Computational-Fluid-Dynamics-Based Kriging Optimization Tool for Aeronautical Combustion Chambers

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Current state-of-the-art in Computational Fluid Dynamics (CFD) provides reasonable reacting flow predictions and is already used in industry to evaluate new concepts of gas turbine engines. In parallel, optimization techniques have reached maturity and several industrial activities benefit from enhanced search algorithms. However, coupling a physical model with an optimization algorithm to yield a decision making tool, needs to be undertaken with care to take advantage of the current computing power while satisfying the gas turbine industrial constraints. Among the many delicate issues for such tools to contribute efficiently to the gas turbine industry, combustion is probably the most challenging and optimization algorithms are not easily applicable to such problems. In our study, a fully encapsulated algorithm addresses the issue by making use of a new multi-objective optimization strategy based on an iteratively enhanced meta-model (Kriging) coupled to a Design of Experiments (DoE) method and a fully parallel three dimensional (3D) CFD solver to model turbulent reacting flows. With this approach, the computer cost needed for thousands of CFD computations is greatly reduced while ensuring an automatic error reduction of the approximated response function. Preliminary assessments of the search algorithm against simple analytical test functions prove the strategy to be efficient and robust. Application to a 3D industrial aeronautical combustion chamber demonstrates the approach to be feasible with currently available computing power. One result of the optimization is that possible design changes can improve performance and durability of the studied engine. With the advent of

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massively parallel architectures, the intersection between these two advanced techniques seems a logical path to yield fully automated decision making tools for the design of gas turbine engines.

Nomenclature

X	Set of optimization parameters
X^{\star}	Global optimum parameters
f(X)	Objective function of optimization parameters X
$\hat{f}(X)$	Approximation of $f(X)$
$\hat{\sigma}_f(X)$	Variance of the approximation $\hat{f}(X)$
f_M	Merit function
Q	Parameter of the merit function
Φ_{eta}	Criterion of spatial homogeneity of samples in a design space
η_c	Combustion efficiency
θ	Parameter of the combustion efficiency
Pr_f^s	Stator thermal stress criterion
T	Temperature
P	Pressure
V	Velocity
Q	Mass flow rate
ρ	Flow density
V_c	Volume of the primary zone
\dot{m}_a	Air mass flow entering the primary zone
σ	Porosity of multi-perforated plates
S	Surface
P_{pi}	Position of primary jets
P_{dt}	Air flow split between the swirler and the multi-perforated plates
P_{mp}	Air flow split between external and internal multi-perforated plates
Subscrip	t
3	Compressor value
4	Plane 4 value
T	Swirler value
MP	Multi-perforation value
Superscr	ript
a	Dimensionless value

b Baseline value

e External value

i Internal value

ref Reference value

I. Introduction

Systematic use of optimization for gas turbine combustion chambers is usually limited due to the substential computing power required by such applications. Furthermore, global optimization strategy remains beyond today's limits and well tuned, targeted search methods (based on knowhow) in a restrained design space are the only viable options. Despite these constraints, numerous domains have seen the advent of fully automated decision-making tools to help the design of new devices. In fluid mechanics, contributions remain quite limited because of the difficulty in obtaining accurate flow estimates and the need for highly computer demanding algorithms. Flow predictions in real applications are usually obtained by Computational Fluid Dynamics (CFD) which necessitate the numerical solution of spatially and temporally dependent partial differential equations. The resolution of this system of equations usually takes four to five hours on modern supercomputers. That non-negligible computational effort accentuates the need for intensive computing facilities especially if optimization is targeted. It also underlines the necessity for very efficient search procedures such as gradient methods using adjoint CFD solvers.¹ Availability of the adjoint CFD solver partly explains why CFD based optimization is mostly developed for purely aerodynamic problems,^{2,3} where the maturity of the CFD codes allows access to the adjoint solvers. Recent applications of such optimization tools to 3D aerodynamic problems have been realized⁴⁻⁷ with success.

A direct application of aerodynamic oriented techniques to fully turbulent reacting flows is not trivial. Indeed, the extended physics implied by turbulent reacting flows involve strong couplings between combustion, mixing and flow dynamics which make the development of CFD adjoint solvers a difficult task. Gradient estimations can still be obtained by finite difference techniques. However, this approach is known to be sensitive to the noise generated by the numerical solution of the system, the grid management as well as all the various transformations introduced by the optimization process.⁸ Direct deterministic search methods⁹ are thereof preferred. The primary reasons are their reliability, ease of implementation, applicability to non-linear and non-differentiable problems where they yield good results when sophisticated approaches fail.¹⁰ These methods are also easy first choices before going into the development of more complex approaches. They are also available in most optimization tools: *i.e.* Nimrod, 11,12 Dakota, 13 Condor, 14 OPT++, 15 iSIGHT, 16 Optimus. 17

In the context of optimization, algorithm design is faced with two conflicting criteria. "Exploration" indicates the capability of a method to search global interesting configurations over the whole design space. On the contrary, "exploitation" indicates the capability of using already known information to rapidly converge to a local optimum. Among the deterministic approaches, zero order models are usually limited to local searches while performing an efficient exploitation of the available data to converge rapidly to an optimum in the neighborhood of the starting point. Exploration remains critical if a global optimum is targeted. Stochastic processes are usually introduced to extend the local search by random identification of several initial search points.¹⁸ Genetic algorithms are the most commonly used stochastic methods.^{8, 19–21} Finally, the coupling of efficient gradient approaches with stochastic methods would ensure efficient local and global search.²²

As pointed out initially, the most important constraint for the development of CFD based optimization tools, is the limitation on CPU resources: the tool should provide an acceptable response time even with CPU demanding applications while respecting industrial constraints. For example, the N3S-Natur CFD code needs approximately four wall-clock hours to provide a flow field estimate in a single sector helicopter combustion chamber. For that specific reason and since most of the cited optimization methods require multiple evaluations of the objective functions, a reduced fidelity model²³ is introduced to limit the number of expensive CFD runs. The primary idea with this approach is to model the optimization function by an estimate based on a limited number of expensive CFD evaluations, thereby decreasing the overall CPU effort and elapsed time. With the algorithm developed in this work and contrarily to conventional approches, the response surface model is iteratively improved to limit the errors introduced with the estimate. The enhancement of the database, on which the approximation is based, is obtained through automatic requests for new CFD based evaluations which thus provide a set of considered exact values of the response function. Note that the new method has the advantage of not requiring any CFD adjoint solver and is directly applicable to turbulent reacting flow configurations as targeted in this work. Similar simpler Kriging based strategies are adopted in other researches²⁴ and prove to be quite successful in their own areas of application.

When faced with industrial problems, engineers have to deal with multi-objective optimization²⁵ and the most appropriate approach consists in providing Pareto-optima to ease decision making.^{26–29} For that type of optimization problems, access to the optimal solutions is of greatest interest to the designer. However, it should not prevent from identifying the tendencies and dependencies of the design to critical parameters which are valuable information for future developments. Design of Experiments (DoE) is in that case mandatory to efficiently sample the design space^{30,31} and provide efficient analyses of the data.³² With the approach presented, that specificity is automatically addressed since the fully automated decision making tool is essentially dedicated to the improvement of the DoE. Indeed, a DoE is used to construct the estimator (Kriging) which gives access to a local uncertainty on the estimate. This uncertainty is thus optimized by locally refining the DoE thanks to new CFD evaluations.

The document is organized as follows. Specific issues pertaining to the tool automatization, the code management and the optimization algorithm are detailed in sections II-A-B, II-C and III respectively. Verifications and illustration of the impact of the relevant optimization parameters are presented and discussed in section IV-A. Finally, an application to a 3D single sector of a real combustion chamber (section IV-B) is analyzed to illustrate the applicability of the procedure to an industrial case. It results from the demonstration that new design points can be proposed to improve performance and durability of the studied engine.

II. Parameterization of CFD and optimization algorithms

Optimization requires the definition of control parameters determining the search space over which the studied configuration has to be improved. In the aeronautical context, the set of design parameters is very large and cannot be used as a whole. For simplicity, only geometrical and inflow conditions are chosen as possible optimization criteria. That is to say that a given combustion chamber is improved acting on a limited set of parameters and not totally designed from scratch. The user defines cost functions on the search space to assess the quality of a given design in that space. All the state variables and the functions are evaluated from CFD runs. In practice, the steps needed for the preparation of a CFD run are linked to the mathematical formulation of a fluid mechanics problem: defining the flow domain, enforcing the initial and boundary conditions and evaluating the solution for the given set of model equations. A CFD run is hence decomposed in three phases:

- Pre-processing: including automatic mesh generation when shape optimization is concerned, initialization of the physical fields and determination of the boundary conditions,
- CFD computation: solution of the turbulent reacting model equations,
- Post-processing: automatic analysis of the CFD prediction for evaluation and optimization.

The integration of CFD in an automatic strategy for an optimization tool requires to encapsulate these three steps in an efficient and robust package with limited user inputs. Some elements concerning the pre- and post-processing phases are given below. Particularities related to the optimization itself are detailed in section III.

The turbulent reacting CFD code used to provide the flow prediction of the aeronautical combustion chambers is N3S-Natur. It is based on a Reynolds Average Navier-Stokes (RANS) approach and determines the mean stationary flow features for two-phase turbulent reacting flows in complex geometries using tetrahedral grids. Details on the turbulent closures, the turbulent combustion models and the two-phase flow solver are available in Ref.³³ For our work, the following options are used: an implicit solver based on a Gauss-Siedel inversion (first order in time with local time stepping) with a MUSCL second order spatial scheme making use of Van Leer limiter. The turbulence model is the standard $k - \varepsilon$ closure. The turbulent combustion closure is the CLE model.^{34,35} If dealing with two-phase reacting flows, as encountered for the real burner application, a Lagrangian model is activated and coupled to the CFD solver. Convergence of the CFD solver is based on flux balances for mass, total enthalpy and kinetic energy. The CFD solution is thus obtained when all flux balance estimates reach values strictly below 1% of the previous estimate. That stop criterion is used for all of our computations unless specified otherwise. Evaluations of the impact on the optimization predictions of that specific criterion was assessed and found to be weak as long as all balances were below that critical value. Validation and verification of the CFD code can be found in Refs. 34,35

A. CFD Pre-processing

The initial combustion chamber design being provided, the computational domain description is assumed to be available through the Computed Aided Design (CAD) parameterization.^{36–40} Therefore, automation of the initial and subsequent computational meshes is not addressed in detail. Only mesh quality is discussed since it is known to be a critical point when solving partial differential equations using numerical methods.⁴¹ Indeed, great care must be taken to generate a computational grid which ensures meaningful CFD predictions. In the context of geometrical optimization, which involves transformations of an initial computational domain, two methods have been implemented and tested. The first one, generally named moving mesh technique,^{42–45} consists in updating an existing discretization to meet the new set of geometrical parameters. It simply means adjusting the initial grid node positions to fit the new design. Although this method is rather simple to implement, it is limited to small control parameter variations to guarantee acceptable mesh qualities. The second technique aims at fully or partially regenerating a new grid for the new given set of geometrical parameters. Once the geometrical parameterization and regeneration processes are well controlled, this method offers numerous possibilities to produce good quality grids even for complex configurations.^{46–48} For the present work, the full regeneration technique is preferred.

The initialization of the physical fields for a given computational domain is also of practical importance. It has a great influence on the time taken by the CFD computation to reach convergence (the only time when the prediction is meaningful and can be post-processed). For our approach, interpolations based on first order spatial Taylor developments are used to project the baseline fields on the new grids. Finally and for most problems, adjustment of the boundary conditions to meet the specified control parameters is trivial.

B. CFD Post-processing

Post-processing steps are of two types in our optimization process. First, it is used to verify the flow prediction provided by the CFD code: *i.e.* to discriminate unphysical solutions potentially obtained with the CFD solver. These verifications are performed through the evaluation of several mass and energy balances as well as analyses of extreme physical quantities. Second, once verified, the CFD results are processed to evaluate cost-functions for the given values of the control parameters. For the specific problems addressed here, these objective function values are computed using local, planar and/or volumetric diagnostics which are easily obtained by manipulation of the CFD prediction and its computational grid.

C. Management of the integrated optimization platform

The fully encapsulated tool is composed of two main components: a) the optimizer and b) the CFD sequences which seek a prediction/approximation of the turbulent reacting flow. Both components are themselves divided in fundamental sequences corresponding to mathematical or geometrical operations and which often rely on specific computer codes. The first consequence of that multi-code environment is the need for an efficient management technique of all the components (some of which are parallelized) as well as the execution of some of the components themselves in parallel. At the same level of importance, one notes the need for an efficient management of the data transfers between elements to ensure a robust and flexible tool. The use of a coupling device is retained to satisfy at best all of these prerequisites. The dynamic parallel code coupler PALM⁴⁹ offers such capabilities and the optimization platform which results from the present developments is based on this device. Within PALM, the application is decomposed in independent units al-

lowing non-hierarchical coding: the different units can be launched competitively or successively according to the general algorithm and units exchange data by parallel MPI protocols, Fig. 1. The optimization application, called MIPTO for Management of an Integrated Platform for auTomatic Optimization, directly inherits from these capabilities and takes advantage of High Performance Computing (HPC) through the use of parallel units (*i.e.* parallel CFD codes) and simultaneous tasks (*i.e.* simultaneous CFD evaluations) management. The efficient CPU management with a device such as PALM also justifies the optimization methods as detailed below. Note that no disk access is necessary as dynamic addressing is fully managed for data transfer between codes/units. If re-meshing techniques necessitate a commercial software (*i.e.* GAMBIT⁵⁰ in the coming example), MIPTO is able to send requests to check for license availability. That software may be accessed on a distant server if not available where the application is operating. Details about the developed and implemented methods in MIPTO can be founded in Ref.⁵¹

III. The optimization process

The optimization methodology is constructed to:

- Provide relationships between control variables and objective functions (mean tendencies, relative importance of optimization parameters),
- Inform about local and global optima for each objective function,
- Detect the conflicts between cost functions by identifying Pareto Fronts.⁵²

The core of the procedure is based on the construction of an approximate model or Meta-Model²³ (MM) for each objective function. The principal advantage of such MMs is to limit the number of computations involving full 3D CFD evaluations that are known to be very computer intensive and time consuming. The sample databases (DBs) used to compute the MMs are initially constructed from a finite set of CFD runs chosen by a Latin Hypercube Sampling (LHS) algorithm.⁵³ The DBs are then iteratively enhanced by adding new samples evaluated by new CFD computations. These evaluations are chosen by parametric operators that give more or less importance to exploration and exploitation. These new samples are chosen based on the uncertainty information contained in the MMs and aim at reducing the uncertainty of the next MMs.

A. The Kriging estimator as meta-models

In the context of optimization, a wide variety of surrogate models are used in the literature to approximate expensive evaluations of fitness functions. The most prominent methods among all approaches are polynomial models,⁵⁴ artificial neural networks,⁵⁵ radial basis function networks⁵⁶ and Gaussian processes (GPs).⁵⁷ Among these empirical models, GPs appear to be the most promising for fitness function approximations. Indeed, GPs combine the following decisive properties and were successively applied for combustion problems:^{24, 58}

- The implementation of GPs is independent of the number of decision variables,
- GPs can accurately approximate arbitrary functions including multi-modalities and discontinuities,
- GPs contain meaningful Hyper-Parameters (HPs) that can be obtained theoretically with an optimization procedure,
- GPs yield an uncertainty measure of the predicted value in the form of a standard deviation.

Two MMs are available in the developed tool. The first one draws inspiration from ordinary Kriging.⁵⁹ The second one aims at enhancing the behavior of the estimator when faced with noisy functions or badly sampled DBs.⁶⁰ Both MMs learn their specific HPs according to the current DBs and yield an estimator $\hat{f}(X)$ of the true function f(X) as well as the standard deviation $\hat{\sigma}_f(X)$ of the predictor for the design point X.

B. The meta-models enhancement operators

For each iteration of the method, the enhancement of the DBs are based on two operators:

• For each objective i, a search of local optima is performed for the merit function f_M^i defined by

$$f_M^i(X) = \hat{f}_i(X) + \rho \,\hat{\sigma}_{f_i}(X),\tag{1}$$

where ρ is a negative user defined parameter. The value of this parameter controls the conflict between exploration and exploitation. When ρ tends to 0, the exploitation is fostered. As ρ decreases, more attention is given to exploration. The local optima are obtained through the use of a multi-start strategy⁵¹ of a gradient algorithm,⁶¹

• The second operator acts in the case of multi-objective studies. It selects the points that belong to the Pareto Front, obtained from the MMs with the genetic algorithm NSGA-II,^{62,63} and which have the highest values of $\hat{\sigma}_{f_i}(X)$. The operator then aims at improving the precision of the predicted Pareto Front.

The two enhancement operators propose a set of new sample points to be evaluated using the CFD solver. In order to optimize the use of computational resources (if the number of new samples is not proportional to the number of simultaneous evaluations), a third party can add other samples based on cross-over genetic type operations.⁶⁴ It is important to underline that for certain sets of control parameters, the CFD solver may not find acceptable solutions. For these points and to avoid penalization of the merit function, the value of $\hat{\sigma}_{f_i}(X)$ at these locations is suppressed if no information about the objective function is provided (failed CFD).^{65,66}

The global algorithm is presented in Fig. 2. The initial DBs (depicted by the item "Observations" in the figure) is obtained by a DoE method. Optimal (in the sense of orthogonality and dispersion^{67,68}) LHS is generally used for this initialization phase. The stopping criteria for that sequence are the total number of CFD evaluations, the number of new samples obtained by the operators or the overall precision of the MMs. The two steps referred to as "Observations and New Observations" in Fig. 2 consist in evaluating independent sets of design parameters. Consequently and depending on the available computing resources, the different evaluations can be done simultaneously. This feature aims at reducing the overall response time of the method while benefiting from HPC.

IV. Algorithm verification and application to a real combustion chamber

A. Methodology verification and assessment

In order to verify the behavior of the implemented optimization method, a simple analytical cost function for a single optimization parameter is considered. For this test and to mimic non-converged or failed CFD computations, the design space contains a Non-Definition Zone (NDZ) where the evaluation of the control parameter is not possible, Eq. (2),

$$f(X) = f(x) = -0.01 \left(200 - (x^2 + 5.5x - 11)^2 - (x^2 + x - 7)^2 \right) - \left[2.5 \exp\left(-(x - 1.5)^2 \right) + 1.3 \exp\left(-(x + 4)^2 \right) \right]$$
(2)
with $x \in [-5, -0.5] \cup [1, 5].$

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The convergence criterion for which the estimator is considered to be a good representation of the target response function, is set to be the L_2 norm of the difference between the analytical target and its estimate. Convergence is in this case set to be below 1% and is kept as such for all of the tests presented in this section. The convergence history of the algorithm is presented in Fig. 3 after the initialization and for the 5 subsequent iterations corresponding to the enrichment of the DBs. With the chosen input parameter, $\rho = -10$, the merit function converges to the estimator in only 4 iterations. The treatment of the NDZ does not disturb the method which yields final DBs composed of samples that cover the decision space with more concentrations at local and global optima.

As already mentioned, the choice of ρ has a great impact on the results provided by the method. In order to analyze more precisely this impact, we consider the analytical multi-modal objective function presented in Fig. 4, and given in Eq. (3),

$$f(X) = f(x, y) = -0.1(x + 4)^{2}$$

- 3(1 - x)²exp(-x² - (y + 1)²)
+ exp($\frac{-(x + 1)^{2} - y^{2}}{3}$)
+ 10($\frac{x}{5} - x^{3} - y^{5}$) exp(-x² - y²)
with $X \in [-2, 2] \times [-2.5, 2.5].$ (3)

The experiment aims at observing the convergence rate towards the global optimum (denoted by X^* in Fig. 4) along with the evolution of the spatial homogeneity of the samples probed in the design space. The last quantity is evidenced by Φ_{β} :⁶⁸

$$\Phi_{\beta}(DB) = \left[\sum_{\substack{X^i \in DB \\ X^j \neq X^i}} d(X^i, X^j)^{-\beta}\right]^{1/\beta}$$
(4)

where $d(X^i, X^j)$ is a distance measure between X^i and X^j over the design space.

The precision of the MM which represents the analytical function is also gauged through the parameter RMSE (Root Mean Square Error⁶⁹) evaluated with an external test database DB^{T} composed of 50 samples:

$$RMSE = \left[\frac{1}{size(DB^{T})} \sum_{X \in DB^{T}} (\hat{f}(X) - f(X))^{2}\right]^{1/2}$$
(5)

The accuracy of the response surface provided by the MM is assessed in Fig. 5a at each iteration and by comparing X_{min} , the set of optimization parameters that lead to the minimum objective

function value from the current DBs, with X^* . The behavior shows convergence of the method to the global optimum X^* in the design space and in the objective function space, Fig. 5b. Clearly small $|\varrho|$ values yield faster convergence of the method toward X^* (emphasis on the exploitation concept). The drawback of a fast convergence to X^* is illustrated in Fig. 6. For small values of $|\varrho|$, the linear increase of Φ_β goes along with a size increase of the current DBs and a homogeneous enhancement of the DBs in the design space. As expected, large values of $|\varrho|$ lead to a homogeneous exploration of the research space. On the contrary, small values tend to a fast decrease of the exploration performances. The case $\varrho = -4$ shows the typical behavior of the method: at the early stage of the enhancement, $\hat{\sigma}_{f_i}(X)$ is rather large when compared to $\hat{f}_i(X)$. It produces a homogeneous sampling of the search space. At latter stages, $\hat{\sigma}_{f_i}(X)$ decreases (the predictor becomes better throughout the design space) and the merit function converges to the predictor. The enhancement essentially focuses on new samples located around local optima which degrade the overall homogeneity of the sample distribution (drastic increase of Φ_β).

Based on the previous set of tests, homogeneity in the search space has a direct consequence on the precision of the MM in the design space: if there are too many samples in a reduced region of the design space, the RMSE quality measure is degraded (RMSE tends to 1). For multi-objective optimization processes homogeneity is preferred and ρ is set to a fixed value equal to -10 in the following example.

B. Application to a real combustion chamber

1. Description of the target configuration

In this section, an application of MIPTO is presented for a TURBOMECA combustion chamber, Fig. 7. The supplied CFD computational domain corresponds to one single sector of the full annular flame tube. Details about the boundary conditions (location and type) as initially supplied are presented in Fig. 8a. The meshes used for this application are unstructured and contain a mean of 210,000 nodes and 1,130,000 tetrahedral cells. The planes used to analyze the CFD predictions as well as illustrations of the turbulent reacting flow within the chamber are presented in Fig. 8b. Note that Plane 4 coincides here with the location of the distributor, which is a critical element of the engine. Indeed the distributor is subject to large thermal stresses due to the temporal and azimuthal temperature variations induced by combustion taking place in the primary zone of the combustion chamber (region delimited by the primary jets as well as the swirled injector). One aim of the current application is to reduce that thermal stress, hence improving the design of the

given chamber for a given operating point.

2. Presentation of the baseline configuration

Figure 9 presents the dimensionless aerodynamic and combustion fields of the reference design. Most of the aerodynamic activity concentrates in the primary zone of the combustion chamber. In this region, a large re-circulation zone located after the air injector characterizes the flow. The external primary jets bring air within the primary zone to ease combustion. The effect of the internal jets on that part of the chamber is not so clear. Flow streamlines in Plane 1 of the chamber show an important re-circulation zone located after the external primary jets. Note that the re-circulating gases of the primary zone lead to fast evaporation and mixing of the liquid fuel injected through the swirler. Combustion, visualized through the reaction rate (Fig. 9d), takes place in the vicinity of the swirler. Fuel that is not burned in the primary zone is consumed near air admission orifices, mostly in the neighborhood of the external primary jets. Finally, temperature maps (non-dimensionalized by the fresh air temperature, T_3) in Planes 1, Fig. 9e and Plane 2, Fig. 10a, underline the trajectories of hot gases when leaving the primary zones.

3. Definition of the optimization problem

The considered optimization process deals with two conflicting objectives. The first one consists of maximizing the combustion efficiency, η_c . Following Lefebvre,⁷⁰ combustion efficiency for the studied system can be expressed in term of a parameter denoted by θ and defined by:

$$\eta_c = f(\theta) = f\left(\frac{P_3^n V_c \exp(T_3/T_{\text{ref}})}{\dot{m}_a}\right),\tag{6}$$

where P_3 and T_3 are the pressure and the temperature of the air supplied by the compressor, \dot{m}_a is the air mass flow entering the primary zone of the flame tube and V_c is its volume. The maximization of V_c (or the minimization of the inverse of θ) leads to a maximization of the combustion efficiency.

The second objective deals with the thermal stress imposed by the hot gases impacting the distributor, Fig. 11. A relevant measure of this stress is the profile factor at the stator location,⁷⁰ Pr_{f}^{s} , given by:

$$Pr_f^s = \frac{\max(T_4(r)) - T_4}{T_4 - T_3},\tag{7}$$

where T_4 is the mean exit chamber temperature, and $\max(T_4(r))$ is the maximum value of the exit radial temperature illustrated in Fig. 11. Note that minimizing the profile factor increases life-expectancy of the stator and the engine.

The control parameters used to minimize the objective functions (Pr_f^s, θ^{-1}) are of two kinds:

- 1. Geometric: the relative distance from the swirler location to the external and internal jets, noted P_{pi} . Note that the axial distance which separates the external and internal jets is kept constant.
- 2. Flow conditions: the total air mass flow is kept constant. Only the air flow split between the internal, external multi-perforated plates and the swirler are changed (*i.e.*: P_{mp}, P_{dt} respectively).

Figure 12 illustrates the optimization parameters. In the following, the superscript b corresponds to the baseline configuration. In order to keep a constant total air mass flow rate entering the flame tube, which ensures the reference operating point, constraints need to be defined on the optimization problem. For our test, the total amount of air flowing through the multi-perforated plates, Q_{MP} , and the swirler, Q_T , are adjusted to satisfy

$$Q_T = Q_T^b P_{dt}, (8)$$

$$Q_{MP} = Q_{MP}^{b} + Q_{T}^{b} (1 - P_{dt}), \qquad (9)$$

$$Q_{MP}^{i} = P_{mp} \left((1 - P_{dt}) Q_{T}^{b} + Q_{MP}^{b} \right),$$
(10)

$$Q_{MP}^{e} = (1 - P_{mp}) \left((1 - P_{dt}) Q_{T}^{b} + Q_{MP}^{b} \right).$$
(11)

Variations at the swirler inflow are imposed using scale similarity (proportionality) on the velocity profiles specified at the swirler inlet boundary condition of the CFD run. Similarly, multi-perforated inflow conditions necessitate the specification of a velocity, V, proportional to the plate's porosity, σ , its surface area, S and the local flow density ρ :

$$Q = \rho S V \sigma, \tag{12}$$

$$\sigma^{i} = \frac{Q^{i}_{MP}(P_{dt}, P_{pi})}{S^{i}(P_{pi})} \times \left(\frac{S^{i}\sigma^{i}}{Q^{i}P^{i}_{MP}}\right)^{b},$$
(13)

$$\sigma^e = \frac{Q^e_{MP}(P_{dt}, P_{pi})}{S^e(P_{pi})} \times \left(\frac{S^e \sigma^e}{Q^e P^e_{MP}}\right)^b.$$
(14)

In the above expressions, the superscripts i and e respectively denote the internal and external surfaces. The quantity in parenthesis with the b superscript refers to a ratio evaluated for the original design.

The optimization problem then looks for the optimal choices of

$$\min \begin{cases} Pr_f^s \\ \theta^{-1} \end{cases} \quad \text{with} \begin{cases} P_{pi} \in [0, P_{pi}^{\max}], \\ P_{dt} \in [P_{dt}^{\min}, 1], \\ P_{mp} \in [P_{mp}^{\min}, P_{mp}^{\max}] \end{cases}$$
(15)

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4. Results of the optimization process

For the problem considered, one CFD evaluation for a given set of control parameters requires approximately 168 CPU-hours which corresponds to 6 wall-clock hours or elapsed time if using 28 processors of a IBM JS1 power 5 1.5 GHz processors. The enhancement of the DBs is limited to 100 CFD computations (714 CPU-days on 28 processors = 25.5 days) and the initialization of the DBs is set to provide 30 samples (210 CPU-days on 28 processors = 7.5 days). From the 100 CFD runs, the algorithm discards 14% of the predictions (unphysical results) to construct the MMs. For confidentiality reasons, the presentation of the results uses dimensionless quantities:

$$(Pr_f^s(P))^a = \frac{Pr_f^s(P) - (Pr_f^s)^b}{(Pr_f^s)^b},$$
(16)

$$(\theta^{-1}(P))^a = \frac{\theta^{-1}(P) - (\theta^{-1})^b}{(\theta^{-1})^b}.$$
(17)

This normalization is to be interpreted with respect to the performance of the reference design and negative values correspond to improved criteria while positive values indicate degradation. Note also that both objective functions depend on $P = (P_{mp}, P_{dt}, P_{pi})$.

The analysis of the data provided by the enhancement process is presented in two steps. The first step, inspired from sensitivity analyses,⁷¹ aims at better understanding relationships between control parameters and objective functions. The second step deals with the actual search for interesting new configurations after going though the optimization process.

When the number of control parameters or objective functions is quite large, sensitivity measures^{72,73} can guide the designer in distinguishing which parameters are the most important. For our application, one can directly analyze the objective function responses through scatter plots, Figs. 13 & 14. The combustion efficiency parameter θ^{-1} is mostly dependent on the position of primary jets, P_{pi} , and to a lesser extent on the air flow split parameter, P_{dt} . These behaviors are explained in light of the mathematical expression of θ^{-1} where V_c depends on P_{pi} and \dot{m}_a on P_{dt} :

$$\theta^{-1} = \frac{\dot{m}_a(P_{dt})}{P_3^n \, V_c(P_{pi}) \, exp(T_3/T_{\text{ref}})}.$$
(18)

The objective Pr_f^s depends on P_{pi} and P_{dt} . The role played by the mass flow passing through the external and internal multi-perforated plates to feed the dilution process is not detectable by Pr_f^s . The dependency of Pr_f^s along the design space is not as trivial as for θ^{-1} . Focusing on the P_{pi} parameter, moving the primary jets downstream leads to an increased volume of the primary zone, which contributes to a more complete combustion. The major drawback is a shortened dilution length and poorer mixing of the hot products by fresh gases. Hence, the Pr_f^s criterion is degraded.

Looking at the effect of P_{dt} , one notes that below a critical value, the excess of fuel in the primary zone is consumed thanks to the air provided by the primary jets. Mixing and dilution of hot gases by these jets is not efficient any more and the small amount of cold air to be injected by the multiperforated plates is insufficient. Once again, the Pr_f^s criterion is degraded. Beyond the critical value of P_{dt} , combustion in the primary zone is more and more complete and the primary jets play their intended role by properly mixing the hot gases: *i.e.* Pr_f^s is improved.

Figure 15 presents the results of the optimization in the objective function space. The Feasible Domain (FD) and the Pareto Front have been determined from the MMs constructed from DBs containing 88 CFD evaluations. Based on the position of the FD in the objective function space, the optimization process allows for potential improvement of the combustion efficiency and degradation of the thermal criterion at the distributor location (Plane 4). Two separate zones are highlighted on the Pareto Front. In the first one, noted Z_1 , it is possible to drastically improve the combustion efficiency without degrading too much Pr_f^s : $(\theta^{-1}, Pr_f^s) \in [-0.35; 0] \times [-0.1; 0.1]$. On the contrary, in the second region, noted Z_2 , small improvements of θ^{-1} lead to large degradation of Pr_f^s . $(\theta^{-1}, Pr_f^s) \in [-0.5; -0.4] \times [0.15; 0.45]$. Figure 15 also underlines the fact that in the second region of the Pareto Front, a large density of samples locates near the Pareto Front. The method has found new interesting compromises during the last iteration and the new approximation of the Pareto Front has not yet been explored by the enhancement operators. This also highlights the fact that the method has not yet converged to the true Pareto Front of the multi-objective problem. However, a compromise between computational time and convergence properties of the results needs to be set for practical applications. Despite the mentioned shortcoming, two new design points detailed in Table 1 seem interesting in the context of the optimization process. These potential new designs are analyzed below, Figs. 16 to 18. The baseline configuration is presented in Figs. 9 & 10 for comparisons.

The first compromise is located along the Pareto Front, between the two identified zones of the response function space. The second design corresponds to an improved combustion efficiency and a degraded Pr_f^s when compared to the previous point. Figure 16 illustrates the main flow topology differences between the two designs, while Fig. 17 concentrates on the fuel repartition and temperature distributions, and Fig. 18 shows the exit temperature maps. As underlined and identified in practice when defining a new combustion chamber,⁷⁰ the flow topology is greatly influenced by the position of the primary and dilution jets. When these jets are moved downstream (away from the swirler), more and more complex flow structures coexist in the primary zone. A second consequence of the primary jet adjustment is the reduction in intensity of the re-circulation

zone positioned behind the external jets and which is responsible for a large part in the mixing of the hot products with fresh gases. The reduction in size of that flow structure goes in hand with the intensification of a second re-circulation zone positioned behind the internal jets and which has a limited impact on mixing. Finally, and as expected with the changes in the primary and dilution zones inferred by the set of parameters, the new designs yield different exit temperature fields, Fig. 18. For the retained cases, spatial heterogeneity of the exit temperature fields is clearly observed when compared to the reference case, the original design being the optimum.

Based on the previous set of results, several rules of design can be inferred to efficiently obtain a combustion chamber that is optimal in term of combustion efficiency and exit temperature profile. Although necessary to shield the chamber walls from the hot product of combustion, the multiperforated plates do not influence the exit temperature profile. The leading parameters are for this specific objective, the jet position and the flow rate of fresh air that is available for injection at this location. The outer jet also plays a critical role for combustion. If not penetrating the primary zone, fuel is burned in the mixing region of the chamber (outside the primary zone) thereby ruining the exit temperature profile. Combustion efficiency essentially depends on the primary zone volume or equivalently the outer jet position. The aim of the designer is thus to have the proper location of the primary jets as well as the air flow split between the swirler and these jets. It also needs to ensure sufficient outer jet penetration in the primary zone which guarrantees complete combustion in this region while allowing proper mixing of the hot product before exiting the chamber. All other parameters seem to have second order effects.

Preliminary conclusions resulting from the application of the optimization tool to full three dimensional multi-phase reacting CFD are as follows:

- 1. optimization using MMs along with initial DBs of 30 CFD runs proves to be feasible with available HPC power and within industrial constraints,
- 2. Although convergence of the extimated cost functions is not fully ensured after 100 CFD evaluations, the tool recovers know-how obtained by experienced engineers on this specific chamber. That is:
 - (a) Multi-perforated plates have a small impact on the profile factor at the stator location and on the combustion efficiency,
 - (b) The primary jet axial position is of foremost importance and increased efficiency usually results in a decreased exit temperature homogeneity for the configuration investigated.

A recapitulation of the computer costs involved by the use of such an optimization tool is given in Table. 2. Projections are added based on the fact that current CFD codes scale almost linearly to further emphasize the potential impact of HPC on today's engineering work in the field of gas turbine engines.

V. Conclusion

Massively parallel architectures give access to huge computing power and provide new possibilities for the development of tools to be used for the definition of new design of industrial products. Among the impacted fields, the design of the aeronautical combustion chambers still relies heavily on engineering know-how and experience. Although turbulent reacting flow predictions by use of CFD applications is extensively used today by industry for the design of the next generation of combustion chambers, the amount of personnel effort and CPU cost required by these computations prevent extensive design testing or improvements. In fact and contrary to the realm of aerodynamics, the constraints are so important that optimization strategies using CFD codes are scarce in the context of combustion. In this work, a preliminary demonstration of the feasibility of a fully automated decision making tool for combustion chambers is provided. The adopted multi-objective optimization strategy relies on turbulent reacting CFD runs and HPC. In order to limit the impact of many evaluations of flow computations which are CPU- and time-consuming, meta-models are introduced along with a DoE approach. The main contribution of this work lies in the search for optima that are obtained from a meta-model which is automatically improved based on new CFD computations and quality estimators detailed above. Specific issues linked to the management of parallel applications for efficient use of HPC are also addressed. Verifications and sensitivity of the proposed strategy are presented for simple optimization problems based on analytical expressions. To conclude, the application of the new tool to a real gas turbine combustion chamber proves to be feasible with available computing power and yields manageable response time. For that industrial combustion chamber, the aim of the optimization is to improve an existing design in terms of engine durability and efficiency. Two potential new candidates are proposed along with a parameter sensitivity analysis and the identification of the Pareto front. Finally, the tool provides design rules in agreement with the know-how gained by experienced engineers for this type of configuration.

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	$\% \left[P_{dt}^{\min};1\right]$	$\% \left[0; P_{pi}^{\max}\right]$	$\% \left[P_{pm}^{\min}; P_{pm}^{\max} \right]$	$(Pr_f^s)^a$	$(\theta^{-1})^a$
Reference: A	100	0	61	0	0
Candidate: B	57	71	100	0.14	-0.39
Candidate: C	0	100	100	0.44	-0.52

Tables

Table 1. Coordinates of the designs analyzed in the document.

Number of processors	16	32	64	128	256	2048	4096
Single sector flame tube [days]	65.6	32.8	16.4	8.2	4.1	0.5	0.25
Single sector flame tube and its casing [days]	85.3	42.6	21.3	10.7	5.3	0.7	0.3
Complete annular chamber [days]	1280	640	320	160	80	10	5

Table 2. Wall clock time for the optimization of different computational domain of an aeronautical gas turbine engine and as a function of the available computing power. Notice that these numbers are obtained provided that the application scales ideally which is not guaranteed for the CFD solver for example.

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Fig. 1. Schematic representation of the optimization environment tool design with PALM.



Fig. 2. Flow chart of the proposed strategy to construct the database used to generate the metamodels (MMs).



Fig. 3. Verification of the meta-model generation for an analytical case with a Non-Defined Zone (NDZ). Successive iterations of the database enrichment: evolution of the merit function, $f_M(X)$, and the meta-model, $\hat{f}(X)$, at the successive iterates.



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Fig. 12. Description of the optimization parameters and the constraints used for the optimization process of the engine.



Fig. 13. Scatter plots of the combustion efficiency as a function of the other parameters and obtained by CFD at the requested points in the search space.



Fig. 14. Scatter plots of the exit temperature profile factor as a function of the other parameters and obtained by CFD at the requested points in the search space.



Fig. 15. Pareto front as identified for the multi-objective optimization along with the reference configuration, A, and two potential candidates, B & C, for an improved engine configuration.



Fig. 16. Flow field visualization in Plane 1 (*cf.* Fig. 8b) of the aerodynamic quantities obtained by CFD for the two potential candidates for an improved engine configuration: left column, point B, right column point C as defined in Fig. 15 and : (a,b) velocity streamlines, (c,d) velocity magnitude. For comparisons with the reference configuration, A, see Fig. 9.



Fig. 17. Flow field visualization in Plane 1 (*cf.* Fig. 8b) of the combustion quantities obtained by CFD for the two potential candidates for an improved engine configuration: left column, point B, right column point C as defined in Fig. 15 and :(a,b) Fuel/Air ratio (FAR) and (c,d) temperature divided by T_3 . For comparisons with the reference configuration, A, see Fig. 9.



Fig. 18. Temperature fields divided T_3 at the exit of the combustion chamber (Plane 4 of Fig. 8b) obtained by CFD for the two potential candidates for an improved engine configuration: a, configuration B and b, configuration C.