

# Large Eddy Simulation of a Stable Supersonic Jet Impinging on a Flat Plate

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This paper describes a numerical study based on Large Eddy Simulation (LES) of the flow induced by a supersonic jet impinging on a flat plate in a stable regime. This flow involves very high velocities, shocks and intense shear layers. Performing LES on such flows remains a challenge because of the shock discontinuities. Here, LES is performed with an explicit third-order compressible solver using a unstructured mesh, a centered scheme, and the Smagorinsky model. Three levels of mesh refinement (from 7 to 22 million cells) are compared in terms of instantaneous and averaged flow fields (shock and recirculation zone positions), averaged flow velocity and pressure fields, wall pressure, RMS pressure fields, and spectral content using one and two point analyses. The effects of numerical dissipation and turbulent viscosity are compared on the three grids and shown to be well controlled. The comparison of LES with experimental data of Henderson et al. shows that the finest grid (a 22 M cell mesh) ensures grid-independent results.

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## Nomenclature

$c$	= sound speed, m/s
$C_p$	= heat capacity at constant pressure, $\text{J.Kg}^{-1}.\text{K}^{-1}$
$d$	= nozzle inner diameter, 2.54 cm
$h$	= wall-nozzle distance, 10.5664 cm
$p$	= pressure, Pa
$r$	= cylindrical coordinate : distance from the axis, m
$R$	= ideal gas constant, $8.314 \text{ J.K}^{-1}.\text{mol}^{-1}$
$t$	= time, s
$T$	= temperature, K
$u, v, w$	= velocity components, m/s
$W$	= molar mass of air, $28.85 \text{ g.mol}^{-1}$
$\vec{x}, x, y, z$	= cartesian coordinates: vector and components, m
$\Delta x$	= typical cell size, m
$\Delta t$	= computational time step, s
$\Delta_{Cell}$	= size of compression-expansion zone, m
$\Phi$	= phase shift, rad
$\mu$	= dynamic viscosity, $\text{Kg.m}^{-1}.\text{s}^{-1}$
$\rho$	= density, $\text{Kg/m}^3$
$-$	= spatial filtering of large eddy simulation
$\sim$	= Favre averaging
$\hat{\phantom{x}}$	= Fourier transform
$\langle \rangle_{mean}$	= mean, in the process of time-averaging
$\langle \rangle_{rms}$	= root mean square or quadratic mean, in the process of time-averaging
$\langle \rangle_{az.}$	= azimuthal averaging

## I. Introduction

In recent years, Large Eddy Simulations (LES) have demonstrated their ability to predict the behavior of weakly compressible turbulent flows [1–5]. LES applications to supersonic flows however remain to be validated. Typical examples of non-reacting supersonic flows with strong industrial interests are impinging jets which appear in leaks of high pressure tanks and pipes or breaches in the outer structure of reentry vehicles [6]. Such jets can exhibit tone-producing modes that depend on the jet exit distance to the wall and nozzle pressure ratio [7–9]. They have been studied for astronautics (multi-stage rocket separation, attitude control thruster), aeronautics (jet-engine exhaust impingement, vertical take-off stability) or turbomachinery (gas turbine blade thermal failure). The work of Donaldson and Snedeker in 1971 [10, 11] describes the physics of circular jets impinging on spherical, normal and oblique surface. A later work from Lamont and Hunt [12] focuses on the near field jet impingement zone, showing the link between the overall load on the plate and the momentum flux at the nozzle lip. A fair amount of studies focused in the eighties and nineties on the unsteadiness of such flows, e.g. Ho and Nosseir [13], Kuo and Dowling [14] .

Computational approaches based on the method of characteristics or Euler solvers are common tools for supersonic jets. Reynolds Averaged Navier-Stokes (RANS) simulations address the viscous features and the effect of turbulence. These numerical models are widely developed and computationally efficient but they are limited to stationary flows. Unsteady phenomena linked to turbulence are best addressed with LES, which provides time-dependent filtered quantities of one flow realization. LES has been used with success to reproduce the intermittent and unsteady tone-producing modes of free jets [15] and cavities [16]. Combined with advanced diagnostics to take advantage of the spatial and temporal description of the problem, LES can provide specific informations about non-linear interactions or causality of phenomena.

From a chronological point of view, specific numerical techniques have been developed for supersonic jets since the fifties. The work of Ferri at NACA [17] illustrates the sophistication of the characteristics approach in the context of transonic aircrafts. The BAMIRAC report of Adamson [18] reviews several approximate methods to calculate the main inviscid features of highly underexpanded jets. Between 1990 and 2000, various Euler or RANS solvers took advantage of adaptive gridding

or multigrid [19–23]. Oscillating impinging jets were simulated by Kuo [14] and Sakakibara [24]. Improvements of turbulence models lead to accurate quantitative predictions [25], and unsteady RANS succeeded in reproducing screech phenomenon [26], in spite of a narrow bandwidth description around the discrete tone. In 2003, Arunajatesan [27] used LES for steady supersonic impingement of jets, to predict the major quantities of the flow such as the averaged pressure loads and centerline velocity. A few studies focused also on planar [28] and rectangular [29] supersonic screech jets. Note that direct computation of noise by LES has been also demonstrated in subsonic configurations [30–32]. More recently, hybrid methods mixing RANS and LES approaches were applied successfully to compressible mixing layers [33] and to supersonic jets [34], to address the problem of wall generated-turbulence, a recurrent LES bottleneck [35]. Note that wall-generated turbulence in the nozzle of a free jet does have a clear impact on the noise generated and special care must be taken (e.g. [15]) for stable regimes, but it is not first-order in the loud discrete tones of unstable impinging jets.

Table 1 presents a selection of numerical works in the last decade related to similar configurations. The degrees of freedom are the total number of unknowns. WENO approaches having a high CPU cost, the simulation domain and related grids are significantly smaller. Structured grids are mainly used since the academic impingement configuration is geometrically simple. On the other hand, unstructured grids are attractive for the short human time spent in the gridding phase, the freedom of refinement/coarsening in any direction, and its ability to deal with complex geometries.

The present work describes a LES of a supersonic jet impinging on a wall using a centered scheme with hyperviscosity at shocks and unstructured grids. The paper is organized as follows. The experimental target configuration is introduced in section II, with a brief flow description. Then, the numerical method is presented and commented with particular stress on shock-capturing technique and high performance computing (section III). The analysis is focused on the validation of time-averaged quantities with respect to experimental data, the observation of several flow features referenced in the literature, and the energy distribution through spectral analysis. A specific attention is devoted to the influence of the mesh on the LES solution. This study concludes with a discussion on the pros and cons of LES applied to such flows.

**Table 1 Selection of previous numerical works**

Case	Name	Year	Method	Mesh	Volume	Size ( d.o.f.)
Imp. jet	Arunajatesan [27]	2002	LES	Struct.	30° sector	$5 \times 2.78 \cdot 10^6$
Mix. lay.	Georgiadis [36]	2003	Hybrid	Struct.	3D slab.	$5 \times 0.87 \cdot 10^6$
Free jet	Cheng [37]	2005	ILES-WENO	Struct.	2D Axisym.	$4 \times 87.5 \cdot 10^3$
Screech. Jet	Berland [15]	2006	LES	Struct.	3D slab	$5 \times 16.3 \cdot 10^6$
Ctrld. Jet*	Chauvet [38, 39]	2007	RANS & Hybrid	Struct.	3D	$5 \times 10.5 \cdot 10^6$
Screech. Jet	Singh [40]	2007	LES-WENO	Struct.	2D Axisym.	$4 \times 210 \cdot 10^3$
Launcher Nozzle	Deck [41]	2009	Hybrid	Struct.	3D	$5 \times 11 \cdot 10^6$
Imp. Jet	Present work	-	LES	Unstruct.	3D	$5 \times 3.8 \cdot 10^6$

\* Complex geometry with mixing enhancement by control jets.

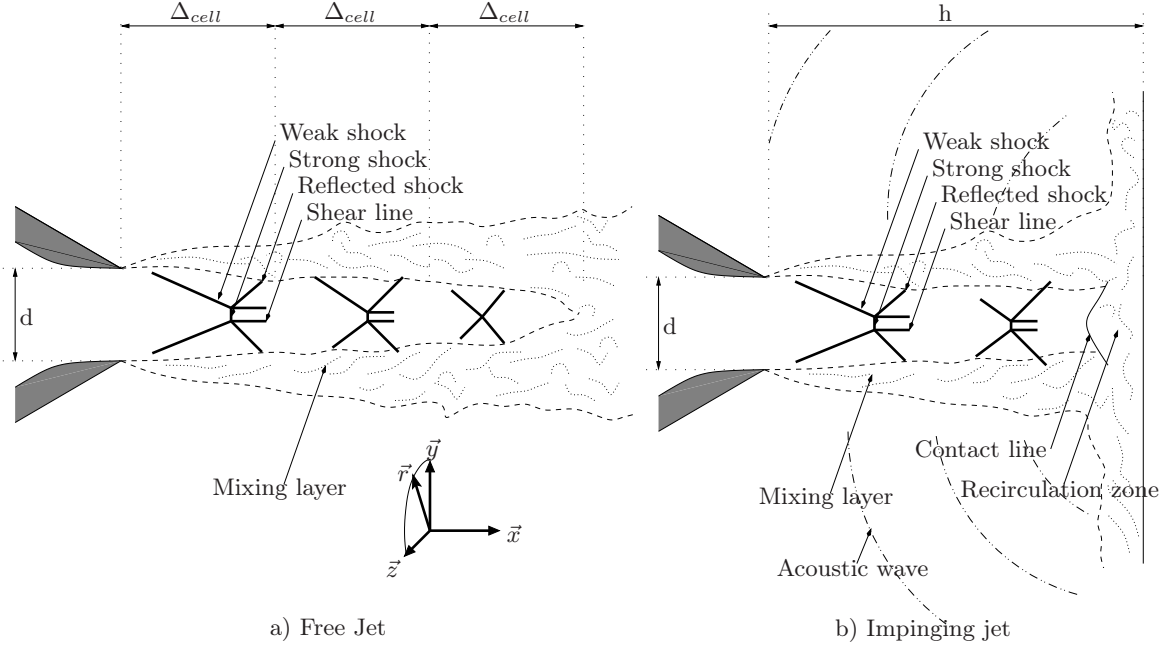
## II. Target Configuration

Standing at the intersection of screech investigation and impinging jets, the experimental work of Henderson [7] is of particular interest to LES. Indeed, the variation of a single geometric parameter, namely the nozzle to plate distance, leads either to silent or tone-producing flows. The current work aims at assessing the validity of LES on one of the stable cases.

### A. Flow Description and Parameterization

The flow parameters governing the response of free supersonic jets are: the convergent nozzle of diameter  $d$  and the Nozzle total Pressure to ambient static pressure Ratio (NPR). For a NPR approaching 4, shock structures take place at the rim of the nozzle. The fluid goes through a Prandtl-Meyer expansion to recover the ambient pressure value. Mach reflections converge to generate a cone-shaped weak shock ended by a disc-shaped strong shock, Fig. 1a. The weak and strong shocks merge into a triple point giving birth to a reflected oblique shock and a shear line. For free jets, this inviscid pattern composed of the weak, strong, reflected shocks and shear lines is repeated several times downstream. The size of this expansion-compression cell  $\Delta_{cell}$  was measured for a wide range of configuration [42]. The succession of cells is the primary mechanism by which the jet inner pressure adapts to reach the ambient state.

For impinging configurations, a large obstacle (usually a wall, Fig. 1b) is blocking the supersonic free jet plume. The most critical configuration appears if the distance  $h$  from the jet nozzle to the wall is shorter than the inviscid core. For that case, the strong pressure drop issued by the strong shock creates a near wall flow recirculation bubble with a contact line separating the main jet and the recirculated flow as shown on Fig. 1b. The main effect of this impact of the jet on the wall is the generation of strong acoustic waves and depending on the NPR, the wall distance  $h/d$  or the plate dimension, discrete acoustic tones may be produced.

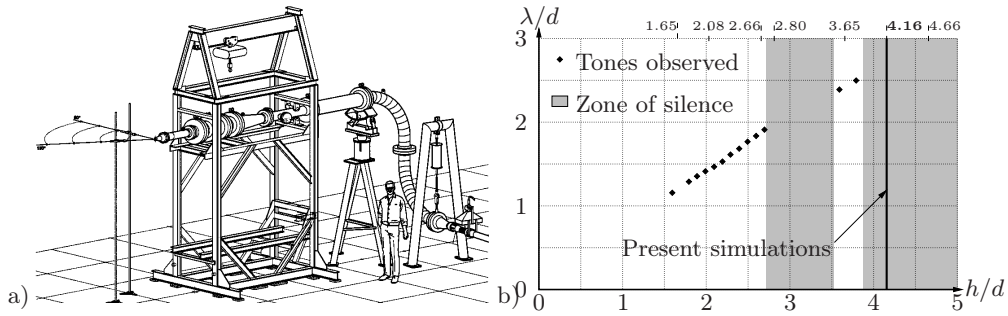


**Fig. 1 Impacting jet on a flat plate: flow description and main geometrical parameters.**

Although the extreme case of the tone dominated impacting jet is of clear interest, a simpler stable case is retained for this study for the LES validation. For this reason the present work focuses on the stable case studied experimentally by Henderson et al. [7] and defined by  $NPR = 4.03$ ,  $h/d = 4.16$ .

## B. Experimental Apparatus and Diagnostics

The experiment performed by Henderson et al. [7] uses the Small Hot Jet Aeroacoustic rig (SH-JAR), a facility of the AeroAcoustic Propulsion Laboratory (AAPL) at the NASA Glenn Research Center, Fig. 2a. In this specific experiment, the convergent nozzle has a one inch exit diameter  $d$ ,



**Fig. 2** Small hot Jet aeroacoustic rig, facility of the aeroacoustic propulsion laboratory, NASA Glenn research center: (a) view of the laboratory set up and (b) the different regimes observed experimentally.

with an external contour of a 30deg. cone, and discharges into ambient air. The NPR equals 4.03 for a jet temperature close to 300K. The outflowing gas is air seeded with olive oil droplets to allow measurements by Digital Particle Imaging Velocimetry (DPIV). Microphone recordings are also available away from the impact, in a region affected by acoustics only.

An aluminium plate orthogonal to the jet axis is mounted in front of the nozzle at a distance  $h$  that can be moved from  $h/d = 1.5$  to 5. By varying the axial distance  $h/d$ , the experimental layout allows different tone-producing regimes, with a fundamental frequency amplitude often 20 dB above the broadband noise. In particular, symmetrical jet oscillations occur for  $h/d < 2.7$ , a zone of silence appears for  $2.7 < h/d < 3.5$  and tone-producing regimes are observed again for  $3.5 < h/d < 3.8$ . There is no more tone-producing regimes in the interval  $3.8 < h/d < 5$  (Fig 2b). The condition studied here corresponds to  $h/d = 4.16$ .

### III. Numerical Approach

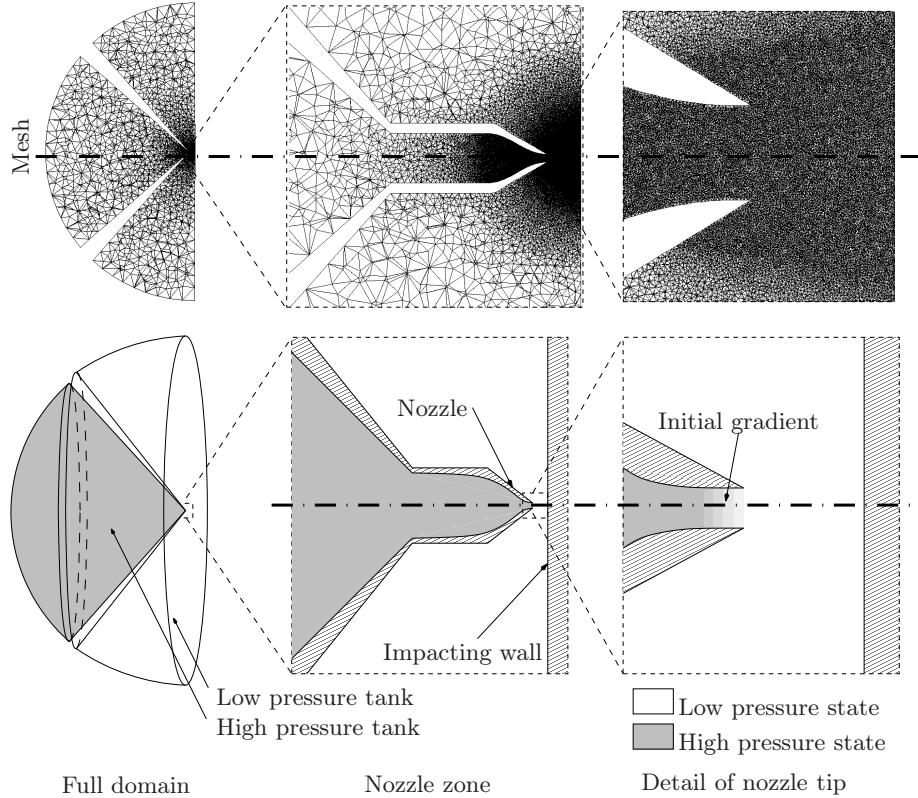
#### A. Computational Domain and Boundary Conditions

To avoid difficulties associated to boundary conditions, the simulation is set up with a numerical "blow-down" configuration in which a very large tank feeds another large tank through a supersonic impinging jet. This allows to use walls on all boundaries (Fig. 3). The sizes of the tanks are taken large enough to make the simulation time negligible with respect to the overall discharge time.

Quantitatively, the tank radii are of the order  $500 \times d$ , leading to approximately  $m_i = 1000$  kg of

mass enclosed initially in the high pressure tank. The mass flow rate at the operating condition being  $\dot{q} = 1.12 \text{ kg/s}$ , the characteristic discharge time approaches  $t_d = m_i/\dot{q} = 1000 \text{ s}$ . The characteristic convective time of the jet is  $t_c = h/u_{jet} \approx 2.5 \cdot 10^{-4} \text{ s}$ , taking  $u_{jet} = 300 \text{ m/s}$ . For proper exploitation of the LES, the simulation discussed below corresponds to the range  $100 \times t_c < t < 120 \times t_c$ , ensuring that no transient effects due to initial or discharge time scales are present: *i.e.*  $t_c \ll t \ll t_d$ .

The computation begins with still conditions, *i.e.* the flow is initially at rest in the low pressure reservoir at  $P_L = 101325 \text{ Pa}$ ,  $\rho_L = 1.15 \text{ kg/m}^3$ , and  $T_L = 300 \text{ K}$ . The high pressure tank is  $P_H = 4.03 \times P_L$ ,  $\rho_H = 4.03 \times \rho_L$ , and  $T_H = T_L$ , (Fig. 3). Unless specified otherwise, all tank boundaries are assumed to be adiabatic no-slip walls. The turbulent boundary layer inside the nozzle is neglected and adiabatic slip-walls are imposed. The impacted flat plate (vertical plane on Fig. 3) is expected to also have a very thin turbulent boundary layer. Since directly computing the eddies in this region would result in extremely fine grids, a crude approximation is adopted and a simple adiabatic slip-wall condition is used.



**Fig. 3** Computational domain and initial conditions



The grid convergence analysis uses three unstructured tetrahedra grids, generated for the same geometry (Table. 2). The leading parameter is the number of nodes in the diameter of the injector. As the solver is explicit, the time step  $\Delta t$  is limited by the smallest cell size  $\Delta x$  and the fastest acoustic propagation speed  $u + c$ , with  $u$  standing for velocity and  $c$  for sound speed (Courant-Friedrichs-Lewy criterion [43]):

$$\Delta t < CFL \frac{\min(\Delta x)}{\max(u + c)} \quad (1)$$

The explicit scheme is third-order in time and space and yields stable numerical solutions provided that  $CFL \leq 0.7$  [44].

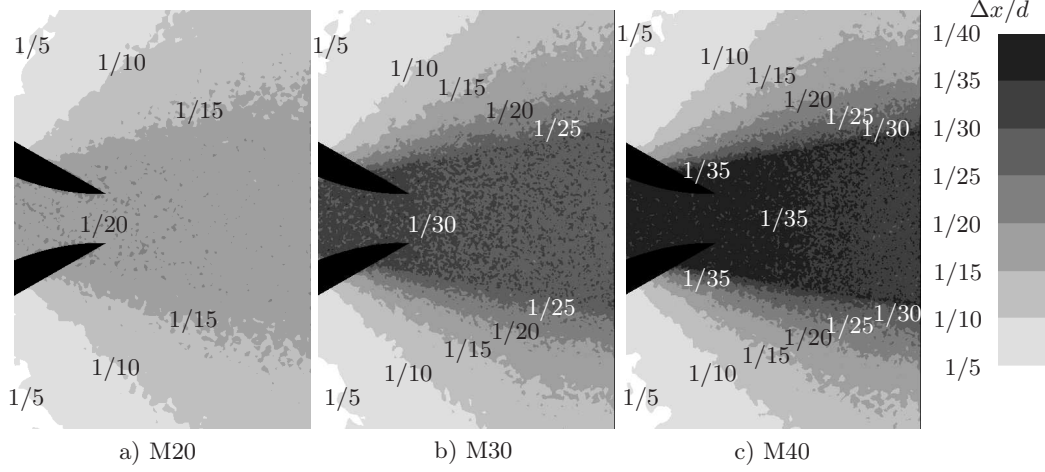
The grid generation package Centaur (<http://www.centaursoft.com>) is used to generate the three meshes. Throughout the following document,  $x, y$ , and  $z$  are respectively used for the axial, transverse, and spanwise components of the position vector  $\vec{x}$ , while  $r$  is used for the radial distance from the jet symmetry axis. Mesh refinement allows a maximum stretching ratio of 1.7 between tetrahedrons and 1.5 between edges. Cell size distribution is depicted in Fig. 4. A uniform grid size is enforced in the region of interest. The jet impact zone is also uniformly discretized for a region defined by  $r/d \leq 2$ . Finally, a buffer zone imposes linear coarsening for the three meshes away from the impact. All grid characteristics are detailed in Table 2 with their computational costs in Table 3. In this last table, Subscripts  $_{cpu}$  (resp.  $_{hum.}$ ) denotes time used by one processor (resp. human time).

**Table 2 Typical mesh characteristics used to assess LES.**

	$\Delta x/d$	Nodes	Tetrahedra	Memory size	Time step
M20	1/20	1 299 149	7 497 557	147MB	0,524 $\mu$ s
M30	1/30	2 763 143	16 026 453	312MB	0,27 $\mu$ s
M40	1/40	3 833 370	22 280 845	433MB	0,214 $\mu$ s

**Table 3 Cpu-time on 512 processors Intel/Nehalem**

	one convective time		Total time used	
	$t_{cpu}^{conv.}$	$t_{hum.}^{conv.}$	$t_{cpu}^{tot.}$	$t_{hum.}^{tot.}$
M20	82.9 hr.	9.7 min.	17 955 hr.	35 hr.
M30	353.8 hr.	41.5 min.	17 412 hr.	34 hr.
M40	662.0 hr.	72.9 min.	19 938 hr.	39 hr.



**Fig. 4 Local view of the typical nozzle to flat plate mesh resolution as obtained for the three grid: a) the coarse, b) the intermediate, and c) the fine meshes. Isocontours and labels corresponds to the field of  $\Delta x/d$ .**

## B. LES Model

LES involves a spatial filtering operation which reduces for locally, temporally invariant and localised filter functions, to:

$$\overline{f(\vec{x}, t)} = \int_{-\infty}^{+\infty} f(\vec{x}', t) G(\vec{x}' - \vec{x}) d\vec{x}' \quad (2)$$

or using favre filtered quantities :

$$f(\widetilde{\vec{x}}, t) = \frac{1}{\overline{\rho(\vec{x}, t)}} \int_{-\infty}^{+\infty} \rho(\vec{x}', t) f(\vec{x}', t) G(\vec{x}' - \vec{x}) d\vec{x}' \quad (3)$$

where  $G$  denotes the filter function and  $f(\widetilde{\vec{x}}, t)$  is the Favre filtered value of the variable  $f(\vec{x}, t)$  [45].

The quantities of interest have spatial  $\vec{x}$  and temporal  $t$  dependencies. The mathematical description of gaseous turbulent flows involves three primary variables : the density  $\rho(\vec{x}, t)$ , the velocity  $\vec{v}(\vec{x}, t)$ ,

and the total energy  $E(\vec{x}, t) \equiv e_s + 1/2\vec{v} \cdot \vec{v}$  where  $e_s$  is the sensible energy.

The fluid follows the ideal gas law :  $p = \rho TR/W_{air}$  and  $e_s = \int_0^T C_p dT - p/\rho$ , where  $T$  is the temperature,  $C_p$  the fluid heat capacity at constant pressure. The viscous stress tensor and the heat diffusion use classical gradient approaches. The fluid viscosity follows Sutherland's law and the heat diffusion follows Fourier's law.

Filtering the instantaneous set of transport equations yields the LES equations, which need closure models [46, 47]. The Sub-Grid-Scale (SGS) turbulent velocity tensor is modelled using the turbulent viscosity model of Smagorinsky and the Boussinesq assumption [48–50]. The eddy diffusivity is also used along with a turbulent Prandtl number. Note that the performance of the models might be improved through the use of a dynamic formulation [50–54] or more advanced type of closures [55–57] which were not tested here.

### C. Shock-capturing Scheme and Artificial Viscosity

The shock pattern present in supersonic jets is one of the main difficulties in LES of high speed flows [58]. LES solvers able to handle shocks rely on either localized artificial viscosity [59–62], or on spatial schemes adapted to handle discontinuities while preserving the positiveness of the solution, giving birth to a wide family of techniques ranging from the initial approximated Riemann solvers (FCT TVD) to the WENO (Weighted Essentially No Oscillatory Schemes [63]). WENO schemes are attractive for LES because of their very high precision [40] but imply longer computational time. In the present work, centered schemes were chosen because of their superior performance (low dissipation) in shock free regions. To preserve the positivity of the solution in regions where strong gradients exist, a hyperviscosity  $\beta$  is introduced in the viscous stress tensor  $\overline{\tau_{ij}}$ , using the approach of Cook and Cabot [62]. This hyperviscosity may be viewed as an additional pressure term that thickens the shock front:

$$\overline{\tau_{ij}}^{modified} = (\beta - \frac{2}{3}\mu) \frac{\partial \tilde{u}_k}{\partial x_k} + 2 \mu \tilde{S}_{ij}, \quad (4)$$

where  $\mu$  is the dynamic viscosity and  $\tilde{S}_{ij}$  is the Favre filtered symmetric strain rate tensor. The bulk viscosity,  $\beta$ , is modelled as,

$$\beta = C(\Delta x)^4 \nabla^2 \|\tilde{S}\| \text{ and } \|\tilde{S}\| = \left( \tilde{S}_{ij} \tilde{S}_{ji} \right)^{1/2}, \quad (5)$$

where  $C$  is fixed to 5 [64]. This viscosity acts on the very sharp velocity gradients characterizing shocks but goes back to zero in zones where the velocity evolves smoothly [65].

#### IV. Results and Discussion

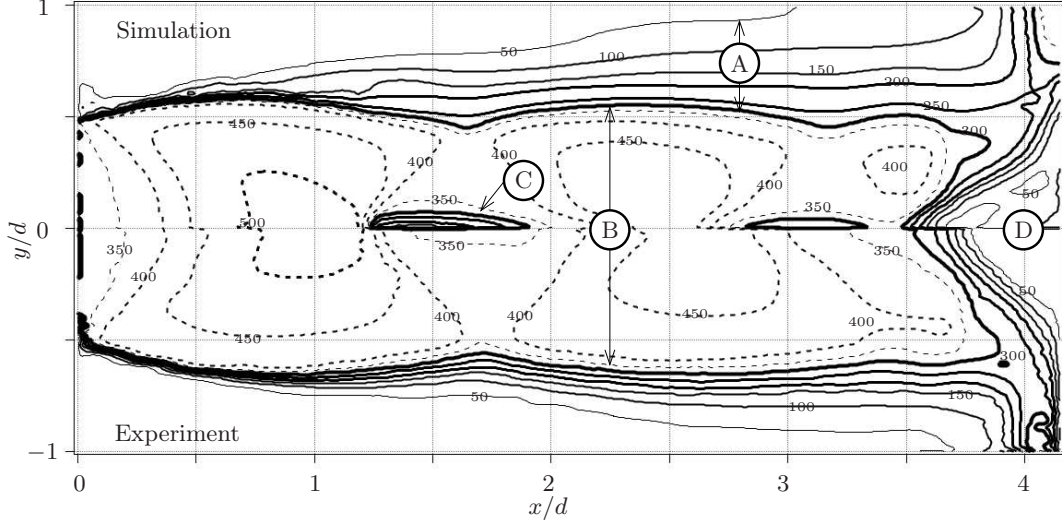
The simulation of this stable case is validated by comparing time-averaged and instantaneous LES fields with experimental results. Impact of the mesh resolution impact on the LES results is studied afterwards. This last part includes the observation of instantaneous flow quantities, time-averaged values, and energy distribution through spectral analysis.

##### A. Mean Flow Topology and Features

Figure 5 presents mean velocity isolines in the median transverse plane ( $\vec{x}\vec{y}$  plane): bottom half is a digital particle imaging velocimetry result by averaging on 400 instantaneous vector fields separated by  $1.2 \mu\text{s}$ , *i.e.* over 0.48 ms. The DPIV field resolution is around 1 mm or  $d/20$ . Top half is a LES result on the finest grid M40 obtained by averaging 160 vector fields separated by  $57 \mu\text{s}$ , *i.e.* over 9.12 ms. Contour lines are drawn using the same procedure, allowing direct quantitative comparisons. Three regions are clearly of interest in this jet configuration. The first region (A) corresponds to the outer part of the jet and is characterized by a low Mach turbulent flow dominated by shear. This subsonic zone, with velocities ranging from 50 to 300 m/s, matches the experiment very well. The LES mixing layer thickness is however slightly larger than the experimental one.

The second jet region (B) is dominated by supersonic flow structures with shocks and locates in the inner part of the jet from  $x/d = 0$  to  $x/d \approx 3.5$ . Here most of the velocity patterns agree, confirming the spatial location of the different weak, strong, and reflected shocks (Fig. 1). The two expansion/compression barrels for  $x/d < 3$  and a small supersonic core near  $x/d = 3.5$  are reproduced by the numerical approach. The size of the expansion-compression cell  $\Delta_{cell}$  is  $1.5d$  in the simulation and  $1.6d$  in the experiment, a value consistent with Powell [42]. A third narrow subsonic region (C) is clearly visible after the Mach disc near the axis in the simulation. This region is mentioned in the experiment: however the DPIV resolution makes its quantification difficult. Note that contours are more diffused in the simulation : a small difference or uncertainty in the NPR between experiment and simulation could cause such discrepancies. The major effect of these local

differences in the inner supersonic and outer subsonic regions is that the overall jet opening angle differs slightly in LES and in the experiment. The fourth zone of interest (D) is the recirculation bubble which forms along the central axis of the jet in the vicinity of the plate. This subsonic region is characterized by an abrupt deceleration and takes on the shape of a cone whose base is located on the vertical wall: it is very well predicted by LES.



**Fig. 5 Comparison of time averaged velocity fields as obtained by the fine mesh simulation (top half) and experiment (bottom half). Contour values in m/s.**

Figure 6 presents an instantaneous iso-surface of unit Mach number obtained by LES: longitudinal streaks are clearly visible on the outer rim of the first supersonic cell. This phenomenon has been studied experimentally by Zapryagaev [66, 67] and Krothapalli [68]. Geometric singularities on the axisymmetric nozzle give birth to small flow perturbations. These perturbations are amplified downstream with a Goertler instability triggered by the curved boundary layer. These streaks are stable in space and time in the range  $0 < x/d < 1.0$ . In the simulation, they stay artificially locked on the perturbations induced spatially by the computational grid. The second observation is the formation of a supersonic hollow disc on the flat wall in the range  $1 < r/d < 3$ , with supersonic flow islands detaching in the circumferential direction  $3 < r/d < 4$ .

Figure 7 shows the instantaneous vortical structures present in LES. The isosurface of  $Q$ -criterion  $Q = 10^{10} \text{ s}^{-2}$  [69] allows to identify typical jet flow vortices : in the jet plume, the size of such

vortices is approximately  $d/5$  (about four computational cells) while on the plate, they reach  $d/10$  (about sixteen computational cells). In the mixing layer, the structures are oblique between the longitudinal axis and the azimuthal direction. Vortices on the wall are aligned azimuthally because of the radial stretching generated by the jet impact on the plate and the flow orientation in this region.

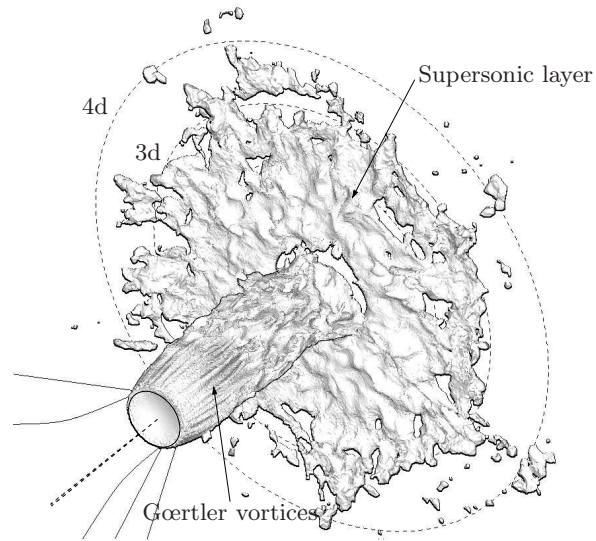
The final zone of interest corresponds to the recirculation bubble in the near wall region and on the jet axis. Donaldson et al. [10] provided a precise visualisation of the recirculation bubble shape on the wall for a similar configuration ( $NPR = 6.76$ ,  $h/d = 5.32$ , diverging nozzle). The typical streamline pattern of the recirculation is reproduced on Fig. 8c, based upon the grease streak picture of Fig. 8d. The limit of the recirculation appears as a dividing ring. This ring is divided into segments by several nodes often aligned with nozzle imperfections in Donaldson's experiments. The division can reduce down to three equal segments for a very smooth nozzle (Fig. 8c).

Time-averaged velocity vectors of the simulation near the wall (Fig. 8a) allow a direct comparison with the experiment. The pattern is nearly symmetric, showing two similar segments. The center of the bubble is on the diameter linking the two nodes. A flow separation line lies on the diameter. The pattern is not perfectly axisymmetric *i.e.* the saddle points are not located in the middle of the segments, and the separation point is not centered. This can be explained either by an averaging time too short compared to the evolution of the bubble, or by the non-perfectly axisymmetric flow upstream, as seen previously on the longitudinal streaks locked on the unstructured mesh.

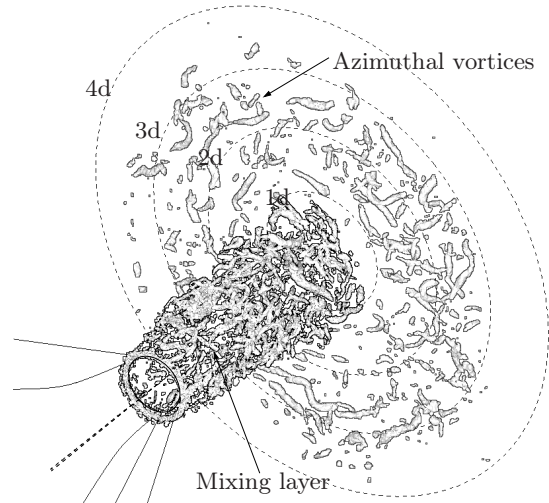
The pattern is very different on the instantaneous picture (Fig. 8b) : the dividing ring moves away from its equilibrium position, in the zone  $1 < r/d < 1.5$ . There are four nodal points and the segments are variable in shape and size : the recirculation bubble oscillates around a regular shape.

## B. Mesh Resolution Effects

Controlling the effects of mesh resolution on LES is a major issue in CFD. As indicated by Pope [70], only statistical results (time-averaged moments for example) are expected to converge when the grid size decreases. Aside from grid resolution, individual realizations in LES depend on rounding errors, compiling options, numbers of processors used on a parallel machine as shown by

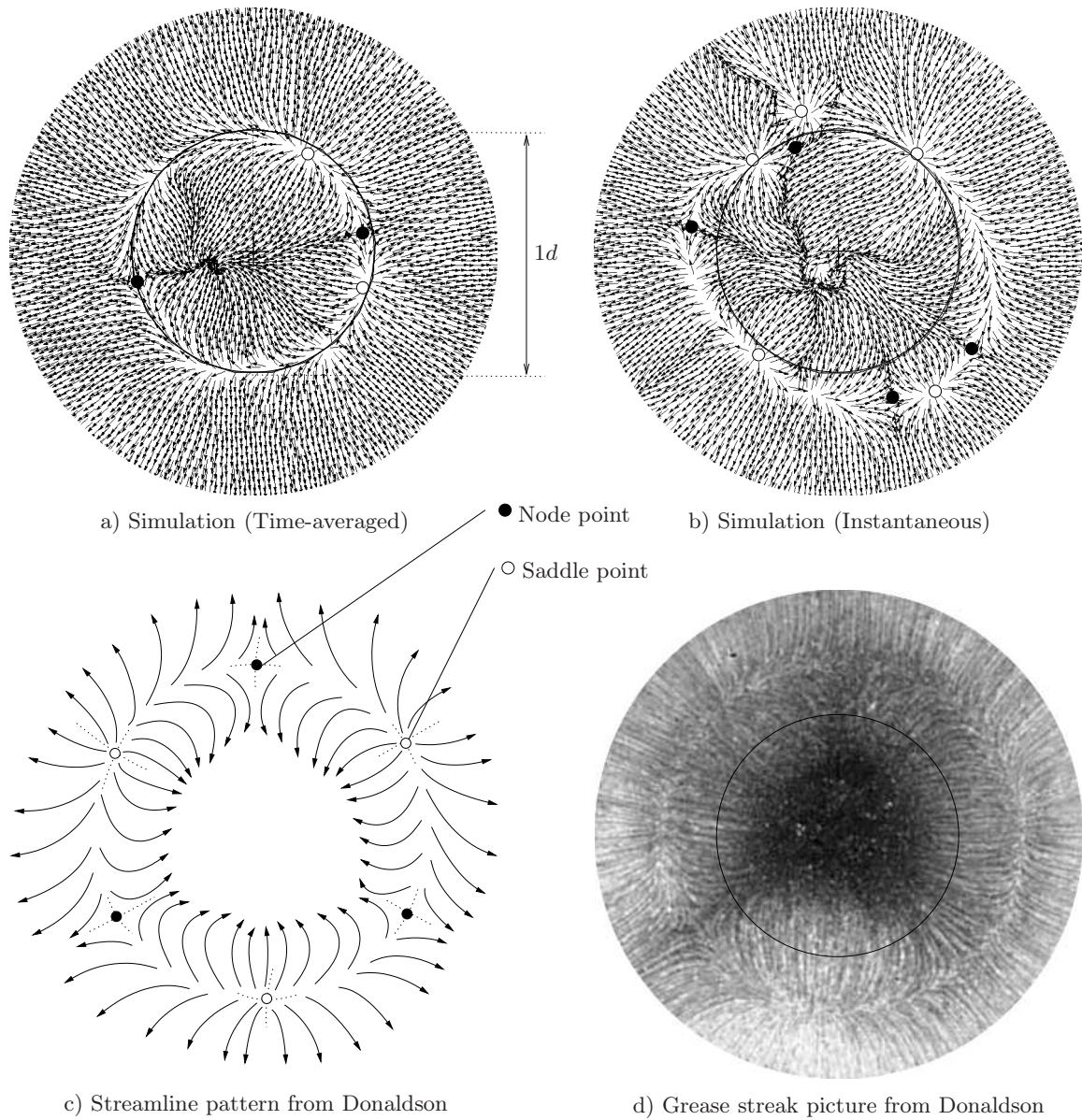


**Fig. 6** Instantaneous isosurface of unit Mach number as obtained by LES.



**Fig. 7** Instantaneous view of the  $Q$  criterium isosurface  $Q = 1 \cdot 10^{10} \text{ s}^{-2}$  obtained by LES.

Senoner et al. [71, 72]. Therefore verifying grid effects in LES is a difficult exercise. In the present case, the influence of mesh is tested by repeating the same simulations on three grids containing 7.5, 16, and 22 M cells. For all grids, time-averaged velocity and pressure fields are compared (Section IV B 2). Power spectral densities are also studied in Section IV B 3.



**Fig. 8** Instantaneous (a) and Time averaged (b) simulated vector fields on the plate, compared to the sketch (c) and grease-streak photograph (d) of Donaldson [10].

### 1. *Instantaneous Fields*

In supersonic jets, different classes of compressible effects are of interest and they all need to be properly evaluated numerically if tone dominated conditions are to be predicted. The first level of compressibility is the generation and location of strong, weak, and reflected shocks. The second level of compressibility can be associated to flow instabilities with the generation of vortical structures and strain. Finally, the third level results from the emission by the flow of acoustic waves propagating



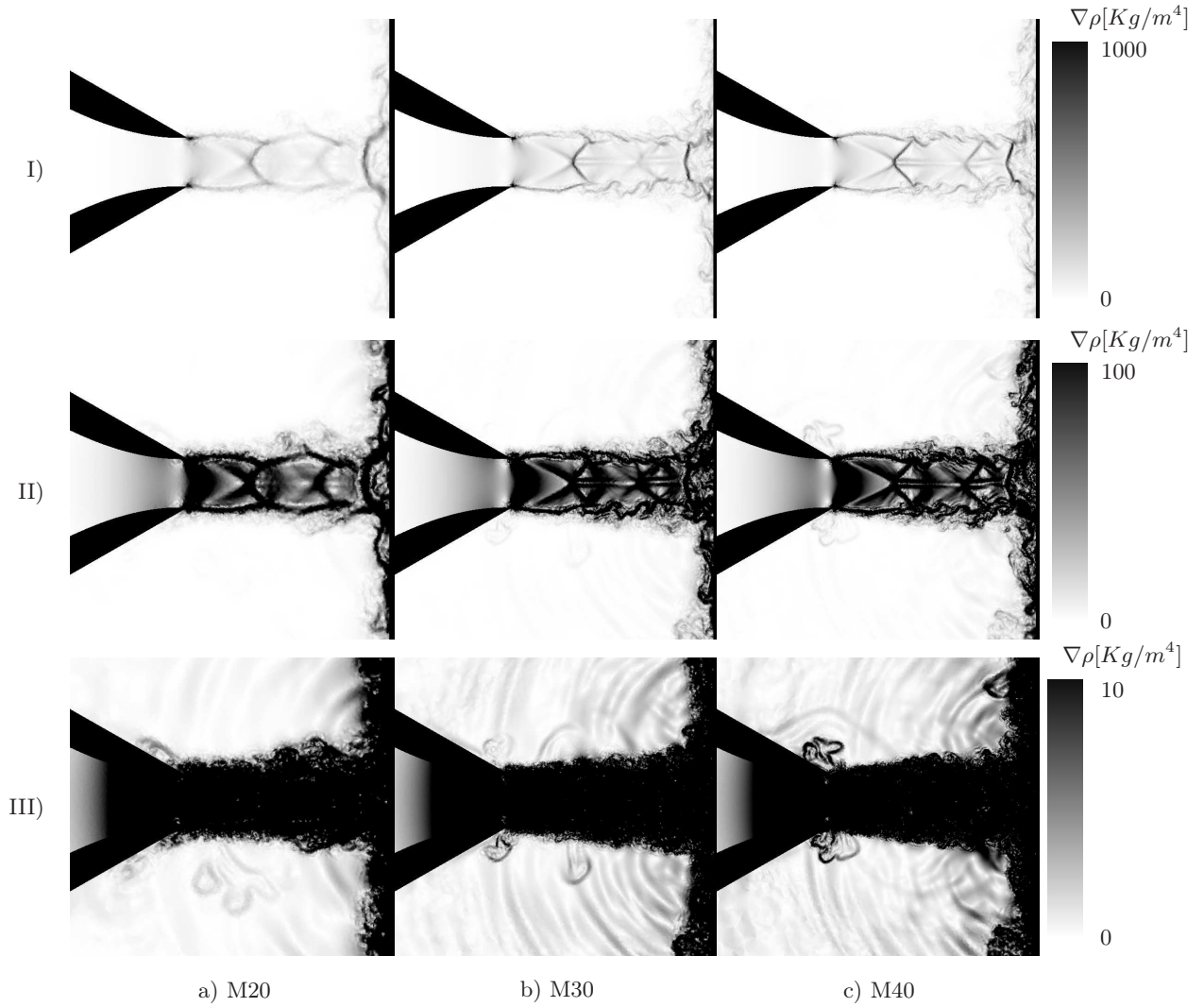
in the medium. These effects can be distinguished by the density gradient  $\|\nabla\rho\|$  which they induce: strong for shocks, medium for vortices and small for acoustics. Three different black and white scales with different orders of magnitude are shown in Fig. 9 for instantaneous LES fields obtained on the three different meshes.

Based on this diagnostic, simulations M30 and M40 (*i.e.* intermediate and fine grid resolutions respectively) appear very similar while simulation M20 (*i.e.* coarse grid) deviates significantly from the two others due to excessive dissipation; in particular the corrugated shear layer (Fig. 12I) and the acoustic waves (Fig. 12III) are excessively smeared on M20 (Fig. 12a).

An illustration of the instantaneous Mach number distribution for the three grid resolutions at the same instant is given in Fig. 10. The coarse grid LES M20 clearly attenuates most of the dynamics. It is able to place properly the different inner jet supersonic structures but produces highly damped unsteady motions in the outer jet mixing layers and along the plate. An increase of resolution improves the local sharpness of the different structures as observed on M30. The fine grid resolution M40 reveals more intermittency in the Mach distribution especially along the plate as well as sharper supersonic patterns in the inner jet region.

As expected with LES, as the local grid resolution increases, more flow structures are explicitly resolved and the resolved flow energy content is increased. It must be compared to dissipation effects due to artificial viscosity, shock-capturing, and flow laminar or turbulent viscosities. These questions are investigated below using instantaneous views of various ratios of viscosities for three grid resolutions.

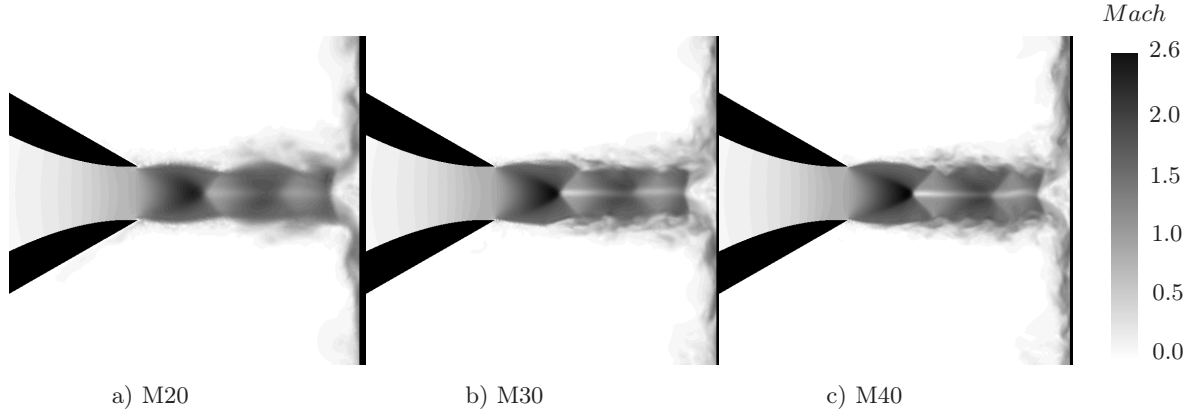
The artificial viscosity ( $\mu_{A.V.}$ ) resulting from the shock-capturing scheme and the necessary numerical stabilization is given in Fig. 11. For all meshes, artificial damping is low and localized along shear layers and shock lines. Compared to the laminar contribution the ratio remains low: it exceeds unity on M40 only in zones of shocks, something which can not be avoided. Simulations M30 and M40 exhibit similar results, with artificial viscosity lower than the laminar one in some parts of the jet (Fig 11b and 11c): in the range 1 – 10 on the shear layer and around 10 on the shock lines. Note nonetheless that LES with M20 yields significantly more damped supersonic structures, in agreement with an artificial to laminar viscosity ratio which saturates around 10 in the core of the



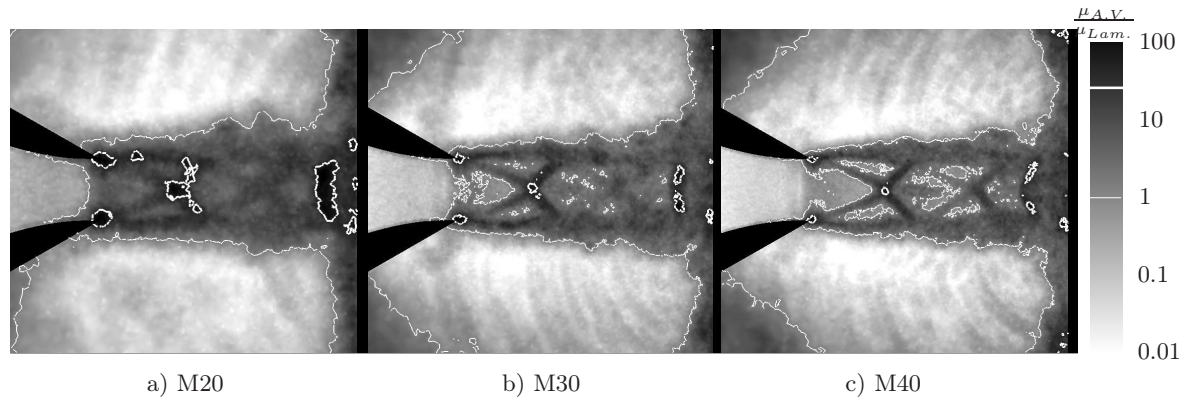
**Fig. 9** Magnitude of the density gradient for three ranges of scale and for the same instantaneous field obtained on the three meshes at the same instant: **Top I**: large values, supersonic patterns; **Middle II**: intermediate values, shear layers; **Bottom III**: small values, acoustic waves. The diagnostic is provided for the  $\bar{x}\bar{y}$  plane going through the central axis of the jet.

jet (Fig. 11a). Nozzle lip, mach disc and top of recirculation bubble are the most damped regions with a ratio close to 30 for all meshes. From a numerical point of view, Fig. 11 confirms the correct behaviour of the strategy devised to handle such flows with LES. As the mesh resolution increases, the zones where artificial viscosity is added reduce in intensity and become highly localized.

The turbulent viscosity ( $\mu_{Turb.}$ ) to laminar viscosity ratio is provided on Fig. 12 and is to be compared to Fig. 11 for the same grids:  $\mu_{Turb.}$  is created by the subgrid scale turbulent model in



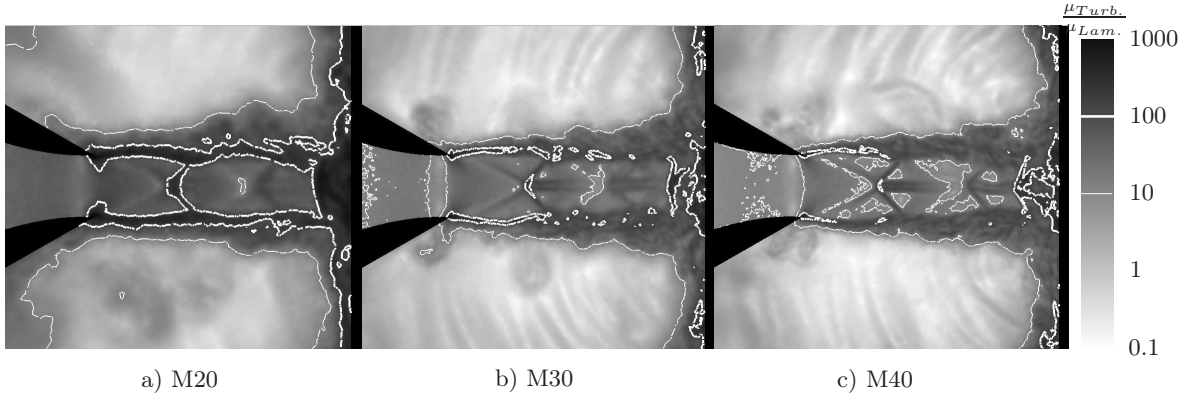
**Fig. 10** Instantaneous Mach number field as obtained on the three meshes. The diagnostic is provided for the  $xy$  plane going through the central axis of the jet.



**Fig. 11** Field of ratio of artificial Viscosity to Laminar Viscosity. White contours are drawn for ratios equal to unity (thin line) and 30 (bold line).

regions of high shear and strain, meeting the requirement of LES. Its intensity reaches values ranging from 10 to 40 times the laminar viscosity in the turbulent zones and is roughly 10 times superior to the artificial viscosity levels of Fig 11. Simulations M30 and M40 yield a ratio  $\mu_{Turb.}/\mu_{Lam.}$  close to 10 in the shear layer of the jet (Fig. 12b and 12c) while simulation M20 shows higher levels of turbulent viscosity (closer to 100) over the entire shear layer (Fig 12a).

To summarize, Mach number and density gradient fields are similar for meshes M30 and M40. Artificial and turbulent viscosity levels are reasonable and localized in shock and shear regions.



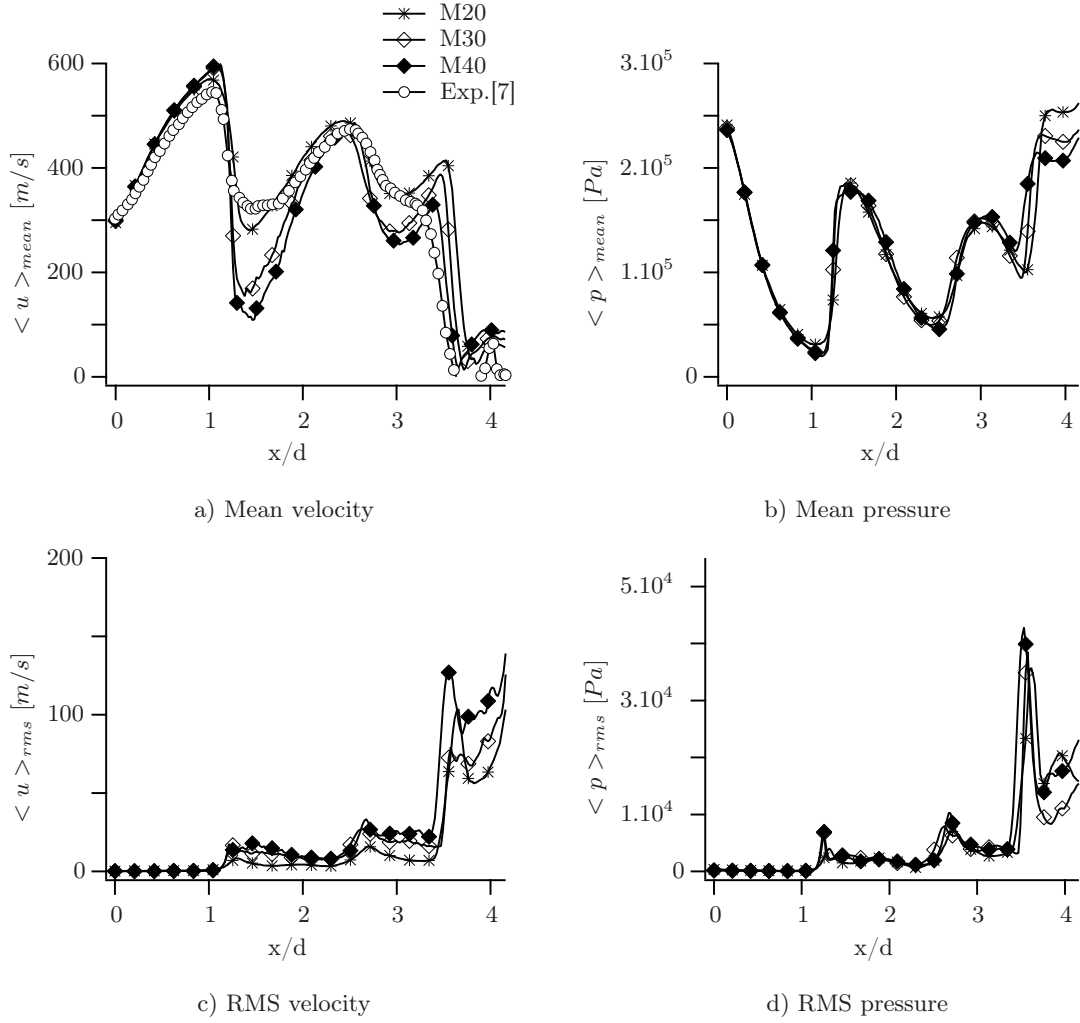
**Fig. 12** Field of ratio of Turbulent Viscosity to Laminar Viscosity. White contour are drawn for ratios equal to 10 (thin line) and 100 (bold line).

## 2. Mean Quantities

Time-averaged values are obtained in the range  $80 < t/t_c < 100$  with a sampling close to  $0.1 \times t_c$  and compared for the three meshes M20, M30, and M40. For notation,  $\langle u \rangle_{mean}$  (resp.  $\langle p \rangle_{mean}$ ) is the time-averaged value of the velocity  $u$  (resp. pressure  $p$ ).  $\langle u \rangle_{rms}$  denotes root mean square velocity of the resolved field, without de-filtering technique or subgrid model inclusion [46].

The longitudinal profiles of pressure and velocity are displayed on Fig. 13. The velocity fields estimated from Digital Particles Imaging Velocimetry (DPIV) in Henderson's experiment [7] are compared to the three simulations in Fig. 13a. The two high velocity zones for  $x/d = 1$  and  $x/d = 2.5$  are well reproduced. Simulations M30 and M40 both show a strong deceleration after these high speed zones, while simulation M20 is apparently closer to experimental observations. This gap can be explained both by the underestimation of the turbulence behind the Mach disc in the simulation and bias of DPIV in strong deceleration and sharp gradients. For example, the subsonic bubble visible in Fig. 5 has a diameter of  $0.1d = 2.54 \text{ mm}$  while the spatial resolution of the DPIV is  $1 \text{ mm}$ : the same profile taken  $1 \text{ mm}$  away from the axis would not show the axial subsonic zone. Time-averaged pressure is decreasing in the recirculation zone : a better resolution of strong shocks yields stronger pressure loss, and a lower static pressure on the wall.

The time-averaged pressure, RMS velocity and RMS pressure (resp. Fig. 13b,13c, 13d) all show a good agreement. In the recirculation zone, for  $x/d > 3.5$ , velocity and pressure fluctuations vary

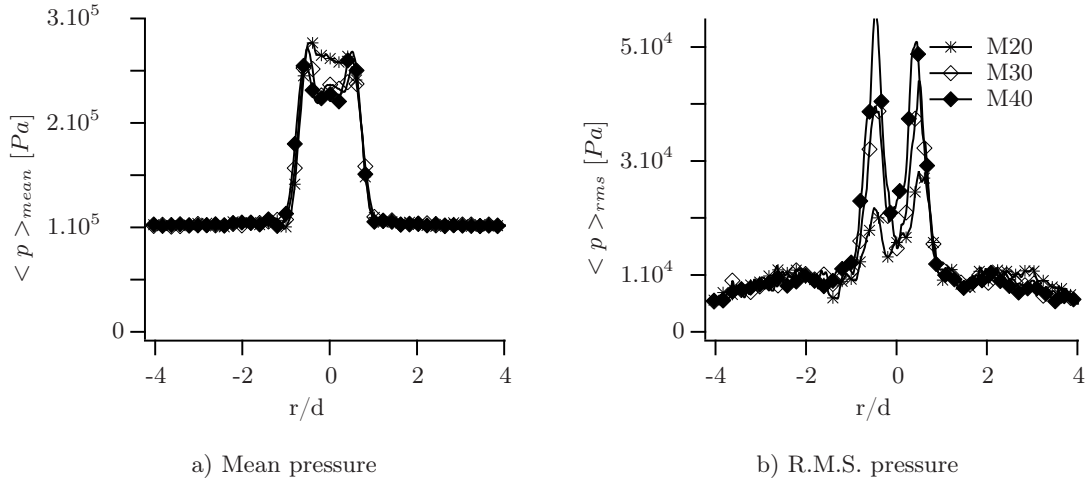


**Fig. 13** Time averaged Velocity (a.), time averaged static pressure (b.), RMS fluctuation of velocity (c.), RMS fluctuation of pressure (d.) along the axis of the jet.

by approximately 20% when the resolution increases.

Time-averaged static pressure distributions on the impinging wall are given on Fig. 14a. It shows a very good grid convergence between simulations M20, M30, and M40. The pressure fluctuation, on Fig. 14b, shows the same convergence, except for the peaks at  $r = \pm 0.5d$  where simulation M40 exhibits fluctuations which are 20% higher than simulation M30.

First-order moments for velocity and pressure show satisfactory agreement with respect to grid resolution (Fig. 13a, 13b, and 14a): a first-order convergence is achieved. Second-order moments converge too but more slowly in the recirculation bubble.



**Fig. 14 Pressure profile at the wall**

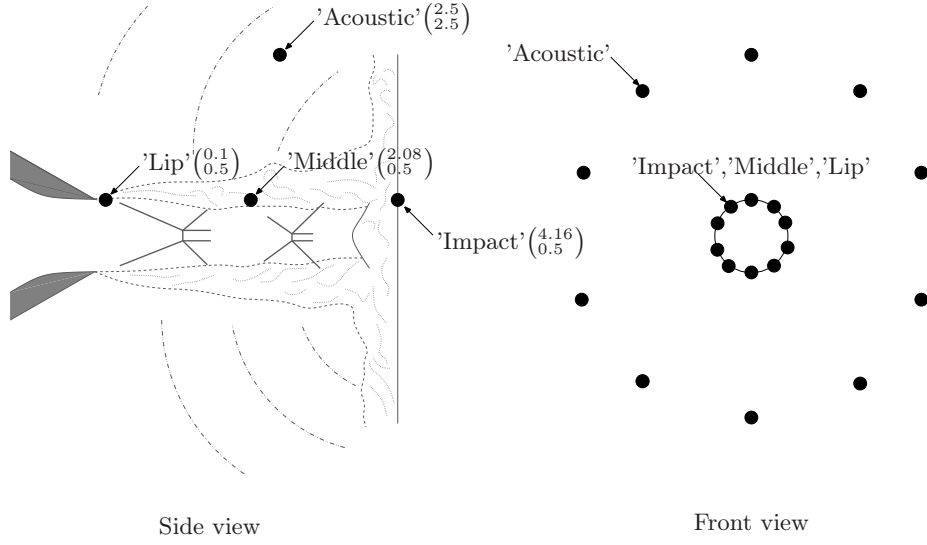
### 3. Spectral Analysis

The energy content of the resolved scales can be quantified by means of spectral estimators to investigate the evolution of the energy distribution with respect to the grid resolution.

A common first-order spectral estimator is the Sound Pressure Level (SPL) distribution of a pressure signal versus the frequency  $f$ . For a location  $\vec{x}$ , the pressure signal is sampled each  $10\mu s$  for ten different radial directions in  $\vec{y}\vec{z}$  planes. The following results are averaged azimuthally over these 10 locations. This averaging process is denoted  $\langle \rangle_{az.}$ . The Fourier transform of pressure is denoted  $\hat{p}_x$ . Note that this transformation involves chronologically i) a resampling of the datasets to ensure a fixed time step, ii) a removal of the linear trends, iii) a low-pass filtering removing frequencies above 40% times the Nyquist frequency, iv) Hanning windowing. The windowing is done for the ten locations before azimuthal averaging  $\langle \rangle_{az.}$ . The Nyquist frequency approaches 100 kHz for all signals. Care has been taken that all segments present the same first and second statistical moments within a 10% tolerance. With the adequate normalization, the norm of the Fourier transform gives the amplitude of each harmonic depending on the frequency  $A_x(f)$ , Eq. (6). Sound pressure level is defined as the root mean square of each harmonic normalized by the reference  $20 \mu Pa$ , and often expressed in decibels, as in Eq. (7).

$$A_x(f) \equiv \left\langle \sqrt{\hat{p}_x \hat{p}_x^*} \right\rangle_{az.} \quad (6)$$

$$SPL_x(f) \equiv 20 \log \left( \frac{A_x \times \sqrt{2}/2}{20 \mu Pa} \right) \quad (7)$$

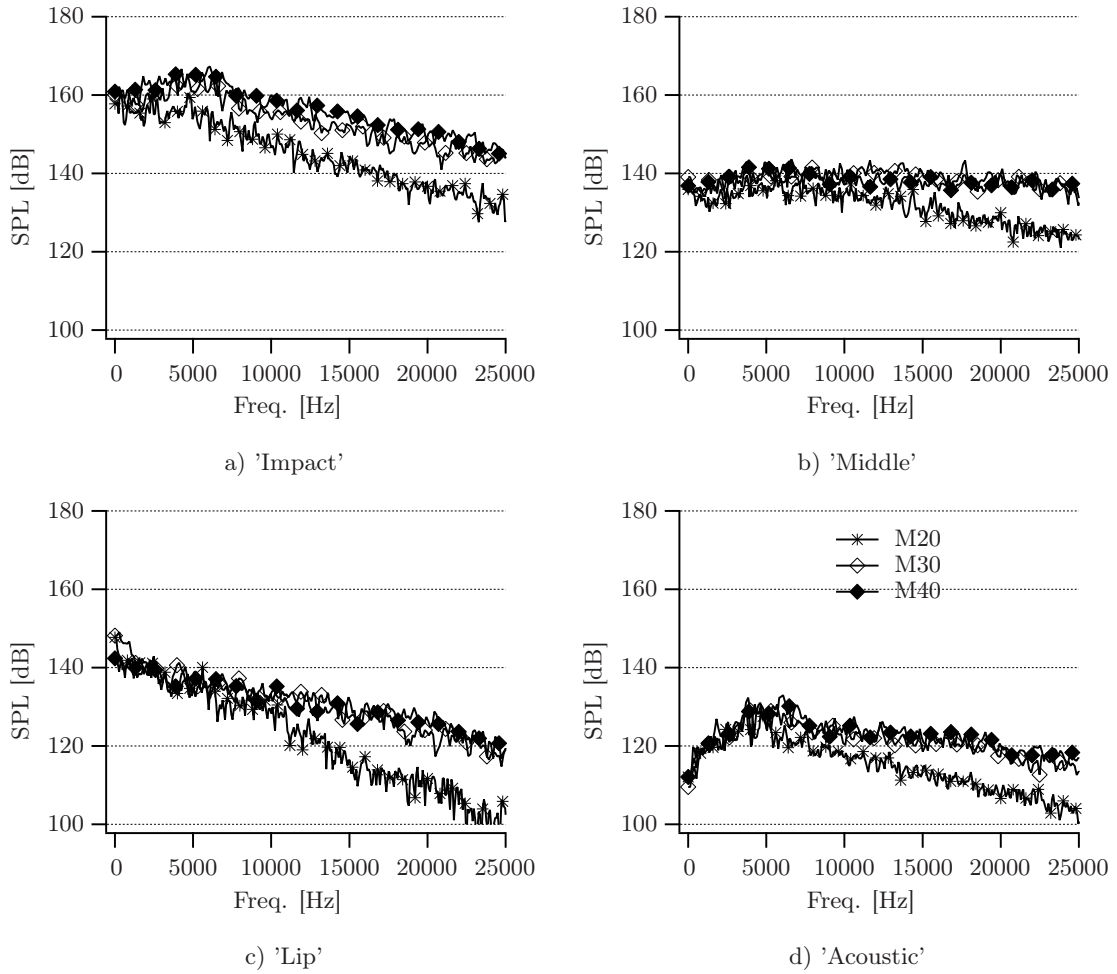


**Fig. 15 Locations used for spectral analysis. Coordinates are specified as  $\begin{pmatrix} x/d \\ r/d \end{pmatrix}$ .**

The spectrum is built for four different locations, 'Impact', 'Middle', 'Lip', and 'Acoustic', detailed in Fig. 15. Going from the highest to the lowest energy content, the spectral analysis yields :

1. 'Impact' : on the impact plate (Fig. 16a) where the strongest pressure fluctuations occur. SPL is above 160 dB in the range 0 – 10 kHz. A weak maximum is found for both mesh M30 and M40 at the frequency 6 kHz, corresponding to a RMS fluctuation near 4000 Pa. The energy content decreases slowly for the high frequencies.
2. 'Middle' : in the middle of the jet, near the Mach disc (Fig. 16b), the energy content is almost constant near 140 dB, i.e. a RMS fluctuation around 200 Pa.
3. 'Lip' : at the nozzle lip (Fig. 16c), the maximum energy starts at 140 dB in the lowest frequency range and decreases smoothly towards 120 dB for 25 kHz (100 dB for mesh M20).
4. 'Acoustic' : In the zone at rest, to isolate propagating acoustic waves (Fig. 16d), the energy distribution is comparable to the impact zone. The SPL rises in the range 0 – 6 kHz from 100 dB to 140 dB, then decreases on the range 6 – 20 kHz down to 120 dB (110 dB for mesh M20).

The spectra are very different in terms of energy distribution and amplitude (Fig. 16). However a clear convergence is established between simulations M30 and M40. The simulation M20 shows in all cases an energetic content 10 dB lower than the two finer grids. The most important discrepancy between simulation M30 and M40 is observed in the impact region, Fig. 16a with a shift of 3 dB/Hz. This shift can be linked to the pressure fluctuations, stronger with the simulation M40. This discrepancy has no impact on the energy spectra at the three other locations. In spite of a maximum amplitude reached for 6 kHz, there is no discrete tone production in this case, as expected from experiments.



**Fig. 16** Averaged Energy spectra

Involving non-linear products, the two-points estimators are part of second-order spectral analyses. The interspectrum  $S_{xy}$  between the pressure signals of locations  $\vec{x}$  and  $\vec{y}$  is normalized by the



source energy  $S_{xx}$ , yielding the coherence estimator:

$$Coh_{xy} \equiv \left\langle \frac{|S_{xy}|^2}{|S_{xx}|^2} \right\rangle_{az.} \equiv \left\langle \frac{|\hat{p}_x \hat{p}_y^*|}{\hat{p}_x \hat{p}_x^*} \right\rangle_{az.} \quad (8)$$

This quantity equals one when the target signal  $p_y$  shows the same amplitude and frequency as the source signal  $p_x$ . Note that such tools are subject to signal to noise sensitivity: coherence between a signal and a white noise showing the same fluctuations amplitude leads to a coherence near unity. As we are dealing with signals of very different amplitudes (cf. Fig 16), the coherence average level is expected to be low. To compare with the SPL levels, the coherence will be expressed in dB. The phase of the interspectrum  $\Phi_{xy}$  defined in Eq. (9) gives the time shift between the signals:

$$\Phi_{xy} \equiv \Phi \langle S_{xy} \rangle_{az.} \equiv \langle \hat{p}_x \hat{p}_y^* \rangle_{az.} \quad (9)$$

Note that when two signals are strongly correlated at all frequencies, the time shift is a linear function of the frequency. The slope of this linear function depends on the propagation speed of the signal and the distance between the two locations.

These tools are first applied to a couple of sensors showing almost no correlation. The fluctuations on the wall at the edge of the recirculation zone are probably one of the strong acoustic sources in this flow. As there is no discrete tone production, the response of the jet to acoustics is expected to be low. Therefore, the locations 'Impact' and 'Lip' are expected to be uncorrelated. The two-point phase estimator shown on Fig. 17a shows strictly no linear trend between the two locations. The coherence estimator illustrated on Fig. 17b starts near unity ( $-0$  dB) in the very low frequency range and quickly reaches an envelope between  $-20$  dB and  $-30$  dB for all three simulations. This quantity shows the difference of SPL between 'Lip' and 'Impact' region (difference between Fig. 16a and 16c). Note that coherence seems to get lower with mesh refinement. However, the results are globally independent of the grid.

The two-point estimators are then applied to a couple of locations expected to be strongly correlated. Indeed, the 'Acoustic' location must be directly perturbed by the 'Impact' zone. A clear linear trend is visible on Fig. 18a. The linear slope is roughly estimated by a phase shift of  $2\pi$  radians each 5000 Hz. This can be compared to an ideal acoustic propagation. The distance between the two locations is  $65.96$  mm and the sound propagates at a constant velocity of  $347$  m/s. Therefore,

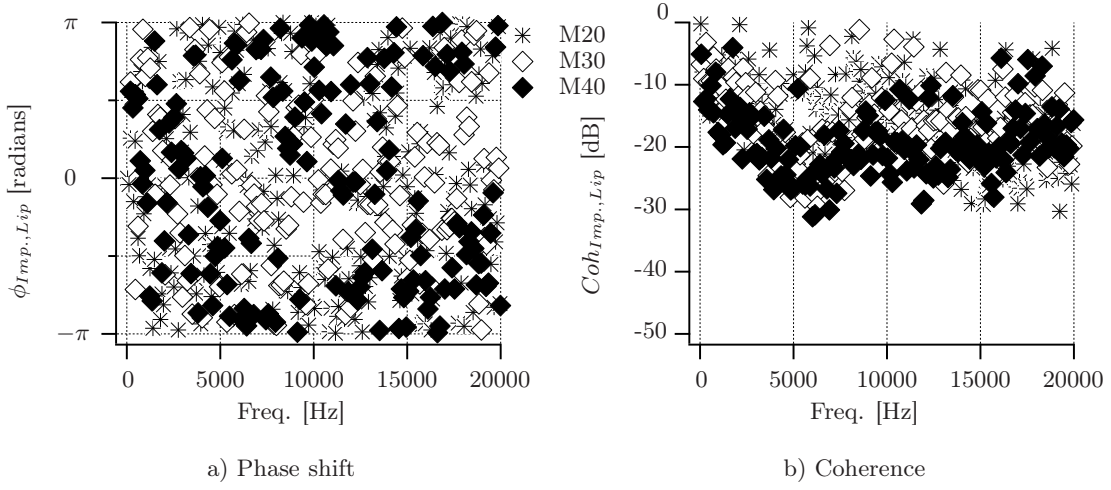


Fig. 17 No correlation : interspectrum between 'Impact' and 'Lip' locations.

the time lag between the two signals is  $189 \mu s$ . In other words, an ideal acoustic propagation gives a  $2\pi$  phase shift each 5275 Hz. This ideal phase shift is superimposed to the numerical results of Fig. 18a. The linear trend gets sharper with mesh resolution, and is in very good agreement with the ideal acoustic propagation. The coherence for this couple of sensors starts very low at low frequencies Fig. 18b to rise slowly from  $-35$  dB to  $-30$  dB in the range 5 to 20 kHz. The coherence is lower with a finer grid. Both mesh M30 and M40 give a similar envelope.

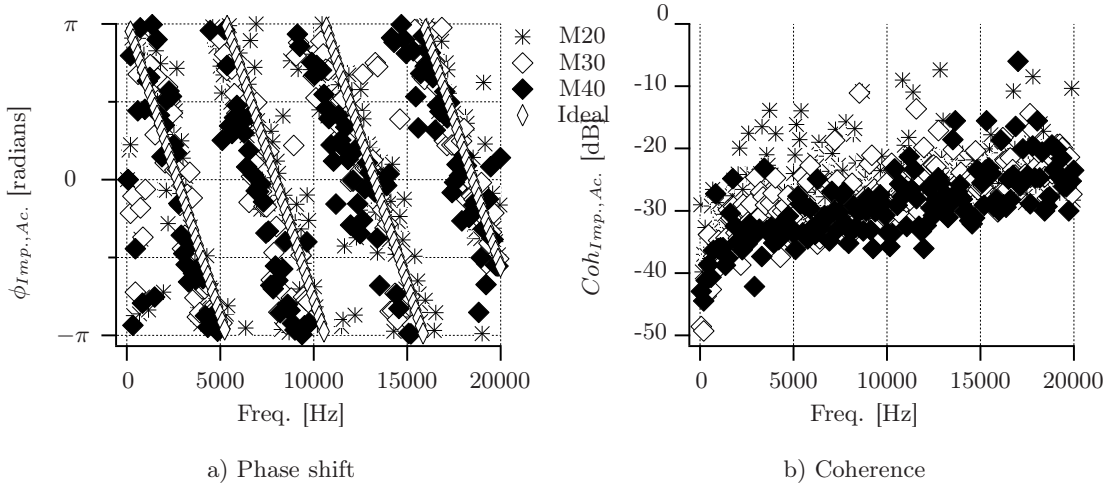


Fig. 18 Strong correlation : interspectrum between 'Impact' and 'Acoustic' locations.

To conclude, the comparison of one-point estimators shows that simulation M30 gives a correct energy distribution. The two-point estimators show a clear interaction between the impact zone

and the acoustic field, in spite of a large difference of intensity between the two signals (30 dB). Moreover, one and two-point estimators results are unchanged between meshes M30 and M40.

## V. Conclusion and perspectives

LES of a supersonic jet impinging on a flat plate in a stable regime have been performed with an explicit third-order compressible solver using an unstructured mesh and the Smagorinsky model. Simulations showed a satisfactory agreement with experiment observations. Three levels of mesh refinement were compared in terms of instantaneous and averaged flow fields (shocks and recirculation zone positions), averaged flow velocity and pressure fields, wall pressure, RMS pressure fields, spectral content. The mid and high-resolution grids yield almost the same results, especially for one- and two-point spectral analyses. The effects of numerical dissipation and turbulent viscosity are also compared on the three grids and shown to be well controlled. The finest grid used here (a 22 Mcell mesh) is sufficient to ensure grid-independent results and offer a coherent framework to use LES to investigate more complex mechanisms on this type of flow, for example self-excited tones.

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