Internship: Parallel solvers for a Discontinuous Galerkin time stepping scheme of the viscoelastic wave equation

About us

Cerfacs is a basic and applied research center, specialized in modeling and numerical simulation. Through its facilities and expertise in High Performance Computing, Cerfacs deals with major scientific and technical research problems of public and industrial interest.

Cerfacs hosts interdisciplinary researchers such as physicists, applied mathematicians, numerical analysts, software engineers who design and develop innovative methods and software solutions to meet the needs of the aeronautics, space, climate, energy and environmental fields. Cerfacs is involved in major national and international projects and is strongly interacting with its seven shareholders : Airbus Group, Cnes, EDF, Météo France, Onera, Safran and Total.

The topic

Coronary artery disease is an increasing medical condition, that often causes a patient experiencing a heart attack. To model the effect of a plaque in a coronary artery, it is observed that the blood flow past the stenosed region becomes turbulent and creates abnormal variations in wall shear stresses. These shears drive low-amplitude acoustic shear waves through the soft tissue in the thorax which appear at the chest wall and can be measured non-invasively by placing sensors on the skin.

A feasibility study for developing such a non-invasive diagnostic tool has been done in a previous international project, where the human tissue is described by a viscoelastic model. The problem at hand is an inverse problem [1], i.e., by measuring the signal at the chest wall, we want to detect the stenosis' location and size. A main ingredient is the forward solver, as it has to be executed up to several hundreds of times in an inverse problem. In [2], a numerical scheme based on a *Higher-order Discontinuous Galerkin* method has been developed. Suppose that the spatial finite element space is of dimension D and polynomials of degree r are used in time, then the block system has dimension $(r + 1) * D$ and is usually regarded as being too large when $r > 1$. With the described method, it is possible to decouple this large matrix and to instead solve $(r + 1)$ linear systems of size D independently from each other. The resulting matrices are complex symmetric.

In this internship, the intern will first familiarize himself with discontinuous Galerkin methods. Then we propose to implement the forward solver using intra- and inter-node level parallelism. We envisage to implement the solver in Python and couple it to the numerical linear algebra library PETSc. Different choices for solving the complex symmetric matrices, representing still an active research domain, shall be first studied and then tested with PETSc (e.g., MUMPS, GMRES). The intern will evaluate the weak and strong scaling of the method and furthermore the interplay of the polynomial order, the accuracy of the solution and the speed of convergence. If time allows, a further step is to implement the solution of the $(r + 1)$ linear systems in parallel on different nodes.

Your profile

Final year Bac+5 with interests in high performance computing, numerical simulations, applied mathematics or related topics. Languages: French and/or English.

Starting date and location

February/March 2024 for a duration 6 months. The internship will take place at Cerfacs, 42 av Gaspard Coriolis, 31057 Toulouse. Under the condition of a valid security clearance, the intern will be integrated in the common team CONCACE, between Cerfacs – Inria – Airbus.

Contact person

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References

[1] H. T. Banks, M. J. Birch, et al. Model validation for a noninvasive arterial stenosis detection problem. *Mathematical Biosciences and Engineering*, 11(3):427{448, 2014. PDF

[2] H. T. Banks, M. J. Birch, et al. High-order space-time finite element schemes for acoustic and viscodynamic wave equations with temporal decoupling. *International Journal for Numerical Methods in Engineering*, 98(2):131{156, 2014. PDF