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Application of the discrete ordinates method to grey media in complex geometries using unstructured meshes

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Abstract

A structured radiative heat transfer code based on the standard discrete ordinates method has been extended to unstructured meshes in order to be coupled with a CFD code. The code has been specially written for unstructured grids using tetrahedrical cells, by trying to avoid complex adaptations that are time consuming. The mesh influence is studied for different cases. Results are compared with those obtained with an accurate method (ray tracing).

1 Introduction

In Computational Fluid Dynamics (**CFD**), the coupling between radiative heat transfer and combustion is based on the solution of the energy equation. The heat source term due to radiation is evaluated by taking into account the temperature and radiating species concentrations profiles, which are obtained from the solution of the aerothermochemistry equations. Among all the numerical methods developed to calculate the radiative heat transfer, the Finite Volume Method (**FVM**) and the Discrete Ordinates Method (**DOM**) offer a good compromise between accuracy and computational requirements. These two methods have been widely used to solve radiative transfer problems in structured three-dimensional geometries using Cartesian or cylindrical coordinates. In particular, the DOM, described by Chandrasekar in 1950 [1], has been deeply studied by Lathrop and Carlson in 60-70's [2] and by Truelove, Fiveland and Jamaluddin in the 80's [3, 4, 5, 6, 7]. Significant improvements have been achieved in the last decade aiming at the reduction of the ray effects and false scattering, more accurate quadratures and the extension to complex geometries [8, 9, 10]. Nevertheless, the coupling between radiative transfer and other physical phenomena, such as combustion and fluid flow at high temperatures, requires the solution of the radiative transfer equation using the same grid employed to solve the other governing equations. Unstructured grids are often used in CFD owing to their geometrical flexibility. In this way, a lot of work has been developed during the last decade to apply the FVM to non-orthogonal structured grids and unstructured grids in three-dimensional enclosures [11, 12, 13, 14, 15]. For the DOM application, Sakami and co-workers proposed an accurate but complex method for the spatial discretization by taking into account the exponential extinction [16, 17, 18]. Liu et al. have used the "step" scheme, equivalent of the "upwind" scheme in CFD [14]. A computer code using unstructured meshes and based on the modelling of radiative transfer using DOM, as well as accounting for non-grey gas radiation, has been developed aiming at a future coupling with a combustion code available at the C.E.R.F.A.C.S. in Toulouse. In this work, this radiative code is described and applied to several test cases for validation purposes. The results are compared with accurate solutions available in

the literature, and obtained using other methods, which may be more accurate, but also too time consuming for industrial applications, such as the ray tracing method.

2 Radiative Transfer Equation (RTE)

2.1 Mathematical formulation

Considering an absorbing-emitting and non-scattering grey medium, the variation of the radiative intensity along a line of sight can be written as:

$$\frac{dI(\vec{s})}{ds} = \kappa I_b - \kappa I(\vec{s}) \tag{1}$$

where $I(\vec{s})$ is the radiative intensity, I_b the radiative intensity of the blackbody, and κ the absorption coefficient. Boundary conditions for diffuse surfaces are taken from the relation giving the intensity leaving the wall I_w as a function of the blackbody intensity of the wall $I_{b,w}$ and of the incident radiative intensity:

$$I_w(\vec{s}) = \epsilon_w I_{b,w} + \frac{\rho_w}{\pi} \int_{\vec{n}.\vec{s'}<0} I_w(\vec{s'}) |\vec{n}.\vec{s'}| d\Omega' = \epsilon_w I_{b,w} + \frac{\rho_w}{\pi} H \tag{2}$$

where ϵ_w is the wall emissivity, ρ_w the wall reflectivity, H the irradiation, $\vec{n}$ the unit vector normal to the wall and $\vec{s}$ the direction of propagation of the incident radiation confined within a solid angle $d\Omega$.

2.2 DOM for unstructured grids

The RTE is solved for every discrete direction $\vec{s}_i$. The integration of the radiative transfer equation over the volume V of an element limited by a surface Σ, and the application of the divergence theorem yields:

$$\int_\Sigma I.\vec{s}.\vec{n} d\Sigma = \int_V (\kappa I_b - \kappa I(\vec{s})) dV \tag{3}$$

The domain is discretized in tetrahedra which is the simplest three-dimensional control volume shape. Indeed, every other form constituting an hybrid mesh can be under-meshed by tetrahedra. Assuming that I_b and $I(\vec{s})$ are constant over the volume and associated with the centre of the cell, P, and that the intensities I_j at the faces are constant over each face and associated with the center of the corresponding face, equation (3) is discretized as:

$$\sum_{j=1}^{N_{face}=4} I_j.(\vec{s}_i.\vec{n}_j).A_j = \kappa V (I_{b,P} - I_P) \tag{4}$$

where $\vec{n}_j$ is the outer unit normal vector of the jth face whose area is A_j. The scalar product of the *i*th discrete direction vector with the normal vector of the *j*th face of the considered tetrahedron is denoted by D_{ij}:

$$D_{ij} = \vec{s}_i.\vec{n_j} = \mu_i n_{xj} + \eta_i n_{yj} + \xi_i n_{zj} \tag{5}$$

In order to calculate the radiation intensities at the cell faces, Ströhle et al. [10] proposed a spatial differencing scheme based on the mean flux that can be very useful in the case of unstructured grids. This scheme relies on the following formulation:

$$I_P = \alpha \overline{I_{out}} + (1-\alpha) \overline{I_{in}} \tag{6}$$

where $\overline{I_{in}}$ and $\overline{I_{out}}$ are cell face averaged intensities entering and leaving the control volume, respectively:

$$\overline{I_{in}} = \frac{\sum\limits_{\substack{j \\ D_{ij}<0}} D_{ij} A_j I_j}{\sum\limits_{\substack{j \\ D_{ij}<0}} D_{ij} A_j} \tag{7}$$

and

$$\overline{I_{out}} = \frac{\sum\limits_{\substack{j \\ D_{ij}>0}} D_{ij} A_j I_j}{\sum\limits_{\substack{j \\ D_{ij}>0}} D_{ij} A_j} \tag{8}$$

Substituting $\overline{I_{in}}$ from equation (6) into equation (4) yields:

$$I_P = \frac{\alpha V \kappa I_b - \Theta \sum\limits_{\substack{j \\ D_{ij}<0}} D_{ij} A_j I_j}{\alpha \kappa V + \sum\limits_{\substack{j \\ D_{ij}>0}} D_{ij} A_j} \tag{9}$$

with:

$$\Theta = \alpha - (1-\alpha) \frac{\sum\limits_{\substack{j \\ D_{ij}>0}} D_{ij} A_j}{\sum\limits_{\substack{j \\ D_{ij}<0}} D_{ij} A_j} \tag{10}$$

The case $\alpha = 1$ corresponds to the step scheme. After calculation of I_P from equation (9), the radiation intensities at cell faces such that $D_{ij} > 0$ are set equal to $\overline{I_{out}}$, obtained from equation (6).

The discretization of the boundary condition (equation (2)) is straightforward:

$$I_w = \epsilon_w I_{b,w} + \frac{1-\epsilon_w}{\pi} \sum_{\vec{n}.\vec{s_i}<0} w_i I_i \mid \vec{n}.\vec{s_i} \mid \tag{11}$$

The DOM is based on the discretization of the solid angle 4π in a chosen number N_{dir} of discrete directions $\vec{s_i}(\mu_i, \eta_i, \xi_i)$ associated with the respective weights w_i. In this way, different angular discretizations can be used, namely the polar/azimuthal discretization typical of the FVM, the S_N and the T_N quadratures, the last two being the most used in the DOM. The results presented below have only used the S_N quadratures.

The control volumes should be treated following a sweeping order such that the radiation intensities at upstream cell faces are known. An algorithm for the optimization of the sweeping order was implemented int the present work. A sweeping order optimization not only avoids too many iterations for the cases with scattering media and/or reflective walls geometries, but also avoids iteration in the case of black walled enclosures without scattering. This order depends on the discrete direction under consideration.

3 Results and discussion

Several simple test cases have been carried out to validate the code for different types of enclosures with transparent or grey media. Four of these test cases are presented here. The influence of the grid size and angular discretization was evaluated.

3.1 Black walled rectangular enclosure

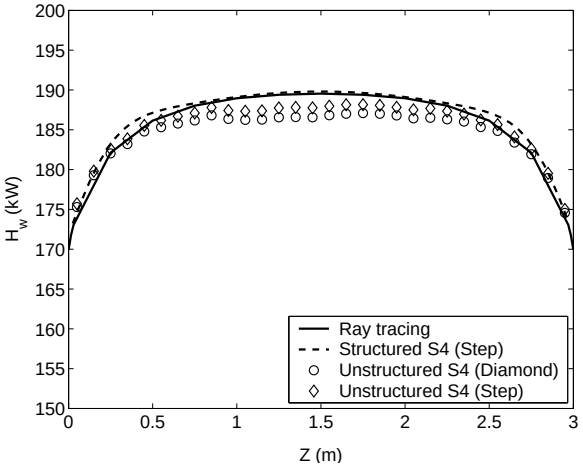
The first test case is made on a box-shaped furnace enclosure (1m x 1m x 3m). The walls are black at 1000°C and the medium is grey with $\kappa = 0.1\text{m}^{-1}$ and at a temperature of 1500°C. Two unstructured grids have been tested. The coarse one has 3222 tetrahedra (case 1) and the fine one 20425 (case 2). The results are compared with those calculated using a cartesian grid with a 20x20x60 mesh and a reference solution solution calculated using ray tracing with 80000 rays (in this way, the results can be considered as numerically quasi-exact). The incident heat flux, H_w, along the centerline of the largest walls is shown in figure 1. The tests have been performed for three different S_N quadratures - S_4, S_8 and S_{12} - and two different spatial differencing schemes - diamond mean flux scheme ($\alpha = 0.5$) and step scheme. The results obtained using an unstructured mesh are in good agreement with the reference solution regardless of the quadrature and the grid refinement. The spatial discretization scheme has a marginal influence on the accuracy of the results. Further calculations were performed using the unstructured fine mesh rotated by 45° around the z-axis (case 3, see figure 1). The S_8 solution is significantly worst than in case 2, contrary to the other solutions. The reason for this is not clear, but may be due to the ray effect.

3.2 L-shaped enclosure containing a grey medium

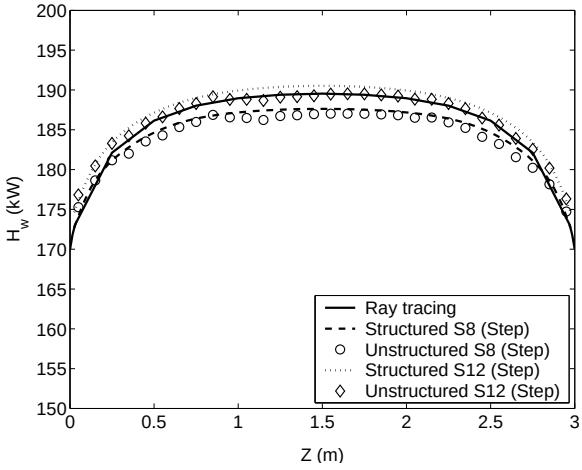
In this test, an L-shaped enclosure containing a grey emitting-absorbing medium at $T = 1000\text{K}$ has been considered (see figure 2). The walls are black at $T_w = 500\text{K}$. The S_4 angular quadrature is used and the number of cells is 17192. The results obtained for four different values of κ are presented in figure 2(c). The results obtained with the step scheme have been compared with those presented by Sakami et al. (DOM using an S_4 quadrature with the exponential spatial discretization scheme applied to a grid comprising 2000 tetrahedra) [19] and by Malalasekera and James (Discrete Transfer Method applied to a non-orthogonal structured grid) [20]. The difference between the present results and those obtained in [19] and [20] is not large. The grid used here is fine, showing that a step scheme used with a fine mesh can give results similar to those of the exponential scheme, which is more accurate.

3.3 Cylindrical enclosure with transparent medium

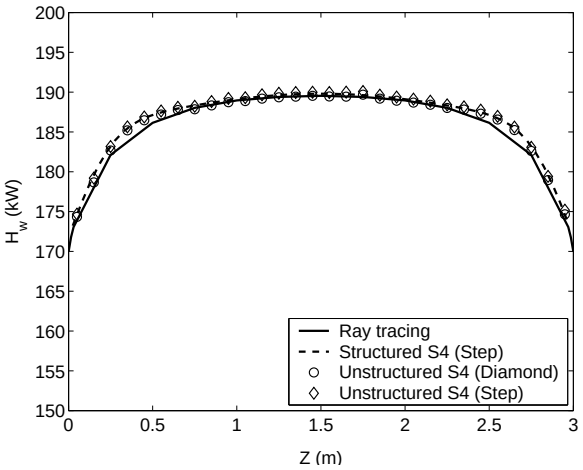
As a third test, a cylindrical black walled enclosure ($h = 2\text{m}$ and $r = 1\text{m}$) with a transparent medium is considered. The top and side walls are cold, while the bottom one is maintained at temperature $T_w = 1000\text{K}$. The unstructured grid has 2591 cells. The ray tracing method is em-



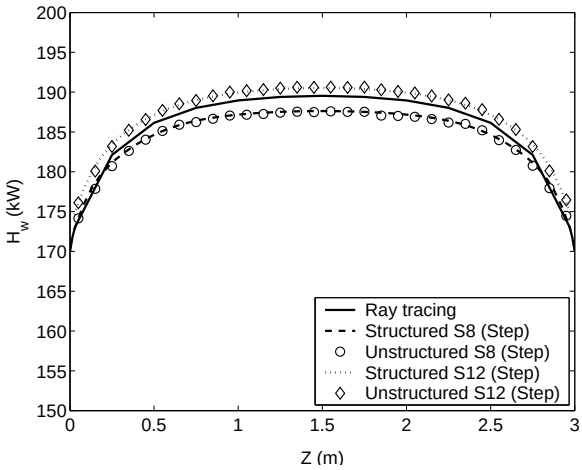
(a) Case 1: Coarse grid (number of control volumes = 3222 tetrahedra)



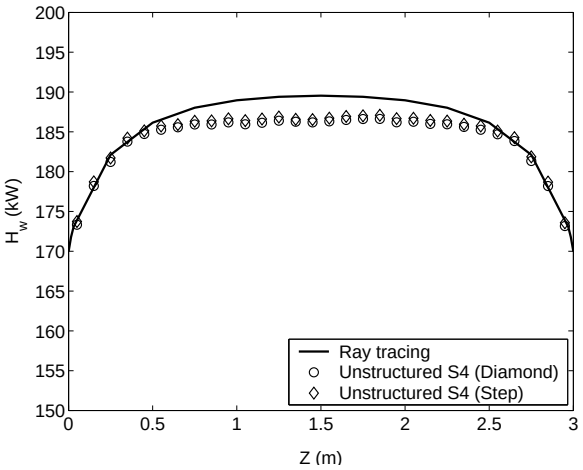
(b) Case 1: Coarse grid (number of control volumes = 3222 tetrahedra)



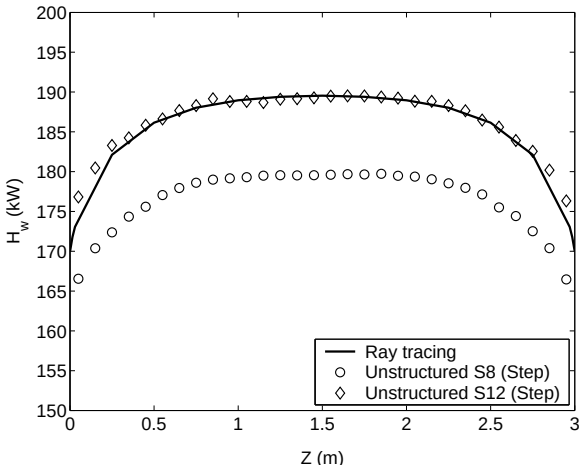
(c) Case 2 : Fine grid (number of control volumes = 20425 tetrahedra)



(d) Case 2 : Fine grid (number of control volumes = 20425 tetrahedra)

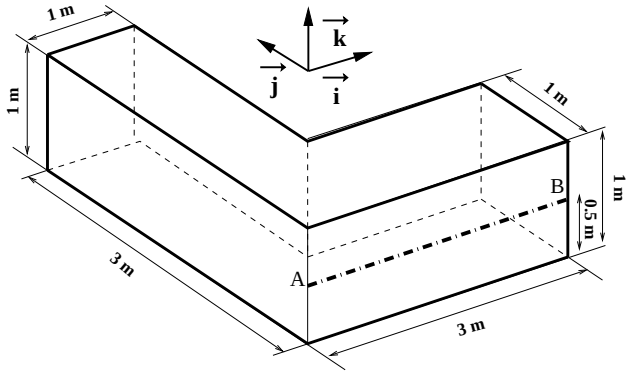


(e) Case 3 : Fine grid 45^o-rotated (number of control volumes = 20425 tetrahedra)

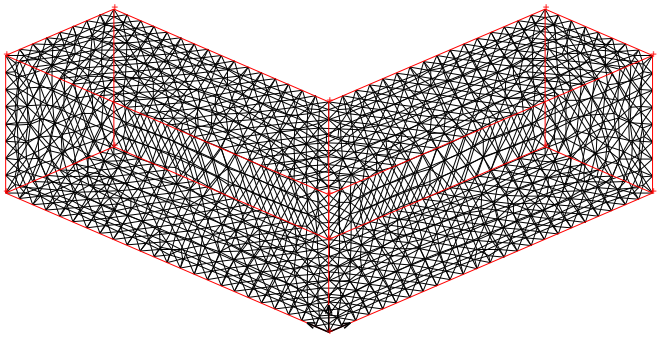


(f) Case 3 : Fine grid 45^o-rotated (number of control volumes = 20425 tetrahedra)

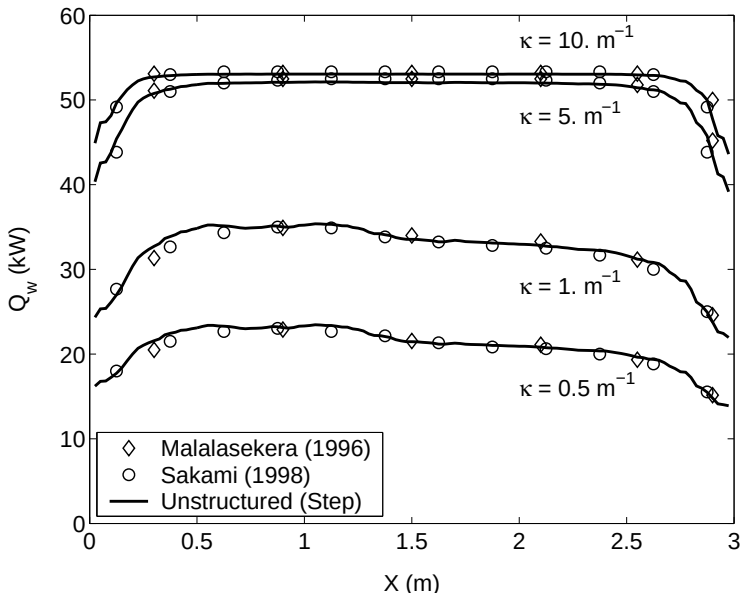
Figure 1: Black walled rectangular enclosure case



(a) The L-shaped geometry



(b) The L-shaped mesh : 17192 cells



(c) Radiative net heat flux at the wall along the line AB of the L-shaped enclosure case using the step scheme and the S_4 quadrature with different values for κ

Figure 2: The **L**-shaped enclosure case

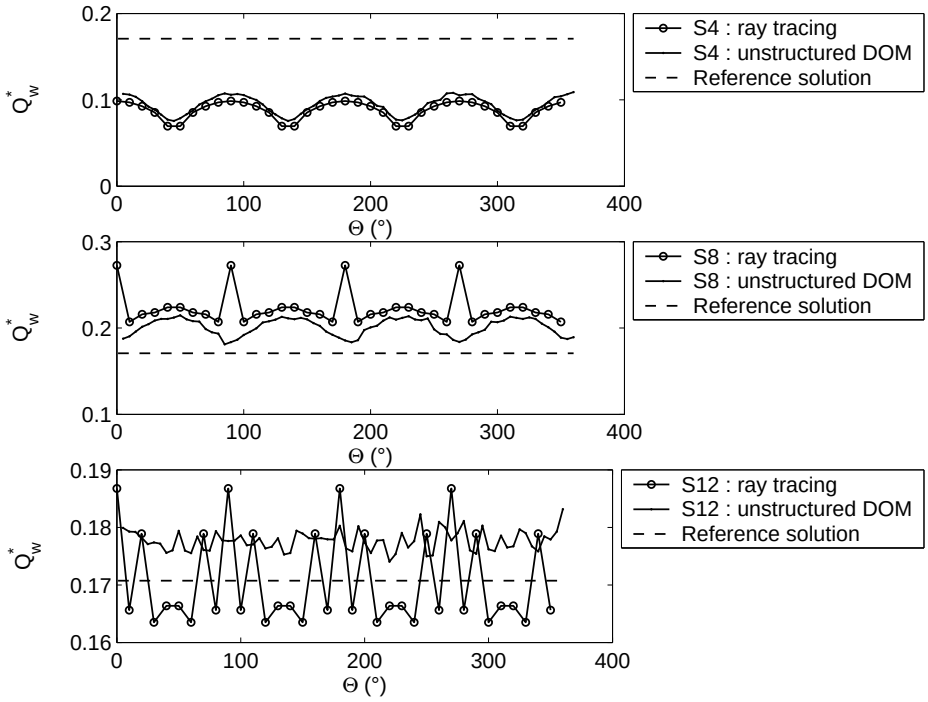


Figure 3: Non-dimensional net heat flux wall along the perimeter of the cylinder at midheight ($z_o = 1$m): comparison ray tracing / unstructured DOM with the same angular quadrature

ployed with a small number of directions, namely the directions defined by the S_N quadratures (those of the quadrature set used in the DOM code). The results obtained using the ray tracing method are independent of the grid, allowing the decoupling of the ray effect due to the angular discretization from the mesh influence. Figure 3 shows the non-dimensional net heat flux normalized by the emissive power of the hot wall, Q_w^*, along the perimeter of the cylinder and at midheight ($z_o = 1$m). The dashed line stands for the exact value, which must be the same along the perimeter. The ray tracing solution exhibits greater oscillations, which are caused by the ray effect. In the DOM, the ray effect is smoothed by the false scattering associated with the spatial discretization.

3.4 Cylindrical enclosure with participating grey medium

Finally, a cylinder ($h = 3$m and $r = 0.5$m) containing a grey isothermal medium at $T = 1200$K is considered. The walls are at $T_w = 300$K. The radiative heat source S_r along the central axis of the cylinder and the radiative net heat flux Q_w at the side wall are obtained with the unstructured code using the step scheme and the S_8 quadrature. The grid used in this test case is relatively coarse (2591 tetrahedra). Results are compared to those obtained with a ray tracing method using 320000 rays (figure 4). In this case of a homogeneous medium, the results of the ray tracing method are also grid independent. The DOM results are in good agreement with the ray tracing solution, regardless of the absorption coefficient of the medium. In the case of optically thin media, the peak of the net heat flux predicted by the DOM is about 10% lower than the ray tracing solution, but the two solutions become closer with the increase of κ. The radiative heat source predicted by the DOM is in excellent agreement with the ray tracing solution for $\kappa = 0.1\text{m}^{-1}$. The increase of the absorption coefficient of the medium yields also an increase of the radiative heat source. However, for optically thick media, $\kappa = 10.\text{m}^{-1}$, the radiative heat source decreases,

because most of the energy emitted by the medium is absorbed within a very short distance. In suce a case, the relative difference between the DOM and ray tracing solutions is significant, since the radiative source approaches zero, but the absolute difference is still small.

4 Conclusion

A DOM code using unstructured grids has been developed. The code has been validated for transparent and grey media. The ray effect influence has been investigated and the compensation between ray effects and false scattering errors has been shown. The method used is relatively accurate. The introduction of an SNB-CK model into the code is in progress in order to take the spectral dependencies into account.

Acknowledgements

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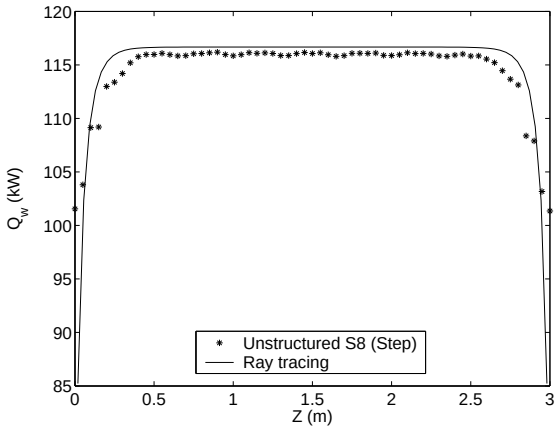
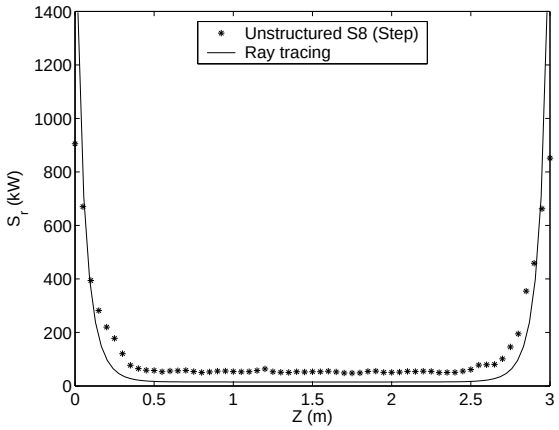
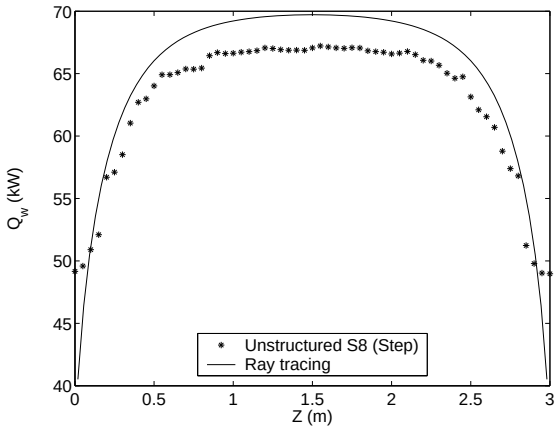


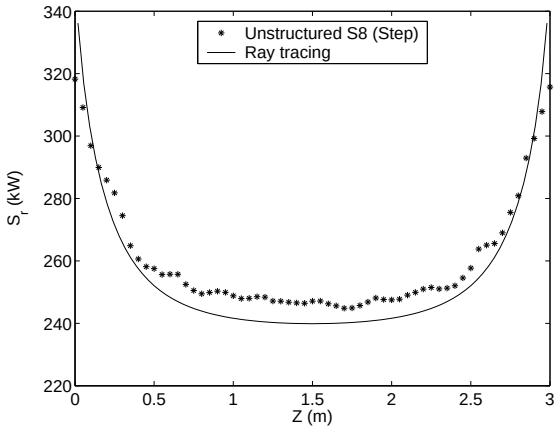
(a) Case 1: Q_w on the side wall for $\kappa = 10.0m^{-1}$



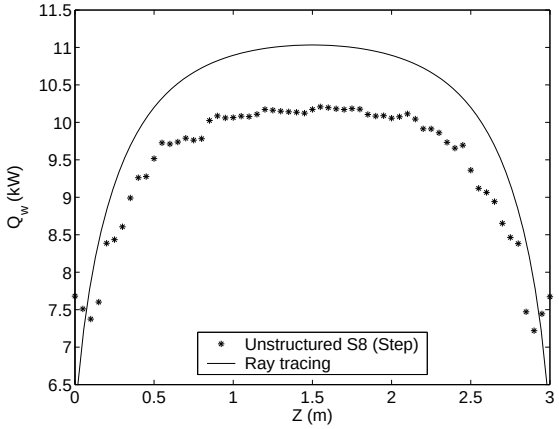
(b) Case 1: S_r on the side wall for $\kappa = 10.0m^{-1}$



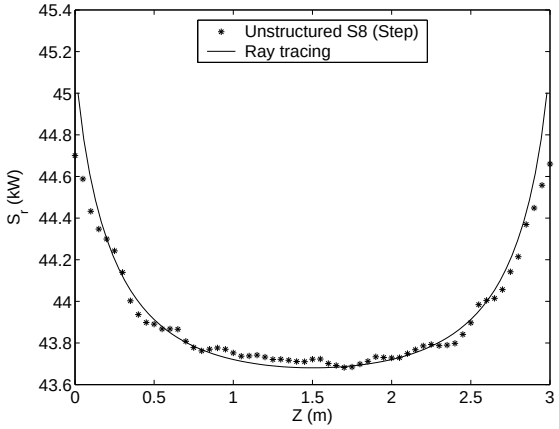
(c) Case 2 : Q_w on the side wall for $\kappa = 1.0m^{-1}$



(d) Case 2 : S_r on the side wall for $\kappa = 1.0m^{-1}$



(e) Case 3 : Q_w on the side wall for $\kappa = 0.1m^{-1}$



(f) Case 3 : S_r on the side wall for $\kappa = 0.1m^{-1}$

Figure 4: Cylindrical enclosure with participating grey medium : unstructured code using a coarse grid (2591 cells) compared to ray tracing.

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