

Lean blowoff dynamics in bluff body stabilized flames: unsupervised classification and balance analysis

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Abstract

Lean blow-out (LBO) is a critical phenomenon in gas turbines. It is enhanced by very to ultra-lean operating conditions which are considered today to decrease the environmental impact of combustion. Despite many studies on the subject, the physical mechanisms leading to global flame extinction are not fully understood. Recently, unsupervised classification has appeared in the literature as an efficient tool to identify key features in reactive flows. In this work, unsupervised classification relying on Principle Component Analysis (PCA) and K-means clustering algorithms is used to investigate the underlying mechanisms of a blowoff event in a bluff body configuration. The unsupervised classification allows to identify and track in time 4 distinct zones: fresh gases, burnt gases, fast reacting flame and preheat zone. To elucidate the blow-out processes, an analysis of mass and energy balances of the different zones is proposed. This analysis describes the temporal evolution of the zones as a result of their interactions, which is the main driver for flame stabilization or blowoff. For the considered blowoff event, the extinction is induced by an imbalance between the various contributions identified by the proposed analysis: while the decrease in fuel mass flow rate modifies both the conductive fluxes and chemistry source terms in the reactive zones, the convective fluxes remain constant over time as the total mass flow rate is kept constant. This work suggests that the proposed methodology is a useful tool to analyze unsteady configurations and understand the main mechanisms at work in such configurations.

Keywords: Lean blow-out; Principal component analysis; K-means; Numerical Combustion

1) Novelty and Significance Statement

The novelty of this research relies in the use of unsupervised classification to study the lean blow-out phenomenon. Unsupervised classification has already been used to study MILD combustion (e.g. Li et al., *Proc. Combust. Inst.* 38, 2021). In this work, the unsupervised classification methodology is extended with a balance analysis to extract physical information on the flow dynamics. This is a significant contribution because it proposes an efficient tool to analyze lean blow-out in complex flows, which remains a challenge and has no equivalent in the literature. Furthermore, the proposed methodology could be extended to study many other combustion processes.

2) Author Contributions

- TL : Conceptualization, Methodology, Formal analysis, Writing - Original Draft
- JW : Conceptualization, Methodology, Writing - Review Editing
- ER : Conceptualization, Writing - Review Editing, Supervision
- BC : Conceptualization, Writing - Review Editing, Supervision
- QD : Conceptualization, Methodology, Formal analysis, Writing - Review Editing, Supervision

1. Introduction

Modern gas turbines aim for lower NOx emissions by operating in very lean conditions, at the price of a higher risk to reach the Lean blow-out (LBO) limit [1] and quench. This raises safety concerns in aeronautical engines and demands costly procedures for land-based power generation turbines. Consequently, important efforts have been made by the community to a better understanding on the blowoff phenomenon [2, 3].

First contributions to the blowoff phenomenon focused on bluff body flames. Early studies primarily relied on experiments to propose semi-empirical correlations from experimental data and Perfectly Stirred Reactor (PSR) assumptions [4]. In the pioneer works on the blowoff phenomenon, two main methodologies to study LBO were derived, either based on PSR [5] or Characteristic Time (CT) [6]. In PSR models, the recirculation zone behind the bluff body is interpreted as a PSR: the fresh mixture coming from the shear layer mixes instantaneously with the mixture already in the recirculation zone. In this framework, LBO occurs when the energy released in the recirculation zone is not sufficient to heat the fresh mixture up to its ignition temperature. On the other hand, CT models consider that LBO occurs if the contact time in the shear layer between the hot recirculating mixture and the fresh mixture is not long enough to heat up the latter to the ignition temperature. More recently, Wang et al. [7] combined PSR and CT approaches to study the LBO of a gas turbine combustor. In their methodology the reaction zone is identified and described as a collection of PSRs.

As pointed out by Shanbhogue et al [2], all these methods can be expressed in terms of a Damköhler number $Da = \tau_{\text{flow}}/\tau_{\text{chem}}$, as they describe the local competition between fluid mechanics and chemical processes, and are particularly well suited to identify local extinctions. However they do not consider the overall energy balance of the flame and therefore cannot predict directly global blowoff.

To study global extinction, Sturgess et al. [8] proposed to describe a combustor with a reactor network, which is a simplification often made in the literature [9, 10]. However, the identification of the different zones corresponding to the individual reactors remains a tedious work, especially when considering their volume and location fixed in time while the combustor dynamics are unsteady for stabilized flames and fully transient during an extinction event. To overcome the limitation of predefined zones, the use of unsupervised machine learning clustering algorithms have been recently proposed in the literature [11, 12].

Unsupervised classification algorithms consist in reducing the dimensionality of the variables and clustering the data [11]. Such methods have already been used to derive combustion model, for example by Savarese et al. [12] to automatically generate chemical reactor networks or by Malik et al. [13] to gen-

erate the manifold variables of the Direct Numerical Simulation of a turbulent premixed NH₃/air flame. These methods also proved to be very useful to identify key features in reactive flows [11]. They were used by Zhang et al. [14], coupled with a neural network, to identify combustion regimes in turbulent non-premixed flames. Doan et al. [15] and Li et al. [16] studied MILD combustion with the help of unsupervised clustering and advanced analysis methods.

In this paper it is proposed to use classification algorithms to study LBO. The methodology is applied to a bluff body configuration, the VOLVO experiment [17], to demonstrate its capability to explain the driving mechanisms leading to flame blowoff. The methodology is in two steps: first, unsupervised clustering is used to identify the key features of the reacting flow; then, a balance analysis on the classification is performed to help understanding the blowoff process.

The remainder of this paper is organized as follows. The details of the methodology are provided in Section 2. The test case, numerical set-up and models are described in Section 3. The results are discussed in Section 4.

2. Methodology

2.1. Unsupervised classification

Unsupervised classification is proposed here to pinpoint coherent flow zones to study complex flows with a large amount of variables in both space and time (high data dimensionality). Dimensionality reduction is first applied. Then clustering divides the control volume into subdomains, providing a way to identify and study the coherent flow zones.

Dimensionality reduction is used to sort data and identify a limited set of variables that can accurately describe the problem. To this purpose, the Principal Component Analysis (PCA) is widely used in the literature [13–15], where the Principal Components (PCs) are built from a linear combination of the original data to form an orthogonal basis. PCA is briefly recalled below.

Consider a data set X of dimension $(n \times p)$ containing n samples of p variables, centered and scaled as :

$$X_{n,p}^c = \frac{X_{n,p} - \overline{X}_p}{\sigma_p} \quad (1)$$

with \overline{X}_p and σ_p the mean and standard deviation of X , PCA provides q linear combinations corresponding to the eigenvectors of a covariance matrix computed from the original data matrix X_c , such that

$$X_q = Z_q A_q^t \quad (2)$$

with X_q ($n \times p$) the approximation of X^c based on the first q eigenvectors, Z_q ($n \times q$) the principal component scores, A_q ($p \times q$) the matrix of the first q eigenvectors and the notation t for the transposed.

1 The clustering is applied in cascade with the PCA 48
 2 in the PC plane. The number of eigenvectors q used 49
 3 for the clustering of the data corresponds to a cumu- 50
 4 lative explained variance of at least 95%. This is 1) 51
 5 necessary to identify and retain only significant vari- 52
 6 ables to reduce biases in the methodology, 2) useful 53
 7 to optimize the performance of the chosen clustering 54
 8 algorithm as its time complexity is $O(n \times q \times i)$, i 55
 9 being the number of iterations for the clustering con- 56
 10 vergence [18]. 57

11 Clustering algorithms are widely spread in the liter-
 12 ature to classify data [12–15]. They are able to
 13 group unlabeled data that show some form of similar-
 14 ity. In this work, the K-means algorithm is chosen. K-
 15 means is a partition-type algorithm that classifies the
 16 data into a prescribed number of clusters, K . Starting
 17 with K randomly defined centroids, the data are asso-
 18 ciated to the cluster with the nearest centroid. Then,
 19 the position of the centroids is iteratively updated to
 20 minimize the mean of the Euclidean distance between
 21 the data in a cluster and its centroid. The objective
 22 function f to be minimized is expressed as:

$$f = \sum_{i=1}^K \sum_{j=1}^m d_{ij}, \text{ with } d_{ij} = \|x_j^i - c_i\|^2 \quad (3)$$

23 where d_{ij} is the Euclidean distance between the j -th
 24 point in the i -th cluster and the centroid of the i -th
 25 cluster.

26 The number of clusters in the K-means algorithm
 27 is prescribed by the user. Statistical metrics have been
 28 developed in the literature to help choose the most
 29 representative number of clusters as, for instance, the
 30 silhouette plot [19]. These metrics typically charac-
 31 terize the data dispersion within clusters, the distance
 32 between clusters or both.

33 2.2. Balances

34 To investigate the dynamics of the main flow fea-
 35 tures, a balance analysis is performed on the clusters.
 36 The rate of change of a conserved quantity q is de-
 37 fined in the differential form as:

$$\frac{\partial q}{\partial t} = -\nabla \cdot (qu) - \nabla \cdot (J) + \dot{S} \quad (4)$$

38 where u is the velocity, $\nabla \cdot (J)$ is the diffusion term
 39 and \dot{S} is a volume source term.

40 The above equation can be integrated over a cluster
 41 volume V_i and transformed with the divergence theo-
 42 rem as:

$$\int_{V_i} \frac{\partial q}{\partial t} dv = - \int_{S_i} (qu + J) \cdot nds + \int_{V_i} \dot{S} dv \quad (5)$$

43 with S_i the surface area of cluster i .

44 The rate of change in the cluster i (LHS of Eq. 5)
 45 can then be explained by the contribution of the dif-
 46 ferent terms in the RHS of Eq. 5. As the cluster vol-
 47 ume V_i varies in time, the contributions are studied

48 by unit volume to avoid biases due to cluster volume
 49 variation. The balance analysis studies particularly
 50 the contributions of the different terms and how they
 51 vary with time as the mean value of the quantities re-
 52 mains, by definition, close to the centroid of the en-
 53 compassing cluster.

54 In this work, the rate of change of the total en-
 55 ergy and species mass fractions are particularly inves-
 56 tigated. The rate of change of the mass fraction Y_k of
 57 species k is:

$$\frac{\partial \rho Y_k}{\partial t} = -\nabla \cdot (\rho u Y_k) - \nabla \cdot (\rho V_{j,k} Y_k) + \dot{\omega}_k \quad (6)$$

58 with $-\nabla \cdot (\rho u Y_k)$ the convective flux, $-\nabla \cdot (\rho V_{j,k} Y_k)$
 59 the diffusive flux, and $\dot{\omega}_k$ the chemical source term of
 60 species k .

61 In the absence of body forces, the rate of change of
 62 the total energy is:

$$\begin{aligned} \frac{\partial \rho E}{\partial t} = & -\nabla \cdot (\rho u E) - \nabla \cdot \left(-\lambda \frac{\partial T}{\partial x_i} \right) \\ & - \nabla \cdot (\rho V_{j,k} h_k) + \dot{\omega}_T \\ & + \nabla \cdot (\sigma_{ij} u_i) \end{aligned} \quad (7)$$

63 with $-\nabla \cdot (\rho u E)$ the convective flux, $-\nabla \cdot \left(-\lambda \frac{\partial T}{\partial x_i} \right)$
 64 the conductive heat flux, $-\nabla \cdot (\rho V_{j,k} h_k)$ the diffusive
 65 flux due to the diffusion of species with different en-
 66 thalpies, $\dot{\omega}_T$ the energy source term, and $\nabla \cdot (\sigma_{ij} u_i)$
 67 the flux coming from the viscous term. As the en-
 68 ergy flux due to the viscous term is found negligible,
 69 it will not be included in the analysis, as usually done
 70 in the literature [20]. This also holds for the diffusive
 71 flux associated to the species diffusion, which will be
 72 therefore omitted in the analysis.

73 The objective of the balance analysis is to identify
 74 which terms in Eqs. 6 and 7 are involved in the flame
 75 stabilization and which are responsible for the LBO.

76 3. VOLVO test case

77 The studied configuration is the VOLVO combus-
 78 tor [17], widely used to evaluate turbulent combustion
 79 models [21]. It is a straight channel with a rectangu-
 80 lar cross-section (0.12 m x 0.24 m). The length of the
 81 channel is 1.55 m. The bluff body has an equilateral
 82 triangular section of 0.04 m in height.

83 The simulation was performed with the code
 84 AVBP developed at Cerfacs [22], which solves the
 85 3D compressible reactive Navier-Stokes equations us-
 86 ing the Large Eddy Simulation (LES) approach. A
 87 central-finite volume Lax-Wendroff scheme [23] of
 88 2nd order both in time and space is used to discretize
 89 convective terms, and a single Runge-Kutta step is
 90 used to explicitly integrate over time. A 2nd order
 91 finite element scheme is used for diffusive terms.
 92 LES equations are closed by the WALE subgrid-scale
 93 model [24]. Subgrid flame-turbulence interactions are
 94 taken into account by the TFLES model with a relax-
 95 ation sensor [25] coupled with the Charlette constant

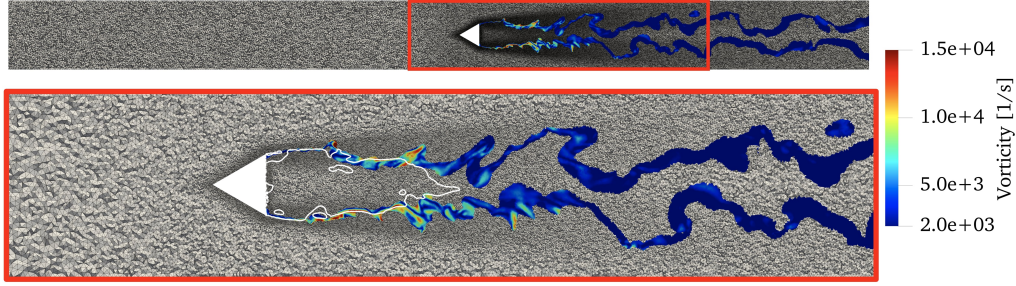


Fig. 1: Computational domain (top) and window of interest for the analysis (bottom) highlighted with a red frame. The vorticity field is shown for reactive zones ($\text{HRR} > 0.1 \text{ MJ/kg/m}^3$). White isoline : $u = 0 \text{ m/s}$.

1 efficiency model [26]. The NSCBC approach [27] is
 2 used for the inlet and outlet boundary conditions and
 3 a no-slip adiabatic condition is set at the walls. The
 4 chemistry, particularly suited to predict laminar flame
 5 velocity and final temperature and products in the lean
 6 combustion regime that is encountered here, is de-
 7 scribed by a 2-step scheme [28]. The above numeri-
 8 cal method was assessed in this configuration by Ro-
 9 chette et al. [29]. They show a reasonable agreement,
 10 though not as precise as higher-order schemes and de-
 11 tailed chemistry. However, they recover most physi-
 12 cal features of the flow, which is enough as our goal is
 13 to demonstrate the analysis methodology's potential
 14 rather than quantitatively characterizing the VOLVO
 15 rig's blowoff limit.

16 The mesh contains 11.9 million nodes and 68.7
 17 million tetrahedral elements. The data that will be
 18 analyzed by unsupervised clustering corresponds to a
 19 window of 0.5 m length around the bluff body, which
 20 contains the entire recirculation zone. The mesh and
 21 the window of interest are displayed in Fig. 1.

22 The procedure to simulate LBO is as follows. A
 23 perfectly premixed mixture of propane-air is injected
 24 at an equivalence ratio of $\phi = 0.65$ to first stabilize
 25 the flame. Then, a step of equivalence ratio is imposed
 26 at the inlet to go from $\phi = 0.65$ to $\phi = 0.50$. This
 27 corresponds to a reduction of the laminar flame speed
 28 by a factor 2 [29]. When the variation of equivalence
 29 ratio reaches the leading edge of the bluff body, solu-
 30 tions are saved at a sampling of 0.7 ms until the flame
 31 blows off. The simulation is performed for a physical
 32 time of 28 ms until the flame has fully disappeared, so
 33 that 40 unsteady 3D solutions are used as a database
 34 for the analysis. Therefore, the entire space-time data
 35 is used for the PCA and, thus, the clustering.

36 4. Results

37 4.1. Application of unsupervised clustering

38 Two sets of variables are used and compared.
 39 The first set only provides thermo-chemical variables,
 40 case A: T and Y_k of the 5 reactive species of the
 41 scheme; N_2 is not provided as it is inert and its mass
 42 fraction can be computed as a linear combination of
 43 the others. The second set, case B, considers all the
 44 variables of the reactive flow equations, i.e., 10 vari-
 45 ables (ρ, u, v, w, Y_k and T in their non-conservative
 46 form). The obtained cumulative explained variance is

47 displayed in Fig. 2 for both cases. For case A, 2 PCs
 48 explain 99.5% of the variance whereas 5 PCs are re-
 49 quired to explain more than 95% of the variance for
 50 case B.

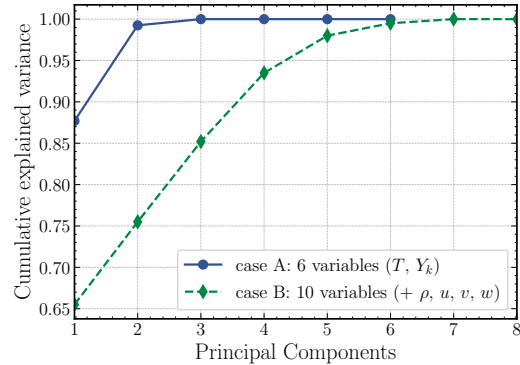


Fig. 2: Cumulative explained variance by the Principal Components for cases A and B.

51 The eigenvectors of the PCs are presented in Table
 52 1 and Table 2 for case A and B respectively. For case
 53 A, the first PC represents a progress variable from the
 54 thermo-chemical point of view: the variables that de-
 55 crease in the flame front have a negative coefficient
 56 value ($Y_{\text{C}_3\text{H}_8}, Y_{\text{O}_2}$) whereas the ones that see an in-
 57 crease in their value have a positive coefficient ($Y_{\text{CO}_2},$
 58 $Y_{\text{H}_2\text{O}}$ and T). The behavior of PC1 for case B is the
 59 same, with the particular feature that the axial veloc-
 60 ity is also correlated to PC1. This can be linked to
 61 the variation in axial velocity as its main change is
 62 a decrease in the recirculation zone and in the wake
 63 behind the bluff body. Both locations contain burnt
 64 gases. Hence, the axial velocity has a global evolu-
 65 tion similar to a reactant. Second and third PCs for
 66 case B are correlated to the transverse velocity com-
 67 ponents. They represent the perturbation induced by
 68 the bluff body and the turbulence as the flow is ini-
 69 tially injected only in the axial direction. Those PCs
 70 are not captured for case A as only thermo-chemical
 71 variables were considered. Finally, the fourth and
 72 fifth PCs for case B are mainly correlated to the axial
 73 velocity and CO mass fraction: these 2 PCs may
 74 be correlated to the intense flame zone where CO is
 75 mainly produced and located in the shear layer where
 76 axial velocity variations exist. A high correlation with

1 CO is also found for the second PC of case A: CO is
 2 the only thermo-chemical variable that has a behavior
 3 slightly different from a progress variable as CO
 4 mass fraction peaks in the reaction zone.

Table 1: Eigenvectors obtained from the PCA for case A.

Variable	PC1	PC2
$Y_{C_3H_8}$	-0.42	0.17
Y_{CO}	0.26	0.96
Y_{CO_2}	0.43	-0.12
Y_{H_2O}	0.43	-0.1
Y_{O_2}	-0.43	0.1
T	0.43	-0.1

Table 2: Eigenvectors obtained from the PCA for case B.

Variable	PC1	PC2	PC3	PC4	PC5
$Y_{C_3H_8}$	-0.38	0	0	0.06	0.14
Y_{CO}	0.22	-0.01	-0.01	0.84	0.5
Y_{CO_2}	0.39	0	0	-0.01	-0.14
Y_{H_2O}	0.39	0	0	0	-0.13
Y_{O_2}	-0.39	0	0	0	0.14
T	0.39	0	0	0	-0.14
ρ	-0.36	0.01	0	0.03	0.16
u	-0.27	-0.01	-0.01	0.54	-0.8
v	0.01	0.65	0.76	0.02	-0.01
w	0	0.76	-0.65	0	0

5 The silhouette scores plotted in Fig. 3 are used to
 6 find the number of clusters for the analysis. The maximum
 7 score is obtained for 2 clusters: the distinction
 8 between fresh and burnt gases is clear in both cases.
 9 However, high silhouette scores are still obtained for
 10 higher numbers of clusters. For case A, 3 and 4 clusters
 11 have a silhouette score close to 2 clusters and may
 12 be considered. For case B, 3 clusters provide a score
 13 close to the maximum. Note also the peak for 5 clusters.
 14 To further understand what the clusters represent, they
 15 are shown in Fig. 4 for both cases and different
 16 numbers of clusters K .

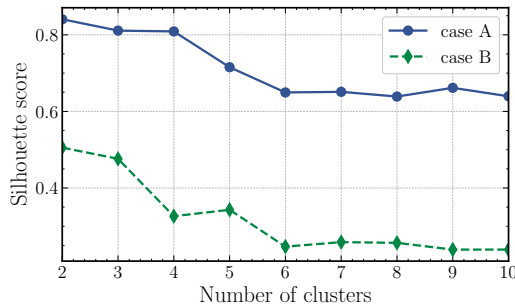


Fig. 3: Silhouette scores for cases A and B.

17 Cases A and B provide similar results for 3 clusters.
 18 Indeed, the 3 clusters correspond to a fresh gases
 19 zone, a burnt gases zone and an intense flame zone,
 20 respectively. This can be related to the fact that in
 21 both cases the majority of the variance is explained
 22 by a progress variable. By introducing more clusters,

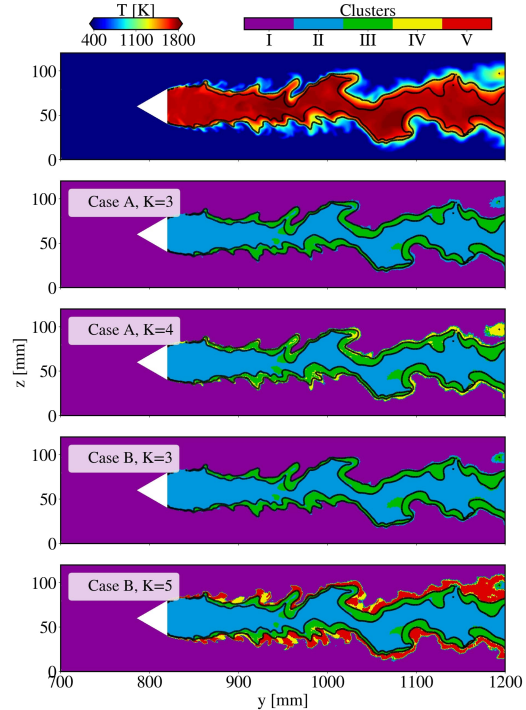


Fig. 4: Top: instantaneous temperature field in a 2D cut.
 Bottom: results of the K-means algorithm at the same instant
 for different cases. Black isoline: $HRR = 0.7 \text{ MJ/kg/m}^3$.

23 differences appear between case A and case B. In both
 24 cases, the new clusters are located between the intense
 25 flame zone and the fresh gases. For case A, the additional
 26 cluster corresponds to a preheat zone where low intensity
 27 heat release rate occurs. On the other hand, clusters IV
 28 and V for case B are correlated with high transverse
 29 velocity, either positive or negative.

30 The previous discussion highlights the importance
 31 of the physical interpretation of the clusters, showing
 32 the additional features that can be retrieved with a
 33 higher number of clusters even with a lower silhouette
 34 score. As case A with 4 clusters presents more
 35 physical features and provides a high silhouette score
 36 close to the maximum, it is retained for the balance
 37 analysis discussed hereafter.

38 4.2. Physical analysis

39 The temporal evolution of the 4 case A-clusters'
 40 location during blowoff is displayed in Fig. 5. Labeling
 41 the clusters is important for the physical analysis. Feature
 42 correlation tools exist in the literature [11] but often
 43 labels are assigned following expert user knowledge [30].
 44 In this work, the labeling is performed following the
 45 study of the mean values of temperature and species
 46 mass fractions in the clusters, reported in Table 3. Cluster
 47 0, corresponding to fresh gases, is characterized by a
 48 low temperature, high reactant mass fractions and low
 49 product mass fractions. Conversely, cluster 2 contains
 50 burnt gases and, thus, has

1 the highest temperature and product mass fractions.
 2 Intermediate zones are either preheat (cluster 1) and
 3 flame (cluster 3), with intermediate values of temper-
 4 ature and major species. Note that the temperature is
 5 higher in the flame than in the preheat zone, and that
 6 CO mass fraction peaks in the flame zone. The label-
 7 ing of the clusters is summarized in Table 4.

Table 3: Clusters' centroids in the physical space

Variables	FG	PF	BG	IF
T [K]	423	1223	1615	1484
Y_{CO}	4.6e-7	4.6e-5	1.3e-5	1.5e-4
Y_{CO_2}	0.010	0.073	0.108	0.094
Y_{O_2}	0.214	0.139	0.097	0.111
Y_{H_2O}	0.005	0.038	0.55	0.050
$Y_{C_3H_8}$	0.029	0.009	0.0	0.007

Table 4: Physical Interpretation of the clusters

Cluster #	Interpretation	Abbrev.
0	Fresh gases	FG
1	Preheat/low intensity flame	PF
2	Burnt gases	BG
3	Intense flame	IF

8 As discussed in the previous section, the initial stable
 9 flame ($t=2.1$ ms) features three main clusters: the
 10 fresh gases (cluster 0 - FG), the burnt gases (cluster
 11 2 - BG) and the intense flame zone (cluster 3 - IF),
 12 plus a preheat zone at the border of the intense flame
 13 zone (cluster 1 - PF). At $t=4.7$ ms, the equivalence
 14 ratio decrease has reached the flame stabilization zone
 15 downstream of the bluff body, and starts to perturb
 16 the flame (cluster 3 - IF), which shrinks and is replaced
 17 by heated, low reacting gases (cluster 1 - PF). This per-
 18 turbation, i.e., switch from IF to PF, then propagates
 19 downstream with the propagation of the equivalence
 20 ratio decrease until $t=11.9$ ms. At this point, the zone
 21 of burnt gases (cluster 2 - BG) divides into two dis-
 22 connected parts. The part of burnt gases upstream,
 23 which persists within the recirculation zone immedi-
 24 ately after the bluff body, is surrounded by PF and
 25 gradually diminishes, while the detached downstream
 26 part of burnt gases swiftly evacuates due to the flow.

27 From this analysis, a first proposition can be made
 28 to define a criterion of extinction. Indeed, in this con-
 29 figuration the flame is anchored in the recirculation
 30 zone downstream the bluff body. In other words if the
 31 flame disappears in this zone, global quenching oc-
 32 curs. This corresponds to no IF in the recirculation
 33 zone, which can be taken as an extinction criterion
 34 and in the present case occurs at $\tau_{LBO} = 13.2$ ms. The
 35 use of clusters therefore allows to easily and clearly
 36 identify global extinction which, as will be seen be-
 37 low, is not possible with the single time evolution of
 38 global quantities [2].

39 This is illustrated in Fig. 6 showing the time evo-
 40 lution of key quantities of interest. After 6 ms, the
 41 thermal power and mean temperature both decrease
 42 linearly with time and completely ignore the extinc-
 43 tion event at $\tau_{LBO}=13.2$ ms. This is due to the fact

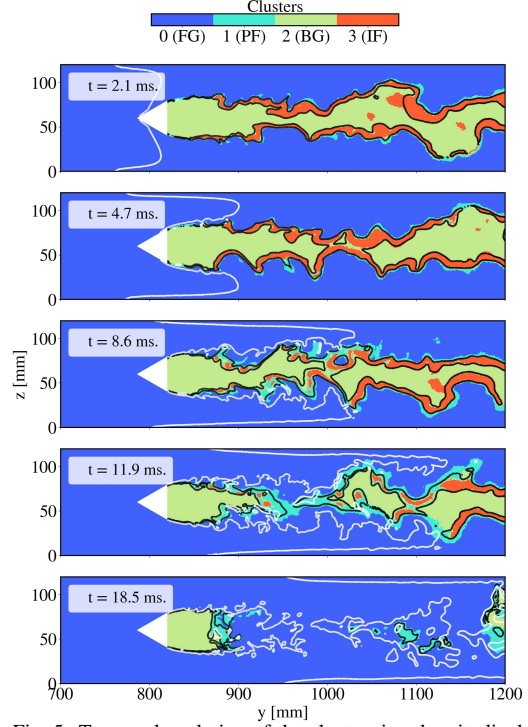


Fig. 5: Temporal evolution of the clusters in a longitudinal cut in case A with 4 clusters. Black isoline: $HRR = 0.7$ MJ/kg/m³. White isoline: $\Phi = 0.51$.

44 that global extinction always starts in a small volume
 45 and is masked by the volumes of the burnt gases and
 46 of the flame, which decrease progressively. It is also
 47 interesting to look at the maximum CO mass fraction
 48 which, as seen above, is a good marker of IF. The CO
 49 mass fraction also decreases with an average constant
 50 slope, but contrary to the two previous quantities it de-
 51 creases from the beginning and, at $t=24$ ms, it abruptly
 52 goes close to zero. This different behavior is due to
 53 the fact that, because CO mostly exists in IF, there is
 54 no masking effect by the rest of the chamber volume.
 55 The sudden final decrease is linked to the sweeping of
 56 the last remaining pocket out of the chamber.

57 Cluster analysis may be used to go deeper in the
 58 understanding of the occurrence of quenching by es-
 59 tablishing energy and mass balances of clusters as
 60 proposed in Section 2.2. The analysis logically fo-
 61 cuses on the first 12 ms, i.e., before extinction occurs.

62 Figure 7 displays the analysis of the rate of change
 63 of total energy in PF for each contribution term de-
 64 scribed in Section 2.2. The net convective flux re-
 65 mains at a constant level over time. This means that
 66 the flow dynamics in the shear layer where PF is lo-
 67 cated is not perturbed by the variation in equivalence
 68 ratio. On the contrary, the balance between conduc-
 69 tive heat flux and chemistry is perturbed after 4 ms
 70 as it starts decreasing. This means that the convective
 71 characteristic time of the flow stays almost con-

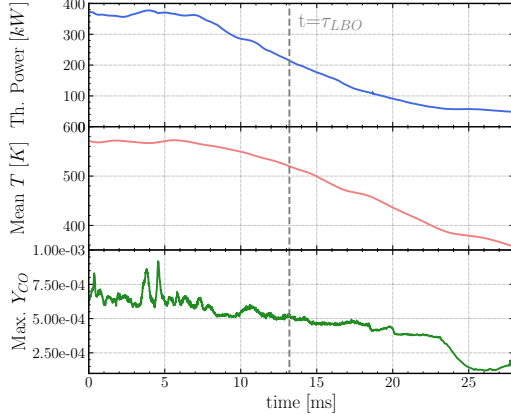


Fig. 6: Temporal evolution of the thermal power, mean temperature and maximum CO mass fraction during extinction.

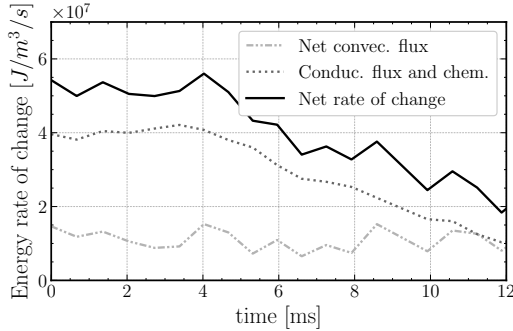


Fig. 7: Temporal evolution of the total energy rate of change by contribution terms for cluster 1 (PF).

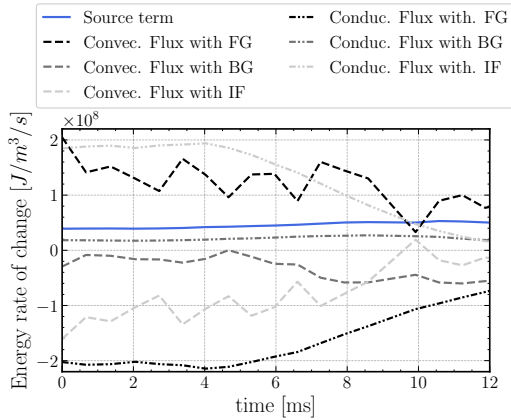


Fig. 8: Temporal evolution of the contributions to the total energy rate of change for cluster 1 (PF).

1 start whereas the characteristic time for chemistry in-
 2 creases. This conclusion is similar to the classical lo-
 3 cal analysis in terms of Damkhöler number but ex-
 4 pressed in a global framework.

5 The rate of change of the total energy can be fur-
 6 ther studied by highlighting the contribution of each

7 cluster, as shown in Fig. 8. This graph clearly shows
 8 that the decrease in the chemistry - conduction bal-
 9 ance in the energy rate of change of Fig. 7 is mainly
 10 due to the sharp reduction of the conductive energy
 11 provided by IF. Even though the net convective flux
 12 remains quasi-constant, the contribution to this flux
 13 from each cluster changes over time. At first, when
 14 the flame is stabilized, PF exchanges mainly with FG
 15 and IF. But during extinction, as IF tends to be re-
 16 placed by PF in the recirculation zone, its contribu-
 17 tion to the convective flux reduces, progressively re-
 18 placed by an increasing convective flux from BG, which
 19 is more and more in contact with PF in the recircula-
 20 tion zone.

21 Finally, a deeper analysis of the changing conduction -
 22 chemistry balance can be made by considering the
 23 time evolution of the CO mass rate of change for
 24 the clusters PF and IF (Fig. 9). Note that the contribu-
 25 tions are normalized by the cluster volume, which
 26 explains why the value of the same flux may change
 27 from one graph to another. PF is the first cluster im-
 28 pacted by the decrease of equivalence ratio at $t=4$ ms,
 29 when both the CO flux from IF and the source term
 30 start decreasing in absolute value. This decrease con-
 31 tinues until 10 ms when the source term changes sign
 32 and becomes positive in PF. This may be associated
 33 to a change of mechanism. In a stable flame, PF
 34 acts mainly as a preheat zone between FG and IF,
 35 where CO produced in IF is consumed. During ex-
 36 tinction, after a certain time (see Fig. 5, $t=8.6$ ms),
 37 an increasing part of PF lies between FG and BG with
 38 lower CO mass fraction and CO may be produced.
 39 The cluster IF reacts to the change of equivalence ra-
 40 tio later than PF, after 6 ms. The source term has a
 41 sharper decrease than the diffusive fluxes, which ex-
 42 plains why IF tends to disappear.

43 The above cluster analysis agrees with previous
 44 results in the literature [2]: when decreasing the
 45 equivalence ratio in the inlet stream, the exchange
 46 through convection is maintained whereas the balance
 47 between the chemistry (studied through the rate of
 48 change of total energy and CO mass fraction com-
 49 ing from the source term) and diffusive terms is per-
 50 turbed. This affects mainly the two clusters located in
 51 the shear layer, cluster 1 (PF) and cluster 3 (IF): PF
 52 sees a reduction in the net rate of change of the total
 53 energy mainly driven by the reduction of the conduc-
 54 tive flux coming from IF.

55 5. Conclusions

56 A balance analysis that relies on unsupervised clas-
 57 sification is applied to study the lean blow-out of
 58 a bluff body configuration, the VOLVO combustor.
 59 The unsupervised classification, coupling PCA and
 60 K-means algorithms, is first discussed with a particu-
 61 lar emphasis on the physical meaning that could be
 62 retrieved from the classification. If fresh gases and
 63 burnt gases are easily recognized, the classification
 64 also identifies reactive zones as key features in the
 65 flow. This allows to identify a blowoff criterion, based

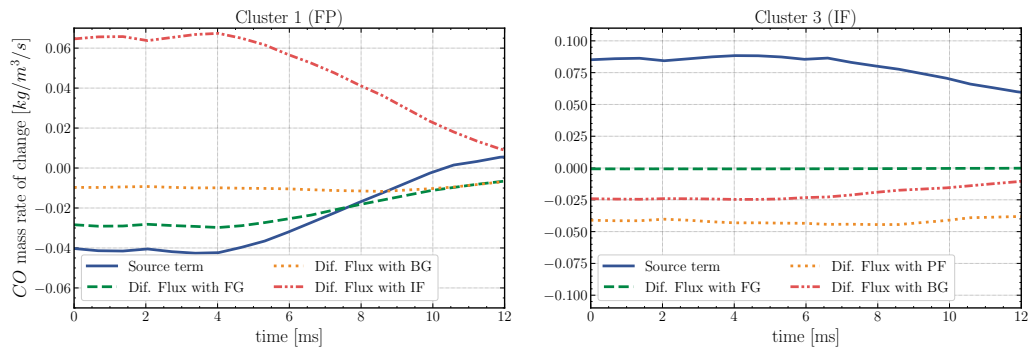


Fig. 9: Temporal evolution of the contributions to the CO mass rate of change for clusters 1 (PF) and 3 (IF).

on the total disappearance of the flame cluster in the recirculation zone downstream the bluff body.

The balance analysis performed on the clusters shows that the decrease of equivalence ratio in an extinction sequence impacts the balance between chemistry and diffusive fluxes in the preheat and flame zones located in the recirculating flow downstream the bluff body whereas the convection contribution remains constant.

Compared to the classical reactor network analysis with non-time-dependent zones, which requires user-dependent, i.e., biased choices, the present work offers a non-biased method to define coherent zones in the flow through the use of unsupervised classification. In addition, the present methodology particularly emphasizes how a change in the system affects the unsteady balance between the zones, which is key for understanding extinction.

This paper demonstrates the capability of the coupling of unsupervised classification and balance analysis to study unsteady phenomena, like lean blow out. This methodology offers a new framework to study and understand the underlying mechanisms of complex unsteady flows and, therefore, opens the perspective to use this method in other contexts such as LBO in swirled turbulent flames, forced ignition, etc.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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