2mgK_{vs1g}^2}{\rho SCL_{\max}} \tag{11}$$

From 10 and 11 it is possible to express L as follow:

$$L = \frac{V^2}{2a} = \frac{2mgK_{vs1g}^2}{\rho SCL_{\max}} \frac{m}{2KF_N} \tag{12}$$

Finally, it has been shown that using a polynomial adjustment can add some precision to the calculation. The aim of computing the polynomial coefficients is to minimize the error between the formula and the available experimental or higher fidelity computational data. Which lead the following formula:

$$L \approx K' \frac{TOW^2}{\rho F_N SCL_{\max}} \tag{13}$$

For the purposes of this study, the magic line factor is used, this factor is recommended by Professor Thierry Drout from ENAC university. The factor states:

$$MG = \frac{TOW^2}{CZ_{max} F_N S \rho} \tag{14}$$

Where, TOW is the takeoff weight, CZ is the drag coefficient, ρ air density, S the wing area, and F_N the engine thrust. Finally, using the magic line factor and calculation of the polinomial coefficients, it is possible to have a final approximation for the takeoff calculation with the following shape:

$$L = 15.5MG + 100 \tag{15}$$

3.3 Uncertainty quantification

Uncertainty quantification (UQ) aims at identifying and quantifying all the relevant sources of uncertainty of a particular quantity of interest (QOI). Figure 13 shows how from the evation of the model, considering the inout uncertainties, it is possible to obtain the probability density function of the QOI. In other words, it aims of quantifying how uncertainty in the inputs translate in uncertainty in the output. Some of these include, uncertainty in the mathematical model, uncertainties on the experimental measurements, uncertainties in the numerical simulation, etc. It is important, in order to perform a robust uncertainty study, to follow some steps. In the first place all the uncertain sources might be identified, followed by characterizing the probability distributions or uncertainty ranges of these sources, propagating them through the model by designing an experiment that draws samples from the uncertain parameter space and finally run the simulations (when performing computational studies). Once the model is evaluated, the impact of the these uncertain sources on the variability of the QOIs is analyzed. In order to perform a comprehensive UQ analysis is important to select carefully not just the model to be used, but also the mathematical and statistical tools. On the other hand, it is possible to take advantage of experimental data while performing UQ, this allows to feed the model with credible distributions of ranges of their parameters[17]. However, if the aim is to quantify the contributions of specific sources to the overall model input uncertainty, it is necessary to perform a sensitivity analysis, that can give accurate information regarding the relative contribution of uncertainties from each source [14] and will be explained in detail in the following section.

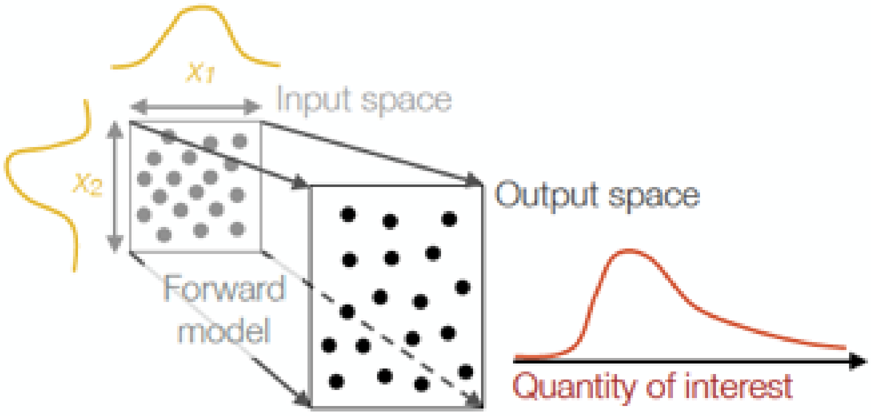


Figure 13: Uncertainty quantification scheme [12]

3.3.1 Sensitivity analysis

Sensitivity analysis (SA) differs UQ as it allows to quantify the relative importance of model input parameters and assumptions. It is used for many different purposes but mainly as a tool to quantify the contribution of model inputs, or sub.groups of inputs, to the uncertainty in the model output. In figure 14 an idealized uncertainty and sensitivity analysis scheme is presented. In here "the uncertainty coming from heterogeneous sources is propagated thought the model to generated an empirical distribution of the output of interest. The uncertainty in the model output, is then decomposed according to source, thus producing a sensitivity analysis" [14].

The good practices while performing sensitivity analysis are crucial while seeing for reliable results. In order to ensure this, it is important to follow the next steps:

- Define the goal of the analysis, from the output function that will answer your question.
- Define the inputs that should be considered as uncertain in the study.
- Estimate or set the probability density function for each of the input factors
  - i Taken from the literature
  - ii Derive from data by fitting an empirical distribution function
  - iii Based on an expert's opinions
  - iv Chosen to be a truncated normal distribution, where truncation serves to avoid sampling outliers
- Select a sensitivity analysis method considering the following:
  - i What is the problem to address?

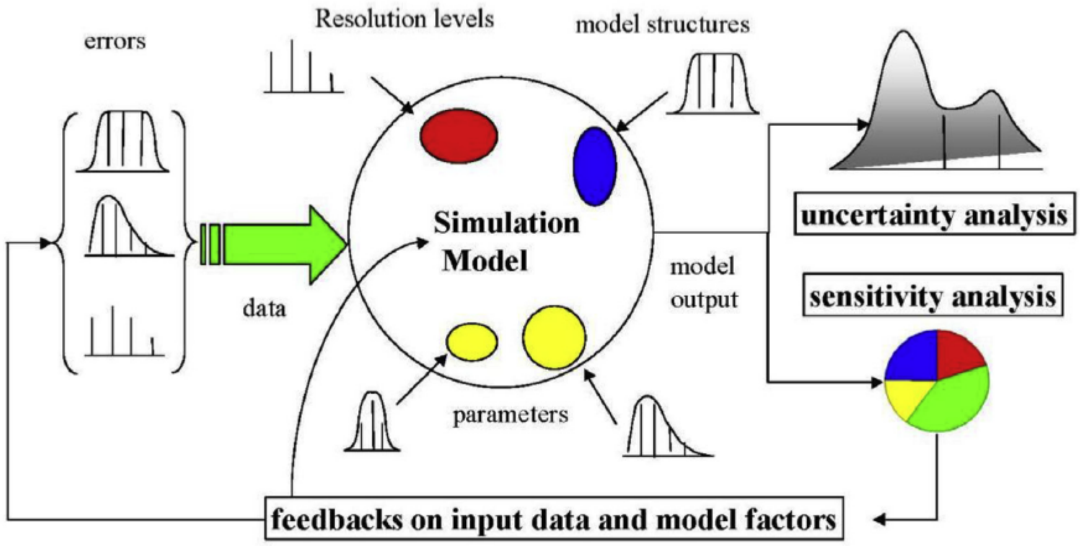


Figure 14: Uncertainty and sensitivity analysis [14]

(a) Screening problem

(b) Need of a quantitative method to be able to answer your final question

ii Identify the affordable number of model evaluations

iii Identify the presence of a correlation structure between input factors

(The methods used in this project will be explained in the following sections)

- Generate the input sample
- Evaluate your model on the generated sample generating the model outputs
- Analyse the model outputs with sensitivity indices and draw the conclusions

3.3.2 Screening methods

Screening methods allow the fast exploration of the model behaviour though out the discretization of the input levels. These methods can be adapted to a large number of inputs however, expertise shows that only a small number of input parameters are influential. That is why these methods are largely used, they aimed to identify the non-influential inputs with a small number of model calls, simplifying the model with realistic hypotheses. One of the most commonly used screening methods used for engineering solutions, is the "One At a Time" (OAT) design, where as stated in the name, each input is varied while fixing the others. It is important to use this method wisely, given that it does not give quantitative information about each input variable importance with respect to the others [7].

For this study, the **Morris method** was selected as a first SA approach. It is know to be the most complete and most costly one. It allows to classify the inputs in three groups: inputs having negligible effects, inputs having large linear effects without interactions and inputs having large non-linear and/or interaction effects. It acts as follow:

i The input design is discretized in d-dimensional grid with n levels by input, to perform a given number of OAT design r (proposed to be between 4 and 10 by Saltelli et al. [13]), as shown in figure 15

ii A starting point is randomly chosen

iii A direction is randomly selected and a new point is obtained given a chosen delta x (dx)

iv The previous process is repeated in an iterative manner. This allows the estimation of the elementary effect for each input

v Sensitivity indies are derived

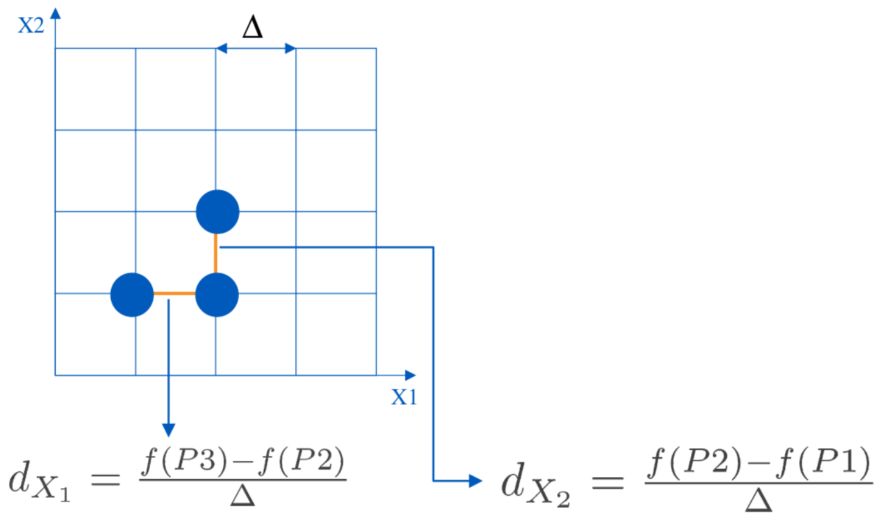


Figure 15: Morris discretization [1]

Let us denoted the $E_j^{(i)}$ the elementary effect of the j-th variable obtained at the i-th repetition as stated in equation 16:

$$E_j^{(i)} = \frac{f\left(\mathbf{X}^{(i)} + \Delta e_j\right) - f\left(\mathbf{X}^{(i)}\right)}{\Delta} \tag{16}$$

where the Δ is a multiple of $frac1n - 1$ already determined and e_j is a vector of the canonical base. Additionally, the coefficients are calculated as follow:

$$\mu_j^* = \frac{1}{r}\sum_{i=1}^{r} {}_j^{(i)} | \tag{17}$$

$$\sigma_j = \sqrt{\frac{1}{r}\sum_{i=1}^{r}\left(E_j^{(i)} - \frac{1}{r}\sum_{i=1}^{r} E_j^{(i)}\right)^2} \tag{18}$$

The interpretation of these indices is:

- Mean absolute value of the elementary effects mu_j^*: corresponds to the measure of the influence of the j-th input on the output. The smaller mu_j^* is, the less the j-th input contributes to the dispersion of the output
- Standard deviation of the elementary effects $sigma_j$: measures the non-linear and/or interaction effects of the j-th input. When $sigma_j$ is large, the elementary effects have big variations on the support of the input and the linearity hypothesis becomes less likely and open the possibility of interaction with at least other variable. On the other hand when it is small, the effect of a perturbation is the same all along the support, which suggests a linear relationship between the studied input and the output [7].

A good way of identifying the three possible groups drawing a graph linking bout indices as shown below in figure 16:

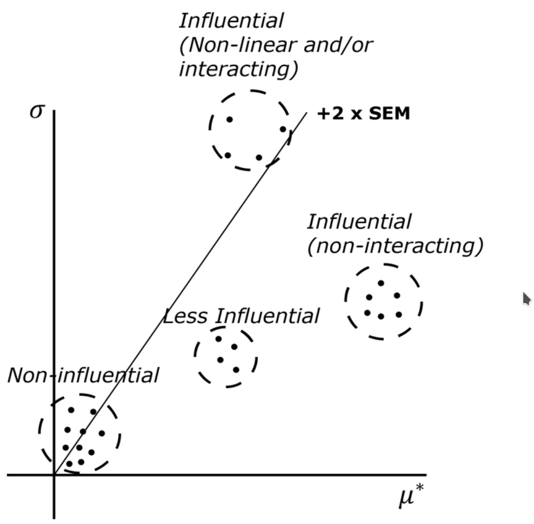


Figure 16: Morris' results interpretation [1]

3.3.3 Variance decomposition and Sobol' indices

Another very well known and used sensitivity analysis method is the computation of Sobol' indices. Contrary to Morris methos, Sobol's calculation allows to quantify the contributions of the different independent inputs $X_1, X_2, \ldots, X_d$ and their interactions in the variance of the output $Y = f(X)$ with $X = (X_1, X_2, \ldots, X_d)$, $E((f(X))^2) < \infty$ and f the model. They are based on the Hoeffding decomposition of f providing 2^{d} terms [13]:

$$\begin{aligned} f(X) =& f_\emptyset + \sum_{i \subseteq I_d} f_i(X_i) \\ &+ \sum_{\substack{\{i,j\} \subseteq I_d^2 \\ j>i}} f_{i,j}(X_i, X_j) \\ &+ \ldots + f_{1,2,\ldots,d}(X_1, X_2, \ldots, X_d) \\ &= \sum_{u \subseteq I_d} f_u(X_u) \end{aligned} \tag{19}$$

where:

- $I_d = \{1, \ldots, d\}$ is the set of input indices,
- $f_\emptyset = E(Y)$ is the expectation of Y,
- $f_i(X_i) = E(Y|X_i) - f_\emptyset$ is the elementary contribution of X_i to $f(X)$, and
- $f_{i,j}(X_i, X_j) = E(Y|X_i, X_j) - f_i(X_i) - f_j(X_j) - f_\emptyset$ is the contribution of the interaction between X_i and X_j to $f(X)$.

From Eq. (19), the variance of Y is:

$$V(Y) = \sum_{i \subseteq I_d} V_i + \sum_{\substack{\{i,j\} \subseteq I_d^2 \\ j>i}} V_{i,j} + \ldots + V_{1,2,\ldots,d} = \sum_{u \subseteq I_d} V_u \tag{20}$$

where:

- $V_i = V(f_i(X_i))$ is the elementary contribution of X_i to $V(Y)$,
- $V_{i,j} = V(f_{i,j}(X_i, X_j))$ is the contribution of the interaction between X_i and X_j to $V(Y)$,

and so on. Dividing Eq. (20) by $V(Y)$ leads to:

$$1 = \sum_{i \subseteq I_d} S_i + \sum_{\substack{\{i,j\} \subseteq I_d^2 \\ j>i}} S_{i,j} + \ldots + S_{1,2,\ldots,d} = \sum_{u \subseteq I_d} S_u \tag{21}$$

where:

- $S_i = \frac{V_i}{V(Y)}$ is the first-order Sobol' index of X_i, that represents the normalized elementary contribution of X_i to $V(Y)$,
- $S_{i,j} = \frac{V_{i,j}}{V(Y)}$ is the second-order Sobol' index of X_i and X_j, that represents the normalized contribution of the interaction between X_i and X_j to $V(Y)$, and so on. However, this order is out of the range for the purposes of this project,

Finally, The total Sobol' index gathering all contributions related to X_i is then defined as:

$$S_{Ti} = S_i + \sum_{\substack{j \in I_d \\ j > i}} S_{i,j} + \ldots + S_{1,2,\ldots,d} = \sum_{\substack{u \subseteq I_d \\ u \ni i}} S_u \tag{22}$$

It should be noted that $\sum_i S_i = 1$ if there is no interaction between the input parameters [13].

3.3.4 Implementation of Sobol' indices computation

The main steps for the estimation of the Sobol' indices for different independent inputs $X_1, X_2, \ldots, X_d$ with $X = (X_1, X_2, \ldots, X_d)$ are explained hereafter, and for more details the reader can refer to [13]. This formulation exposed here is the one developed by Saltelli [13] and it was selected because it offers a very efficient convergence rate.

i Step 1: generation of two ensembles of size N_e for the normalized input parameters set $(x_1, x_2, \ldots, x_d)$. The first (resp. second) (N_e, d) matrix is denoted by A (resp. B) (Eq. (23) (resp. (Eq. (24)))). The space filling strategy is carried out with a Sobol' sequence rather than a classical Monte-Carlo strategy:

$$A = \begin{pmatrix} x_1^{(1)} & x_2^{(1)} & \cdots & x_i^{(1)} & \cdots & x_d^{(1)} \\ x_1^{(2)} & x_2^{(2)} & \cdots & x_i^{(2)} & \cdots & x_d^{(2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_1^{(Ne-1)} & x_2^{(Ne-1)} & \cdots & x_i^{(Ne-1)} & \cdots & x_d^{(Ne-1)} \\ x_1^{(Ne)} & x_2^{(Ne)} & \cdots & x_i^{(Ne)} & \cdots & x_d^{(Ne)} \end{pmatrix} \tag{23}$$

$$B = \begin{pmatrix} x_1^{(Ne+1)} & x_2^{(Ne+1)} & \cdots & x_i^{(Ne+1)} & \cdots & x_d^{(Ne+1)} \\ x_1^{(Ne+2)} & x_2^{(Ne+2)} & \cdots & x_i^{(Ne+2)} & \cdots & x_d^{(Ne+2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ x_1^{(2Ne-1)} & x_2^{(2Ne-1)} & \cdots & x_i^{(2Ne-1)} & \cdots & x_d^{(2Ne-1)} \\ x_1^{(2Ne)} & x_2^{(2Ne)} & \cdots & x_i^{(2Ne)} & \cdots & x_d^{(2Ne)} \end{pmatrix} \tag{24}$$

ii Step 2: definition of d matrices C_i formed by all columns of A except the i^{th} column taken from B [13].

iii Step 3: computation of the model output (and QoI) y_A (resp. y_B and y_{C_i} with $i = 1, \ldots, d$), for all the input values in the sample matrix A (resp. B and the d matrices C_i), obtaining $(d+2)$ vector outputs of dimension N_e: $y_A = f(A)$, $y_B = f(B)$, $y_{C_i} = f(C_i)$ with $i = 1, \ldots, d$.

Additionally, the calculation of the first and total Sobol' indices for an ensemble of size N requires the integration of two independent samples. Given d uncertain variables and N_e perturbed members for each variable, the total number of simulations is thus $N = N_e(d+2)$ [13].

To sump up, sensitivity analysis provides information on the relative importance of the model input parameters assumptions. It analyses how the different values of a set of independent variables affect a specific dependent variable under certain specific conditions. It is important to make the difference with the uncertainty quantification that does not provide the quantification of the impact of each one of the variables uncertainty on the model response.

Finally, it is important to add that there are two types of sensitivity analysis methods, depending on how the problems need to be approached and how detailed the information is needed. There are local sensitivity analysis, which provides the sensitivity of the model outputs with respect to reference values for the model inputs. On the other hand, the global sensitivity analysis provides the contribution to the uncertainty quantities of interest from the input parameter, when varying over the whole input parameter space. It requires the integration of a large number of simulations with a direct model, however, it identifies the most significant parameters and improves the understanding of the model behavior.

4 Methodology

The methodology has been divided in three main parts. First one focus on the problem definition,the second one the engine tool adaptation and calibration and the third one the sensitivity analysis. This section aims to explain step by step what has been done during the internship period, and shows the methodology foll lowed to fulfill the research objectives.

4.1 Problem definition

When starting a new research project, it is important to define some goals and main objectives in order to focus the effort and the time in fulfilling them. At the beginning of this internship, the big question was clear: How climate change will affect aircraft performance?. However, this is a very big and open question. That is the reason why, the very first big task was to define the problem. This means, understanding the state of the art of the subject, identifying the possibilities in terms of time, cost, tools available and the expertise f the researcher. This section will explain how the problem definition was for this study and at the end will present the conclusions and the path to follow. First of all, a very rigorous literature review was performed in order to understand what have been studied before on the subject and what could be useful for the present work. Additionally, it is important to state the first objective of the project was to take advantage of both engine and climate models. In order to reach the objective, it was necessary also to spend some time on understanding climate modelling and climate change models basic theory. While reading all the work that has been done (that is already summarized in section 2) it was possible to identify two main questions.. First question was What aspect of climate change is it going to be studied?. This question has endless possibilities, starting with temperature rise, changes in pressure and humidity, which period of time?, turbulence, extreme climate events, which climate model to be used?, etc. Second question was What aspect of engine performance is it going to be studied?, this question similar to the first one, has many possibilities. It is possible to study the thrust generation, the takeoff weight as already studied for other researchers, the takeoff distance, etc. At this point of the project it was possible to identify that even understanding the basic function of the climate models it was going to be difficult to obtain and treat climate data without a proper formation by the hand of experts. The time was limited and the conditions at the beginning of the internship (COVID-19 health crisis) made difficult the possibility of having a proper formation and guide for the climate experts. That is the reason why, after some discussions and contemplating all the possibilities, it was decided to focus on the study of temperature based on already register data. On the other hand, for the engine modelling, it was possible to have access to a tool that allows the computation of takeoff distance as a function of the temperature. Further engine modelling was planned, however, already explained unexpected external factor, did not allow to access the tools at the beginning of the study. That is the reason why, it was decided to focus on the takeoff distance code applied with sensitivity analysis tools to create a good methodology for further studies. Figure 17 shows a schematic summary of what was explained in this section.

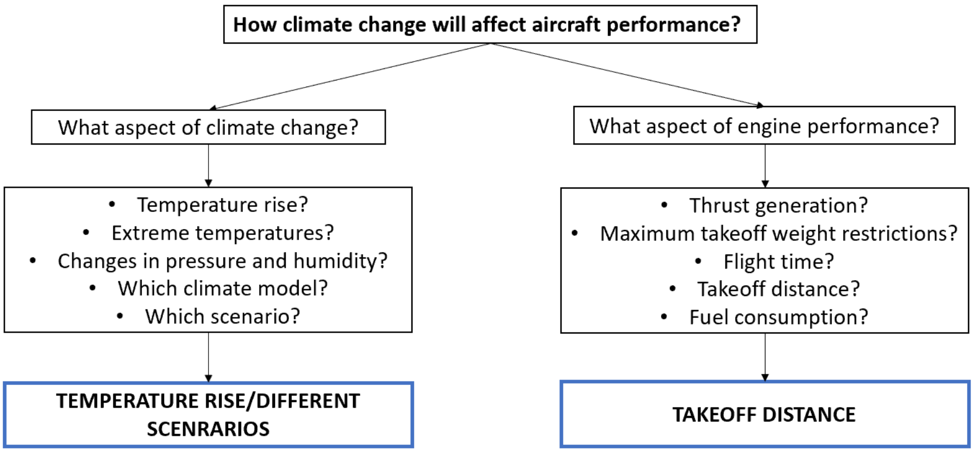


Figure 17: Summary problem definition

4.2 Engine tool calibration

Once the problem was defined, it was time to focus on the engine tool. At an early stage of the project, a code that calculates all the engine characteristics at each stage was proposed as a potential main tool. This tools allows to model different kind of engines while changing the main characteristics such as temperatures, ratios, efficiencies, etc. However, it was decided after defining the problem, that the reach of such a tool was not in accordance with the project objective and it was discarded. Even though the tool had to be discarded as the main tool for the research, it was use full to understand the behaviour of the engine thrust generation while changing the outside temperature and it was good as a python training to start with.

Later on, it was possible to have access to the a takeoff distance code developed by Professor Thierry Druot, which used the magic line theory already explained in 3.2.2. This tool can be calibrated for different aircraft models and engine types if the right information is introduced. Among the factors that are possible to calibrate there are the thrust generation, the maximum weight, the lift to drag ratio, the wing configuration and size, the engine number and the airport altitude. With all this factors together and well defined, it is possible to have an idea of which aircraft are we studying and by extension which kind of engine. This section will present the calibration process and how the aircraft was defined and why.

To start with it was necessary to define the aircraft family to be study, the selected one needed to be a common one and additionally count with online and public information regarding its aerodynamic characteristics. It is known that aircraft information is not available easily online due to all the industry confidential aspects behind aircraft design. For this reason the Airbus A320 family was selected, it is a very well know aircraft and there are some available information regarding its configuration and it has been widely studied for different researchers that aims to know more details about its aerodynamics. All the details that were calibrated for the purpose of this study will be explain here after.

First thing on the list was the thrust generation, it is a very important factor that determines, somehow, the engine performance. At this point it was already known that the thrust has a relationship with the temperature, however the exact relationship was unknown and it is not documented in previews studies. To clarify this topic and aiming to find a simplified, yet accurate enough, relationship between the engine thrust and the temperature, it was possible to have access to some empirical relationship from an Airbus tutor's contact. The initial questions was "Is there any law, or any formula that describes the thrust as a function of the outside temperate at the moment of takeoff?". To answer this question, it was possible to establish a private conversation with an Airbus engineer who accept to give to the project the information about the main engine behaviour behaviour with respect to the outside temperature. This information is based on AIRBUS'S observations of engines with different architectures and sizes. Such information is summarize below:

- With temperatures below ISA+15K the engines thrust is constant for a given Mach and a given altitude.
- With Temperatures above ISA+15K the engines thurst decreases approximately linear with the temperature rise. When the outside temperature is ISA+30K, the thrust is known to be aproximately 87 percernt of the thurst at ISA+15

Based on the previews information, an empirical law was formulated. It is drawn in image 18 and states as follow:

$$F_N(T) = dthrust * F_{Nmax} \tag{25}$$

where T is the outside temperature, dthrust is the percentage of thrust decrease that is equal to 0.87 when the T is equal to 45 in Celsius and F_{Nmax} is assumed to be the maximum engine thrust.

The original takeoff distance code, that included a different approximation for the takeoff thrust, was calibrated with this empirical law and the values of the dthrust were varied within a range that will be explained later.

Next step on the calibration process was to find the correct polar drag that defines the Airbus A320 aerodynamics. In order to do so, a extensive literature review was performed until it was possible to find a study from DELFT university which aimed to estimate the aircraft drag polar based on a stochastic hierarchical model. As the present study does not aim to calculate the exact value fro the takeoff distance, but to have an idea of the

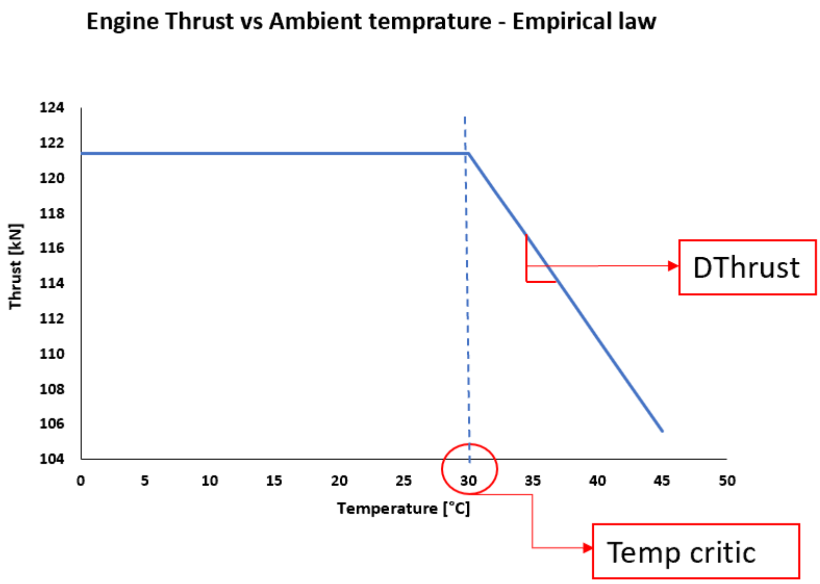


Figure 18: Empirical law for takeoff thrust

magnitude, this study seemed to fulfill the requirements. As already mention above, the aerodynamic features of aircraft, such as the aerodynamic coefficients that determine a crucial part of the aircraft performance, are well protected by the manufactures. That is why, Sun et all used for their study the simplified point-mass aircraft performance model, usually used in air traffic management related research; for more details in their methodology refer to reference [15].

The drag polar in the point-mass model is summarized here. Drag and lift are aerodynamic forces, While the drag is produced by the airflow interacting with the aircraft, the lift is produced due to the difference of pressure between the upper and lower surface of the wings and while traveling in the free stream they can be defined as follow:

$$
\begin{aligned}
L &= C_L \frac{1}{2} \rho V^2 S \\
D &= C_D \frac{1}{2} \rho V^2 S
\end{aligned}
\tag{26}
$$

Where C_L is the lift coefficient and C_D is the drag one. These coefficients can be modeled as a function of the angle of attack (α) the Mach number (M) and the flap deflection (δ_f) as shown here:

$$
\begin{aligned}
C_L &= f_{cl}\left(\alpha, M, \delta_f\right) \\
C_D &= f_{cd}\left(\alpha, M, \delta_f\right)
\end{aligned}
\tag{27}
$$

EIn point-mass models the relationship between the aerodynamic coefficeints C_D and

C_L is reduced to the drag polar. It is simplified function of second order that can be express either as equation 28 or as equation 29:

$$C_D = C_{D0} + \frac{C_L^2}{\pi A e} \tag{28}$$

$$C_D = C_{D0} + kC_L^2 \tag{29}$$

with

$$k = \frac{1}{\pi A e} \tag{30}$$

where A is the aspect ratio of the wing, that is the span divided by the average chord, and e is the Oswald factor that lies in between 0.7 and 0.9. From 29 the parameter k is the lift induced drag coefficient and the C_{D0} us the zero lift drag coefficient. When considering a specific aircraft configuration this coefficients are constant and DELFT study aimed to found them for different aircraft models. Sun et all estimated those coefficients based on a novel stochastic total energy model using Bayesian computing and Markov chain Monte Carlo sampling. The method is based on the stochastic hierarchical modeling approach. With this model and sufficiently accurate flight data and some basic knowledge of aircraft and their engines, the drag polar could be estimated and the values for the particular case of the Airbus A320 are presented below:

$$\begin{aligned} k &= 0.0334 \\ C_{D0} &= 0.078 \end{aligned} \tag{31}$$

Finally, to completed the takeoff distance model calibration, it was necessary to fix some other details as the maximum takeoff weight MTOW and the airport altitude; the Madrid airport was selected as a reference point. The values defined as follow:

- MTOW: 73500 kg
- Madrid airport Altitude: 606m

4.3 Sensitivity analysis experimental settings

4.3.1 Identification of the Uncertain factors

One very important step while performing sensitivity analysis is the identification of the uncertainty parameters among all the factors that might have an influence on the model response. First of all this study aims to investigate the influence that temperature changes will have on the takeoff distance, that is why the first uncertainty parameter is the **outside temperature**. Secondly, as the engine thrust was defined based on an empirical law, it seemed prudent to vary the **percentage of thrust decrease**, as the information proportioned by AIRBUS is based on average values for different kind of engines. Lastly and similar related to the empirical law mentioned before, the **critical temperature** at which the thrust starts decreasing, was selected as the third uncertain parameter. Second step, after defining the uncertainty input parameters was defined their distribution function for each one of them. For the temperature it was selected a normal distribution based on literature review. On the other hand, for the percentage of thrust decrease and the critical temperature it was assumed a uniform distribution. The details of these distributions are summarized here:

- Outside temperature: Normal distribution with mean=35°C and Standard Deviation =±3
- Percentage of thrust decrease *dthrus*: Uniform distribution from 0.8% and 0.9%
- Critical temperature *Tcritic*: Uniform distribution from 25°C to 35°C

For the temperature data, the mean corresponds to the mean of maximum summer temperatures for Spain in the last decades. However, it is important to remark that the distribution was selected as normal even when the temperature data is a maximum referent. This is why, as explained before, the principal objective of this study is not having the most accurate data, but to identify the trends and the impact of increasing the temperature. The data of the maximum temperature and the standard deviation was selected from the official Spanish climate website. On the other hand, the ranges for the uniform distribution of the dthrust and the Tcritic were selected like that as an exercise to understand how much influence do they have on the model response. They were proposed within a possible range of occurrence and it is important to have in mind that the results will strongly depend on the selection of the ranges.

4.3.2 Linearity study

After the engine tool calibration and the BATMAN training it was time to start performing some studies to understand the behavior of the problem. It was obvious from the definition of the takeoff distance length that it will have a linear relation with the temperature as possible to see in 14. However, a linearity study was proposed to see picture the relationship between the input parameters and the model response and additionally a linear regression was performed and the R-squared factor was calculated.To perform this analysis each parameter was varied while keeping the other constants, as a first approximation to one at a time experiment. For this only the outside temperature and the percentage of thrust decrease were studied. The inputs were varied as follow:

- $26^\circ \leq$ Outside temperature $\leq 56^\circ$
- $0.8 \leq$ dthrust ≤ 0.9

The linearity study was conducted, and the strongly linear relation between the input parameters were confirmed, as it will be presented in the results section. Even though the study could have ended there due to the strong linearity of the problem, as a part of the objective it was decided to quantify the influence of these parameters uncertainty on the model response. That is why a sensitivity analysis was conducted to rank the parameters' importance and later on to quantified it.

4.3.3 Sensitivity analysis tools and software OPENTURNS-BATMAN and SALib

After performing the linearity study, it was time to try the One at a time method for sensitivity analysis via the Morris method. This method was explained in detail in section 3.3.2, however this section aim to explain the tools used for this purpose and the mythology followed to implement them. Two tools were used to implement the Morris methods to the takeoff distance model. The first one was the Openturns Morris module, open turns is an open source initiative for the Treatment of Uncertainties, Risks'N Statistics. The second one the library SALib from the MIT, which similar to Openturns, is an open source written in Python for performing sensitivity analysis. For both tools, a sample was generated with 4 design of experiments and 16 repetitions. This sample is generated considering the distributions already mentioned before for the three selected uncertainty parameters. Once the sample was generated the model was evaluated with it and the elementary effect of the j-th variable obtained at each repetition was calculated and with that it was possible to obtain the mean and the standard deviation of the elementary effects. It is important to clarify that using the tools mentioned above, all this calculations are done with a single command if it well defined, reason why they are very useful and easy to use for people with different backgrounds than statistics or mathematics. Finally, from this calculation is was possible to plot the mean against the standard deviation of the elementary effects aiming to identify which of the three parameters had influence on the model and which had not.

Once the one at time study was performed it was decided to apply one sensitivity analysis method that allowed to quantify the influence of each one of the influential parameters subtracted from the Morris study. However, for this purpose it was necessary to learn who to used a new tool. That is why the next step on this methodology was to train in Batman, that is a Bayesian Analysis Tool for Modelling and uncertainty quantification. It is an open source Python package dedicated to statistical analysis based on non-intrusive ensemble experiment.Batman library provides a convenient, modular and efficient framework for design of experiments, surrogate model and uncertainty quantification. It relies on open source python packages dedicated to statistics (openTURNS and scikit-learn). It also implements advanced methods for resampling, robust optimization and uncertainty visualization [16]

Scenario	Temperature rise pretiction by 2100	Mean Summer temperature	Standart deviation
RCP 2.6	1 $°C\pm$ 0.7	36 $°C$	3.7
RCP 4.5	1.8 $°C\pm$ 0.8	36.8 $°C$	3.8
RCP 6.0	2.2 $°C\pm$ 0.9	37.2 $°C$	3.9
RCP 8.5	3.7 $°C\pm$ 1.1	39.7$°C$	4.1

Table 2: Probability density function for the different RCP scenarios

4.3.4 Sensitivity indices calculation

Finally, once the number of samples were defined it was time to calculate the Sobol' indices for the two influential variables. To do so a Batman code was written and from its uncertainty quantification module it was possible to calculate not just the model response in all the sampled data, but also the surface response and of course the sensitivity indices. The methodology followed to calculate the Sobol' indices is already explained in the section 3.3.4. To sum up, it was necessary to generate the 1000 samples considering the probability distribution for each variable and to evaluated the model with them. To generate the sample it was used the Saltelli strategy (for more details in Saltelli's method refer to [14]).

Once the experiment is generated, the model is evaluated with it. It is important to remark that, it was used an ensemble approach that means that it is a non intrusive method, and no change was performed in the original model. What Batman does in this case, is to re-run the model as many times as samples were generated.

Additionally, once the 2020's data was analysed it was time to contemplate the different scenarios of climate change described in section 3.1.5. The methodology followed for the 4 different scenarios was the same as the one already explained and the probability functions for each case as presented in table 2. To construct this table the data from the IPCC latest report was extracted. The limits were considered and summed up to the already known mean temperature for the summer in Madrid recorded by 2020 (already explained to be the mean of the highest temperatures in summer). The probability density functions were constructed considering the worst case scenario, meaning the highest predicted temperature that could reach the Madrid Airport in the year 2100 for each of the scenarios.

5 Results

This section presents the study results followed by their respective analysis and thoughts. Additionally, a description of the project difficulties will be presented and discussed. First results presented here are the ones from the linearity study. The results for the temperature (figure 22a) and the percentage of thrust decrease (figure 22b) are the first ones.

First thing that was performed was the design of experiments (doe)that is shown in 19, where X_0 is the outside temperature and X_1 is the dthrust. As explained before once the experiment is generated and evaluated it was possible to generate the probability density function of the output as seen in figure 20, an evaluate the influence of each of the parameters uncertainties on the output uncertainty. Figure 21 displays the surface response for takeoff distance with respect to dthrust and temperature. The takeoff distance is normalized with the longest runway of Madrid's airport that is 3500m. The normalization of the takeoff distance allows to have a clearer idea of how much changes on the inputs will affect the model response. When analyzing this response, it is possible to appreciate the linearity effect of both variables. It was expected from the takeoff definition 14, where the length has a linear direction with the density, that is translated into a linear relation with the temperature. However, once the temperature increases too much the effect get lost somehow and it is no clear how the variables behave at this point. Finally, it is possible to identify that the takeoff distance strongly depends on the temperature, however, it has a weak dependence on the dthrust factor. Both responses seems to be linear, and this is widely discussed later on in this section.

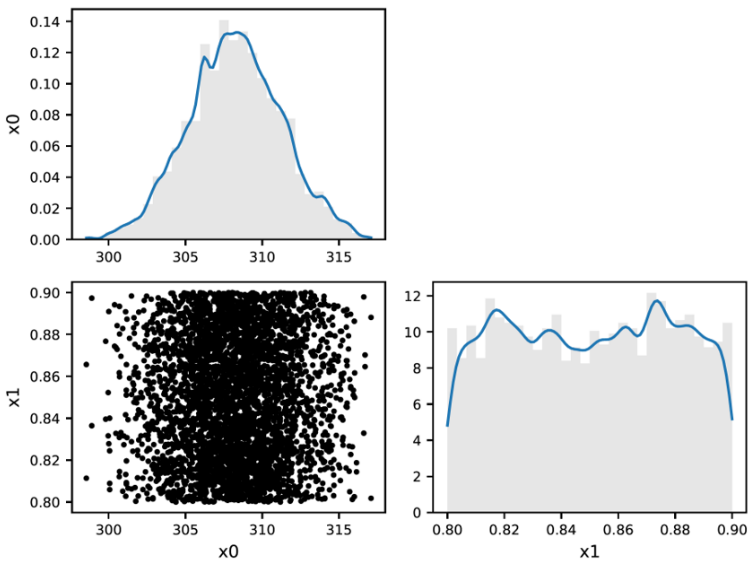


Figure 19: 2020's data probability density functions (sampling data)

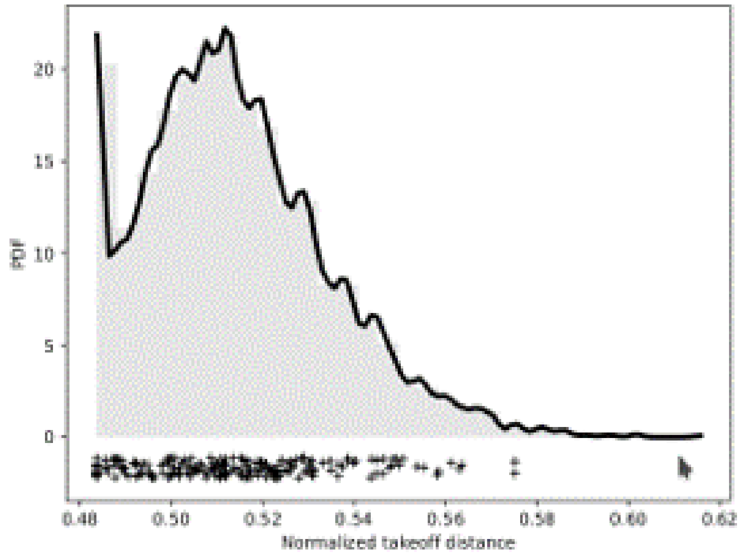


Figure 20: Model response 2020'data

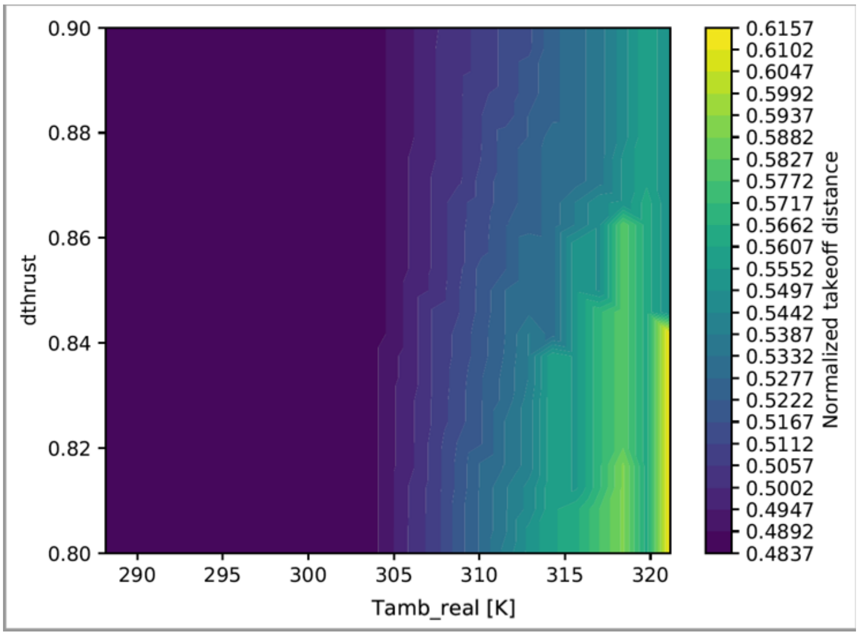
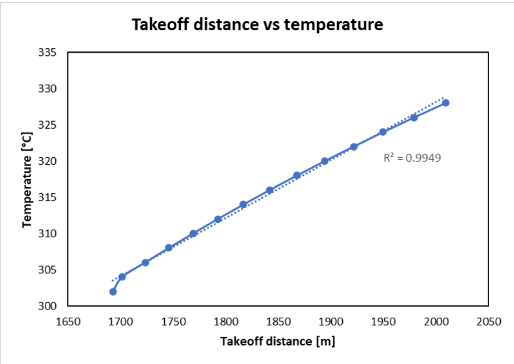


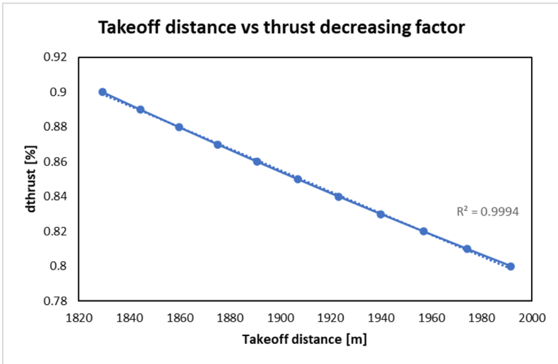
Figure 21: Surface response todays'data

5.1 Linearity

In Figure 22 the results for the linearity study are shown. Figure 22a shows how the takeoff distance increases linearly with the increase of the outside temperature with a R-squared factor of 0.9949. This means that the takeoff distance is linear with respect to the temperature. The result is expected as the definition of the takeoff distance has a linear relation with the density; which is directly related with the air temperature. On the other hand, figure 22b shows how the takeoff is also linear when the percentage of thrust decrease changes. The takeoff distance increases while decreasing the percentage dthrust with an R-squared factor of 0.9994. Again this was expected from the definition of the magic line factor. It was shown that the takeoff distance is linear with respect to both variables. However, as explained before, the aim of the project was to develop a methodology for sensitivity analysis with more complicated models in further studies. This is why, more relevant studies were performed.



(a) Temperature



(b) dthrust

Figure 22: Linearity study results

5.2 Morris study

Figure 23 shows the results for the Morris methods with both tools already mention in the section 4.3.3. Figure 23 displays the mean of the elementary effect against their standard deviation It is important to remember that for the Morris experiment three variables were consider instead of two, this time the critical temperature were studied also. It is possible to see from the graphs that both tools lead to the same conclusion: the most influential variables is the outside temperature (X_0) followed by the percentage of thrust decrease (X_1) and finally the critical temperature (X_2) with no influence at all on the takeoff distance. It is important to clarify that the results for the mean and the standard deviation of the elementary effects had no similar numerical results. The difference in the results is because of the methodology followed to generate the sample in each case. Morris method, as explained before, generates the sample randomly, which means that each time that the sample is generated, the values vary in dimension keeping always the same conclusion.

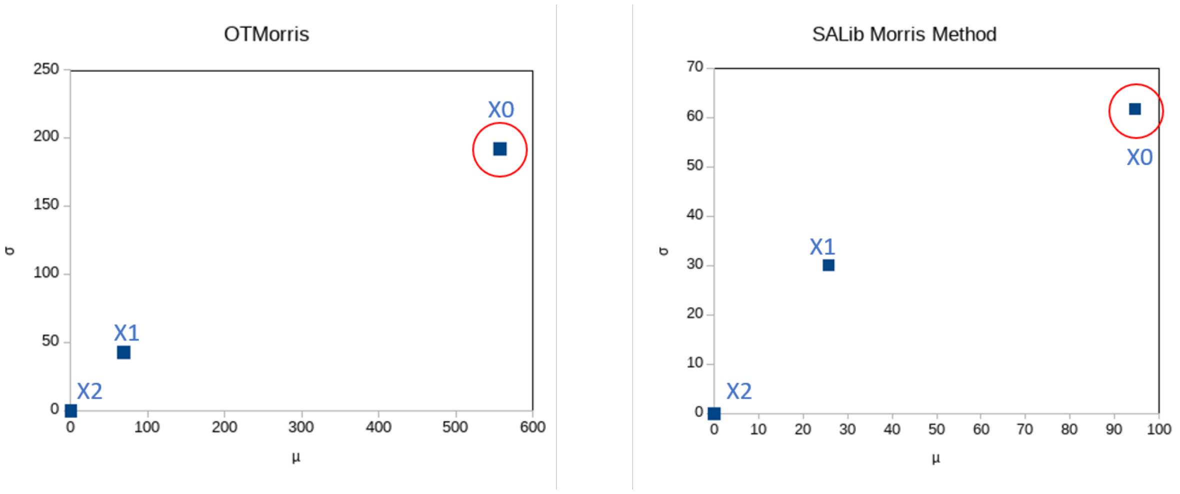


Figure 23: Morris results

5.2.1 Convergence study

It was time to calculate the sensitivity indices using Batman. First thing to do while using Batman is to export the model of interest into the Batman code so it can be accessible from there. The takeoff distance model is written in Python which was an advantage at the moment of implementing it in on Batman, that remember is a Python package. Once the model was callable from Batman it was possible to configure all the details regarding the inputs distributions, the type of sampling, the number of samples and so on. Batman allows to compute the sensitivity indices even if the person that is working on it has no statistics background, which makes it really useful for applied studies in different subjects such as the one of the present report. To define the number of samplings that were necessary, a Sobol' convergence study was performed and the number of samples was varied from very few to 2500. Figure 24 shows the result for this study, as it is possible to see only two variables were used here. The reason why lies on the results from the Morris study that will be presented later on. To summarize, one of the first three variables ended up not being influential on the model response, which is why from now on only the outside temperate X_0 and the dthrust X_1 were studied.

5.3 Sobol' indices

Figure 25 shows a bar diagram with the uncertainties ranges. This bars represent the Sobol' indices of first and total order. Looking at the first order Sobol' index it is possible to see how for the outside temperature the value is notably higher than for the dthrust factor. However, when looking to the total order index that determines the existence of interaction between the variables, it is possible to appreciate that both variable are interacting between each other. This was already know from the thrust definition however, it is a good sign

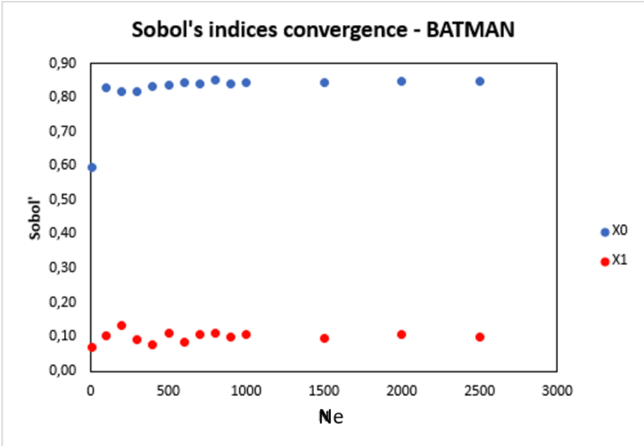


Figure 24: Sobols' convergence study

that the computation was done properly. In the graph the red color represents the first order sobol' and the blue on the Total Sobol' indices.

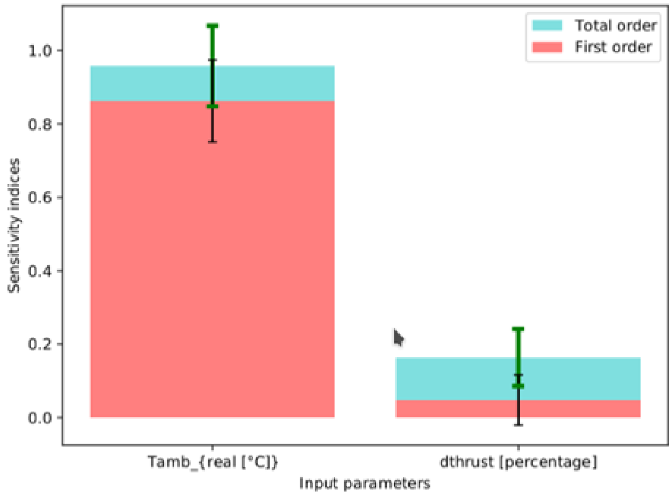
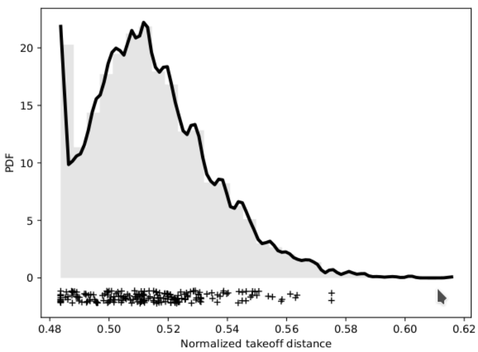


Figure 25: Sobol' indices for 2020's data

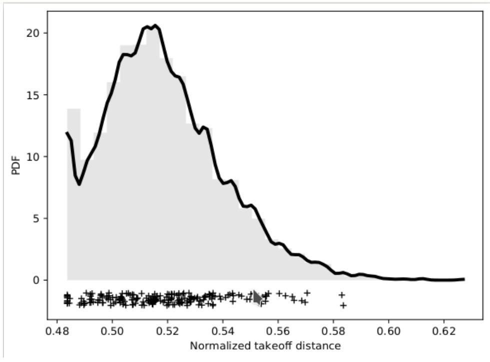
5.3.1 SA under climate change scenarios

Additionally, the results for the Sobol' calculation for the different climate scenarios are shown from figure 26 to figure 27. Starting with the PDF of the response for each case, continuing with the Sobol' indices graphic representation and finally a comparison between the fourth scenarios values and 2020's data.

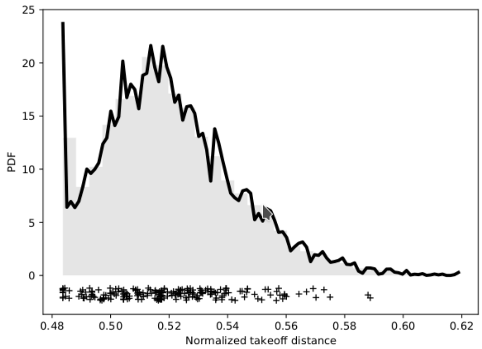
PDF RCP scenarios In Figure 26 the PDF for the four RCP scenarios is presented. It is important to remark that the takeoff distance is expected to increase as the temperature prediction is higher. From the PDF is possible to extract that the higher the scenario the higher the probability to have larger runways needs. Finally, from the enlargement of the PDF it is possible to conclude that there there are more incertitude at higher scenarios.



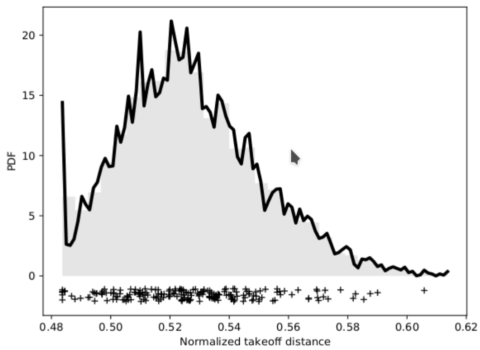
(a) PDF RCP2.6



(b) PDF RCP4.5



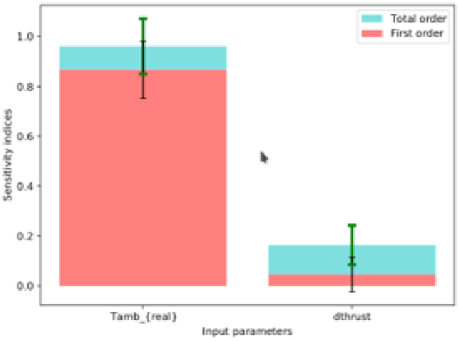
(c) PDF RCP6



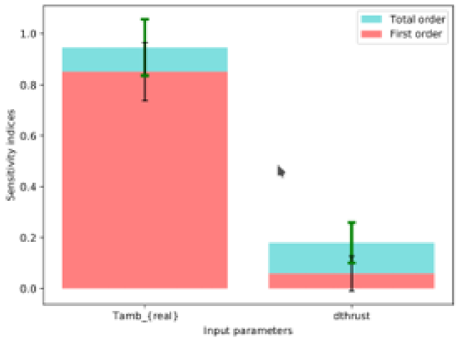
(d) PDF RCP8.5

Figure 26: PDF RCP scenarios

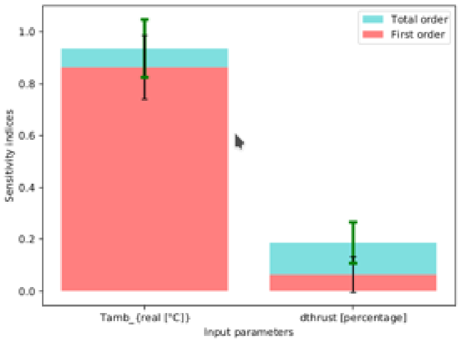
Sobol's indices RCP scenarios Following with the Sobol' calculation, figure 27 shows the results for the four scenarios. As it is possible to see from the bars the sensitivity does nos vary from one scenario to the other. The temperature is in every case the most influential factor as represented for the first order Sobol' index for each case. However, it is possible to perceive a slightly change on the box size for both the first order and the total Sobol' indices. This change is easier to see when analyzing the numbers and that is why they will be presented in Tables 3 and 4 and additionally in figure 28.



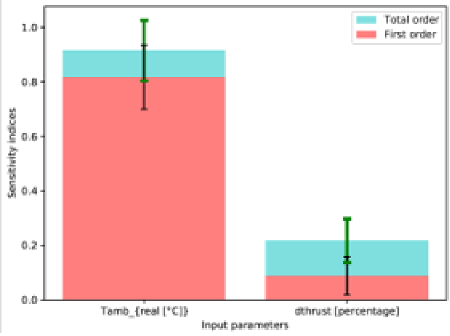
(a) Sobol' indices RCP2.6



(b) Sobol' indices RCP4.5



(c) Sobol' indices RCP6



(d) Sobol' indices RCP8.5

Figure 27: Sobol' indices RCP scenarios

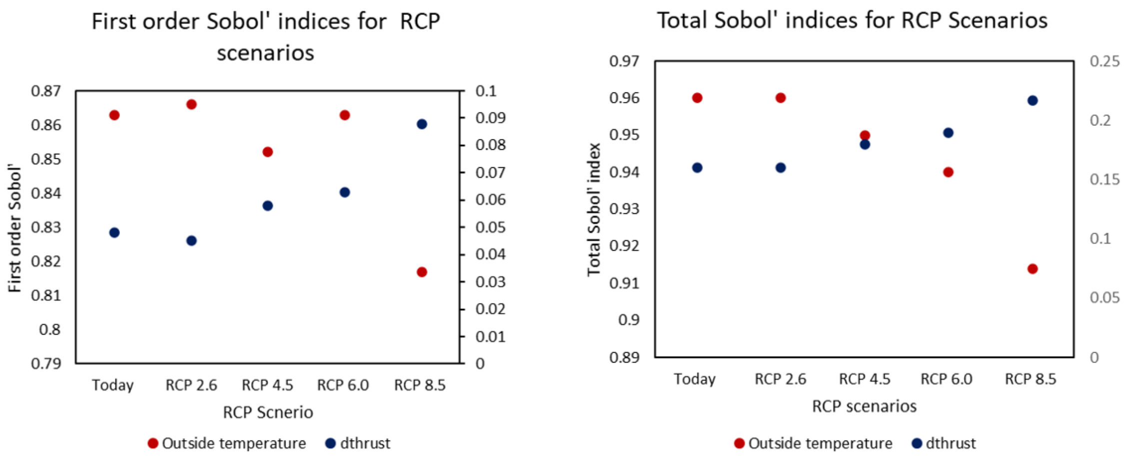
Tables 3 and 4 present the numerical results for the first order and the total Sobol' indices for 2020's and RCP scenarios. Additionally, figure 28 picture the evolution of the Sobol' indices starting with 2020's temperature to the worst case scenario for the year 2100. It is possible to distinguish a small decrease on the first order and the total Sobol's indeces for the outside temperature, from one scenario to the other, however the change is really small an it is not possible to say it is significant. However, the temperature remains the most influential parameter for the takeoff distance model. On the other hand, the dthrust seem to gain some influence when the temperature increases. That goes hand by hand with the definition of the empirical law used here for the thrust generation. The thrust starts decreasing when the temperature is higher than 30 degrees Celsius and the higher the temperature the lees efficient the engine. That is why dthrust is expected to have more influence and interaction with the temperature for the most critical scenarios presented here.

First order	2020's	RCP2.6	RCP4.5	RCP6	RCP8.5
Outside temperature	0.863	0.866	0.852	0.863	0.817
dthrust	0.048	0.045	0.058	0.063	0.088

Table 3: First order Sobol' indices

Total	2020's	RCP2.6	RCP4.5	RCP6	RCP8.5
Outside temperature	0.96	0.96	0.95	0.94	0.914
dthrust	0.16	0.16	0.18	0.19	0.217

Table 4: Total Sobol' indices



(a) First order Sobol' indices

(b) Total Sobol' indices

Figure 28: Sobol' indices for RCP scenarios and 2020's data

6 Discussion and Conclusion

6.1 Discussion

6.1.1 Sensitivity to the assumtion of input parameter distribution

Finally, an additional study is presented in the results and it aims to show how the calculations performed here are susceptible to the input data assumptions. For instance, it was decided to vary the range of the dthrus parameter. Now it will varies between 0.7 and 0.9 instead of 0.8 and 0.9 keeping the 2020's scenario data. The objective is to study how sensible is the model to this input modification and to quantify the effects. Figure **??** and tables **??** and 6 show the result for what was described below. First, in the figure is is possible to see how the thrust gained some influence when its range is bigger. It is also possible to appreciate that the temperature first order Sobol' index is lower than in the previews cases. As a comparison exercise, in the tables above it is possible to see the data from this study case compared with the first results obtained for the Sensitivity analysis.

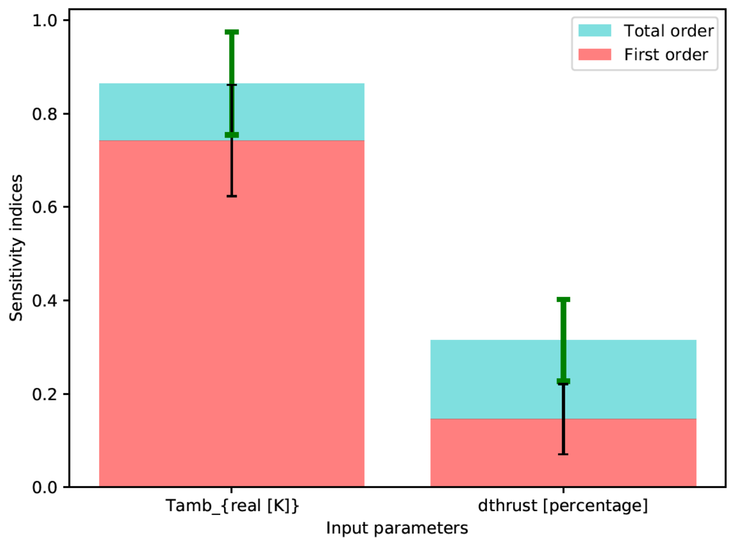


Figure 29: Sobol' indices different dthrust range

First order	**Outside temperature**	**dthrust**
2020's	0.863	0.048
Example	0.742	0.146
Difference	14%	204%

Table 5: First order Sobol' indices 2020's data vs example case

Total	**Outside temperature**	**dthrust**
2020's	0.96	0.16
Example	0.864	0.314
Difference	10%	96.25%

Table 6: Total Sobol's order 2020's data vs example case

6.2 Conclusion

The study was lead with a very simplified engine tool that allowed to find a good methodology to perform sensitivity analysis on the impact of climate change on aviation. Thought out all the process it was possible to try different statistical methods and tools to perform sensitivity analysis and it was possible to integrate different areas of study in one single project. In order to accomplish the objectives, several literature review was needed to acquire the knowledge needed form fields different than aeronautics. Performing a first sensitivity analysis on this subject opens the door to future ICCA researchers to continue evaluating this impacts, and concluding on the impact of atmospheric changes on the future of the commercial aviation.

It was possible to go from the identification of the potential sources of uncertainty, the study on the linearity of the model, the calculation of one a time experiment ending with a Sobol' indices computation that allowed to quantify the influence of each one of the parameters. Some assumptions were also made regarding the input parameters probability distributions and ranges of occurrence. Reason why it is important to state that the sensitivity analysis computation is strongly related with the initial assumptions and on the inputs distribution. From this analysis it is possible to obtain very important and significant information. However, it is also possible to obtain results with no significant meaning if the input parameters are not defined properly.

It was possible to identify that the Takeoff distance has a lineal relationship with respect to the outside temperature and with the percentage of thrust decreasing. The study could have ended there as from a linear case it is already easy to identify how the parameters influence the response. However, it was decided to seek for a good methodology to perform in the future the same study with a more complex model and to try several methods that allow the computation of sensitivity analysis.

It was found that Morris and Sobol' methods give the same conclusion: The take off distance is strongly influenced by the outside temperature. However the percentage in which the thrust may decrease due to temperature, does not affect the takeoff distance significantly. Even when the both methods yield to the same conclusion, with Morris it is not possible to quantify the effect of each one of the parameters, but is useful to identify which parameters are important an which are less or have no importance at all. Additionally, as stated before, Sensitivity analysis strongly depend on the input distribution definition and the hypothesis made at the beginning of the problem. Any change on the input uncertainty will have an impact on the output sensitivity which was proven in the section 6.1.1.

As a final remark, it is important to consider that Higher temperatures will bring more difficulties than the increasing of the takeoff distance. At some point, when the temperature is too high the aircraft is not capable to reach the velocities needed to cross the different pathways segments (regulatory effects). However, this restrictions where not considered for this study.

6.3 Perspectives

This project was a great opportunity to start the uncertainty quantification path on the ICCA program. It has as a result a feasibility study of the sensitivity analysis needed to evaluate the influence of the some parameters on the takeoff distance calculation. From now on, it is possible to follow the same methodology applied with more specialized and accurate engine models to draw some conclusions translatable to an economic impact for the aviation industry. Additionally, the humidity and the pressure changes due to global warming should be included in future works, with the purpose of evaluating their impact on the engine performance. The original objective of this project was to take advantage of the climate models in order to have more accurate results. However, the actual world situation did not allow a proper training on the acquisition of this climate data, and the available time for the training position was not enough. This is the reason why, it is recommended, as future work, to include the climate prediction data to be able to built the real probability density functions without so many assumptions. This does not mean that the results from the present work are nor worth of attention. In fact, they gave an idea of what is expected, and it a good star and guide point for future works.

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