CERFACS
CENTRE EUROPEEN DE RECHERCHE ET DE FORMATION AVANCEE EN CALCUL SCIENTIFIQUE

Covariance modelling for variational ocean data assimilation

This 6-month internship will take place in the **Parallel Algorithms (ALGO)** team at **CERFACS**, located at the Météopole in Toulouse.

The principal supervisors will be **Anthony Weaver** (CERFACS-CECI/CNRS; weaver@cerfacs.fr) and **Olivier Goux** (CERFACS-CECI/CNRS; goux@cerfacs.fr).

1. General context

Data assimilation is the procedure for combining observations (e.g., from satellites and *in situ* networks) of the state of a system with a prior (background) estimate of the state in order to produce the best possible estimate of the state. In variational data assimilation (DA), the optimal estimate is obtained iteratively by minimizing a nonlinear weighted least-squares cost function that is the sum of two terms: one measuring the model fit to the background state; the other measuring the model fit to the observations, subject to constraints that relate the model state to the observations. The weighting matrices for these terms are defined in terms of background- and observation-error covariance matrices. The procedure effectively extracts and combines the "best" information from each type of measurement available, given information about their respective uncertainties. Variational DA is widely used for operational state estimation in meteorology and oceanography.

The work in this internship will contribute to the development of the background-error covariance model of the NEMOVAR ocean DA software. NEMOVAR is used operationally for ocean, weather and climate applications at the European Centre for Medium-Range Weather Forecasts (ECMWF) and the UK Met Office. It is developed jointly by CERFACS, ECMWF, UK Met Office and INRIA. Collaborative interactions with scientists at those institutes is expected during the internship. This work will also contribute to an ocean DA project funded by the Copernicus Climate Change Service.

2. Specific work

There will be two strands to the work.

1. **Anisotropic correlations**. In NEMOVAR, background-error correlations are modelled using a diffusion operator, formulated as the solution of a diffusion equation discretized with an implicit, Euler backward scheme (Mirouze and Weaver 2010; Weaver and Mirouze 2013; Weaver *et al.* 2016). The diffusion tensor controls the spatial smoothing properties of the correlation model. In the ECMWF implementation of NEMOVAR, the diffusion tensor for each control variable is estimated from an Ensemble of Data Assimilations (Weaver *et al.* 2021; Chrust *et al.* 2021). The estimation process provides estimates of both the diagonal and off-diagonal elements of the tensor. Currently, however, the off-diagonal elements are neglected.

A more effective way to exploit the ensemble information is to generalize the diffusion operator to account for the cross-derivative terms that result from the use of a non-diagonal tensor. The challenge is to introduce a numerical discretization that generates an accurate anisotropic response, while preserving the self-adjoint and positive-definite attributes of the discretized diffusion operator. Self-adjointness and positive-definiteness are crucial properties that must be satisfied in order for the diffusion operator to yield a valid (symmetric and positive-definite; SPD) correlation matrix. In this internship, the student will investigate the *symmetric* finite-difference scheme of van Es *et al*. (2014) to account for horizontal anisotropic correlations in NEMOVAR.

2. **Improved correlations near bathymetry**. The ocean model NEMO (Madec *et al.* 2023), on which NEMOVAR is based, uses a vertical *z*-coordinate system, with a “partial step” in the bottom ocean level for improved representation of bathymetry. With partial bottom cells, quantities in horizontally adjacent cells are defined at different depths. In order to compute horizontal derivatives required in the model equations (e.g., in the diffusion term), the quantity in the deeper level is first interpolated to the level of the shallower level for improved accuracy. A *z*-coordinate system with partial steps is also used in NEMOVAR. However, the diffusion operator used for modelling background-error correlations does not include any special treatment of the horizontal derivatives in the active bottom cells, thereby leading to spurious correlations in regions of steep topography (e.g., near the shelf seas). In this internship, the student will adapt the correlation model to include the interpolation procedure used in NEMO. As with the anisotropic diffusion scheme, the challenge will be to implement a numerical (interpolation) scheme that preserves the SPD attribute of the correlation matrix.

The intern will be expected to implement partial steps and 2D horizontal anisotropy in the diffusion model of NEMOVAR, and to validate the numerical implementation using standard unit tests. Single observation and point-correlation experiments will be used to illustrate the impact of these new features. Longer duration DA experiments will be conducted in collaboration with the NEMOVAR partners to evaluate the impact of the new features in a realistic, global ocean model framework. The main development and testing will be done using computing infrastructure at CERFACS. Software development will involve programming in Fortran and python.

References

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